

# Transition Variable-Based Fault Estimation and Intermittent Control for Nonlinear Multiagent Systems with Multiple Disturbances

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**Abstract:** In this paper, the fault estimation and intermittent control of nonlinear multiagent systems with multiple disturbances based on transition variable are studied. This paper realizes fault estimation by designing a transition variable-based estimator. Considering the working intensity of the actuator and the nonlinear and multiple state related disturbance in a non-ideal state, intermittent control is introduced to extend the service life of the controller, and the system state observer is designed. Then, an augmented system model is established, and a sufficient condition is derived. Next, system state observer gain, transition variable-based estimator gain, and controller gain are given. Finally, the simulation results show that the proposed method can make the multiagent system achieve consensus, and the designed intermittent observer and transition variable-based fault estimator are effective.

**Keywords:** Multiagent systems (MASs), Fault estimation, Intermittent control, Transition variable estimator

## 1. Introduction

Multiagent systems (MASs) have attracted the interest of many researchers in various research fields such as multi-vehicle systems<sup>[1]</sup>, smart grids<sup>[2,3]</sup>, complex networks<sup>[4,5]</sup>, and so on. In recent years, more fault-tolerant control has been studied in the field of MASs. Deng<sup>[6]</sup> studied the fault-tolerant formation control problem for a class of nonlinear MASs with actuator/sensor faults under directed and switched network topologies. Dong<sup>[7]</sup> studied the output synchronization problem of event-triggered communication schemes for a class of heterogeneous MASs under DoS attacks and actuator failures.

Considering the practical application in industry, the actuator working intensity and the non-linearity and multiple state disturbance of the MASs model under non-ideal conditions should also be considered.

## 2. Problem Formulation

### 2.1. System Model

Consider a continuous multiagent system with  $N$  nodes, which communicate through the topology graph  $\mathcal{G}$ , the adjacency matrix  $H$ , the degree diagonal matrix  $A$ , and the Laplacian matrix  $L = A - H$ , the model is as follows:

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + B[u_i(t) + a_i(t)] + D_1x_i(t)\omega_1(t) + Ff(x_i(t), t) \\ y_i(t) = Cx_i(t) + D_2x_i(t)\omega_2(t) \end{cases} \quad (1)$$

where  $x_i(t) \in \mathbb{R}^n$ ,  $y_i(t) \in \mathbb{R}^q$ ,  $u_i(t)$ ,  $a_i(t) \in \mathbb{R}^m$  are the state, measured output, control input, and fault respectively.  $\omega_1(t)$ ,  $\omega_2(t) \in \mathbb{R}$  are state-dependent disturbances, which follow a probability distribution with the expectation of 0 and variance of 1.  $f(x_i(t), t)$  is the nonlinear dynamic term.  $A$ ,  $B$ ,  $C$ ,  $D_1$ ,  $D_2$ , and  $F$  are known matrices,

Considering the intermittent control of the system, the time set is divided into the working period and

rest period as Figure 1 shows. Define  $[T_k, T_{k+1})$  as a work cycle, in which there is a minimum working period lower bound  $\bar{v} = \lim_{k \rightarrow \infty} \inf p_k > 0$  and a maximum rest period upper bound  $\tilde{v} = \lim_{k \rightarrow \infty} \sup p_k > 0$  where  $p_k = S_k - T_k$ ,  $q_k = T_{k+1} - S_k$ ,  $\gamma_k = q_k / (p_k + q_k)^{-1}$  is defined as the ratio of the intermittent period to the working cycle, and  $\bar{\gamma} = \lim_{k \rightarrow \infty} \sup \gamma_k$  is the maximum intermittent proportion.

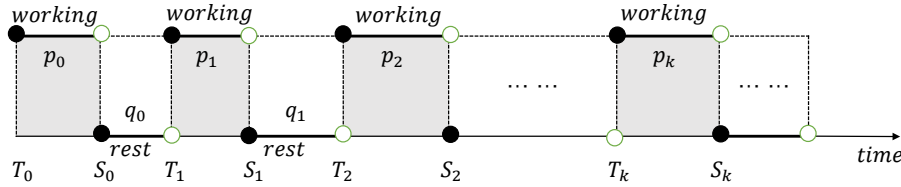


Figure 1: Schematic diagram of control and rest periods

**Assumption 1** The communication topology graph is connected.

**Assumption 2** For all vectors  $x_j(t) \in \mathbb{R}^n$ ,  $x_k(t) \in \mathbb{R}^n$ , the nonlinear function  $f(x_i(t), t)$  satisfies

$$\|f(x_j(t), t) - f(x_k(t), t)\| \leq \Lambda \|x_j(t) - x_k(t)\| \quad (2)$$

where  $\Lambda$  is a known matrix.

**Lemma 1** Given a matrix  $X \in \mathbb{R}^{n_p \times n}$ ,  $rank(X) = n_p$ , which has the following SVD description  $X = U \begin{bmatrix} X_1^\top & 0 \end{bmatrix}^\top V^\top$ , where  $U \in \mathbb{R}^{n_p \times n_p}$ ,  $V \in \mathbb{R}^{n \times n}$  is the unitary matrix, and  $X_1 \in \mathbb{R}^{n_p \times n_p}$  is the singular value diagonal matrix. Suppose that there exists a positive definite matrix  $P \in \mathbb{R}^{n \times n}$ , such that  $PX = X\bar{P}$  is satisfied if and only if the following holds:  $P = U \text{diag}\{P_1, P_2\} U^\top$ , where  $P_1 \in \mathbb{R}^{n_p \times n_p}$ ,  $P_2 \in \mathbb{R}^{(n-n_p) \times (n-n_p)}$ .

## 2.2. Fault Model and Fault Estimator

In (1), based on Chen<sup>[8]</sup> and Guo<sup>[9]</sup>, variables  $\xi_i(t)$  are introduced here to complete the establishment of the actuator fault model.

$$\begin{cases} \dot{\xi}_i(t) = E_1 \xi_i(t) \\ a_i(t) = E_2 \xi_i(t) \end{cases} \quad (3)$$

where  $\xi_i(t) \in \mathbb{R}^r$ ,  $E_1 \in \mathbb{R}^{r \times r}$ ,  $E_2 \in \mathbb{R}^{m \times r}$ .

The system state observer in this section is designed as follows.

$$\begin{cases} \hat{x}_i(t) = \begin{cases} A\hat{x}_i(t) + B[u_i(t) + a_i(t)] + D_1\hat{x}_1(t)\omega_1(t) + Ff(\hat{x}_{i(t)}, t) + G_1[\hat{y}_i(t) - y_i(t)] & , t \in \mathfrak{R}_1 \\ A\hat{x}_i(t) + B[u_i(t) + a_i(t)] + D_1\hat{x}_1(t)\omega_1(t) + Ff(\hat{x}_{i(t)}, t) & , t \in \mathfrak{R}_2 \end{cases} \\ \hat{y}_i(t) = C\hat{x}_i(t) + D_2\hat{x}_i(t)\omega_2(t) \end{cases} \quad (4)$$

where,  $\hat{x}_i(t) \in \mathbb{R}^n$ ,  $\hat{y}_i(t) \in \mathbb{R}^q$  are the estimated values of  $x_i(t)$ ,  $y_i(t)$ , respectively, and  $G_1$  is the observation gain. Inspired by Shi<sup>[10]</sup>, the transition variable  $\tau_i(t)$  is constructed.

$$\tau_i(t) = \begin{cases} \xi_i(t) - G_2\hat{x}_i(t) & , t \in \mathfrak{R}_1 \\ \xi_i(t) & , t \in \mathfrak{R}_2 \end{cases} \quad (5)$$

where  $G_2$  is the transition variable gain. If the actuator fault model with initial value  $\xi_i(0) = \xi_{i0}$  is the unique solution of (3),  $\tau_i = \tau_i(t)$  is the unique solution of the following system (6):

$$\begin{cases} \tau_i(0) = \tau_{i,0} \triangleq \xi_{i,0} - G_2 \hat{x}_i(0) \\ \dot{\tau}_i(t) = \begin{cases} (E_1 - G_2 B E_2) \tau_i(t) + (E_1 G_2 - G_2 A - G_2 B E_2 G_2) \hat{x}_i(t) - L B u_i(t) \\ -G_2 G_1 [\hat{y}(t) - y(t)] - G_2 F f(\hat{x}_i(t), t) - G_2 D_1 \hat{x}_i(t) \omega_2(t) \\ E_1 \tau_i(t) \end{cases} \end{cases} \quad \begin{matrix} , t \in \mathfrak{R}_1 \\ , t \in \mathfrak{R}_2 \end{matrix} \quad (6)$$

Correspondingly, if  $\tau_i = \tau_i(t)$  is the unique solution of the above system (6),  $\xi_i = \xi_i(t)$  is also the unique solution of the actuator fault model, and  $\xi_i(t)$  can be expressed as follows:

$$\xi_i(t) = \begin{cases} \tau_i(t) - G_2 \hat{x}_i(t) & , t \in \mathfrak{R}_1 \\ \tau_i(t) & , t \in \mathfrak{R}_2 \end{cases} \quad (7)$$

And we have the initial value of  $\xi_i(t)$ :  $\xi_i(0) = \xi_{i0}$ . The model of intermittent control observer with intermediate and transition variables is as follows:

$$\begin{cases} \dot{\hat{\tau}}_i(t) = \begin{cases} (E_1 - G_2 B E_2) \hat{\tau}_i(t) + (E_1 G_2 - G_2 A - G_2 B E_2 G_2) \hat{x}_i(t) \\ -G_2 B u_i(t) - G_2 G_1 [\hat{y}(t) - y(t)] - G_2 F f(\hat{x}_i(t), t) - G_2 D_1 \hat{x}_i(t) \omega_2(t) \\ E_1 \hat{\tau}_i(t) \end{cases} \\ \hat{\xi}_i(t) = \begin{cases} \hat{\tau}_i(t) + G_2 \hat{x}_i(t) & , t \in \mathfrak{R}_1 \\ \hat{\tau}_i(t) & , t \in \mathfrak{R}_2 \end{cases} \end{cases} \quad \begin{matrix} , t \in \mathfrak{R}_1 \\ , t \in \mathfrak{R}_2 \end{matrix} \quad (8)$$

Define system state estimation error:  $e_{xi}(t) = x_i(t) - \hat{x}_i(t)$ , transition variable estimation error:  $e_{\xi_i}(t) = \xi_i(t) - \hat{\xi}_i(t)$ . The augmented system can be obtained as follows.:

$$e_x(t) = \begin{cases} [I_N \otimes (A + G_1 C)] e_{x(t)} + (I_N \otimes F) f(e_x(t), t) \\ + (I_N \otimes D_1) e_x(t) \omega_1(t) + (I_N \otimes G D_2) e_x(t) \omega_2(t) \\ (I_N \otimes A) e_x(t) + (I_N \otimes F) f(e_x(t), t) + (I_N \otimes D_1) e_x(t) \omega_1(t) \end{cases} \quad \begin{matrix} , t \in \mathfrak{R}_1 \\ , t \in \mathfrak{R}_2 \end{matrix} \quad (9)$$

$$\dot{e}_{\xi}(t) = \begin{cases} (I_N \otimes E_1) e_{\xi}(t) - (I_n \otimes G_2 B E_2) e_{\xi}(t) & , t \in \mathfrak{R}_1 \\ (I_n \otimes E_1) e_{\xi}(t) & , t \in \mathfrak{R}_2 \end{cases} \quad (10)$$

where  $e_x(t) = \text{col}\{e_{xi}(t)\}$ ,  $f(e_x(t)) = \text{col}\{f(e_{xi}(t))\}$ ,  $f(e_{xi}(t)) = f(x_i(t)) - f(\hat{x}_i(t))$ ,  $e_{\xi}(t) = \text{col}\{e_{\xi_i}(t)\}$ .

### 2.3. Closed-loop System and Controller Design

Taking intermittent control into account, the controller in complete time can be designed as:

$$u_i(t) = \begin{cases} cK \sum_{j \in \mathcal{N}_i} l_{ij} (\hat{x}_j(t) - \hat{x}_i(t)) - \hat{a}_i(t) & , t \in \mathfrak{R}_1 \\ -a_i(t) & , t \in \mathfrak{R}_2 \end{cases} \quad (11)$$

where  $\hat{a}_i(t)$  is the estimation of  $a_i(t)$  and has the formula  $\hat{a}_i(t) = E_2 \hat{\xi}_i(t)$ .

Define  $\bar{x} = 1/N \sum_{i=1}^N x_i(t)$ ,  $\delta_i(t) = x_i(t) - \bar{x}$ , which represents the difference between the agent  $i$  and the average state of the agent and is called the consensus state error of the agent  $i$ . Let  $\delta(t) = \text{col}\{\delta_i(t)\}$ , combining (11), (4) and (1), the following consistent state error augmented system is obtained:

$$\dot{\delta}(t) = \begin{cases} (I_N \otimes A - cL \otimes BK)\delta(t) + (I_N \otimes D_1)\delta(t)\omega_1(t) \\ \quad + cL \otimes BKe_x(t) + \mathbb{N} \otimes BE_2e_\xi(t) + \mathbb{N} \otimes Ff(\delta(t)) & , t \in \mathfrak{R}_1 \\ (I_N \otimes A)\delta(t) + (I_N \otimes D_1)\delta(t)\omega_1(t) + \mathbb{N} \otimes Ff(\delta(t)) & , t \in \mathfrak{R}_2 \end{cases} \quad (12)$$

where  $f(\delta(t)) = \text{col}\{f(\delta_i(t))\}$  ,  $f(\delta_i(t)) = f(x_i(t)) - f(\bar{x}(t))$  ,  $\mathbb{N} = [n_{ij}]_{N \times N}$  , and  $n_{ij} = \begin{cases} 1-1/N & i = j \\ -1/N & i \neq j \end{cases}$ .

### 3. Main Results

Next, according to the designed observer and controller, the sufficient conditions to ensure MASs consensus are first derived, and then a set of solutions of the observer and controller are designed to achieve the expected consensus performance.

#### 3.1. Consensus Analysis

**Theorem 1** Given communication topology graph  $\mathcal{G}$ , given control gain  $K$ , state observation gain  $G_1$ , transition variable gain  $G_2$ , given positive scalars  $\alpha, \beta$ . If there exists positive definite matrices  $P, Q, S$ , satisfying:

$$\begin{bmatrix} \Xi_{11} & cL \otimes PBK & \mathbb{N} \otimes PBE_2 & I_N \otimes PF & 0 \\ * & \Xi_{22} & 0 & 0 & I_N \otimes QF \\ * & * & \Xi_{33} & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \quad (13)$$

$$\begin{bmatrix} \Sigma_{11} & 0 & 0 & I_N \otimes PF & 0 \\ * & \Sigma_{22} & 0 & 0 & I_N \otimes QF \\ * & * & \Sigma_{33} & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \quad (14)$$

$$\bar{\gamma} < \frac{\alpha}{\alpha + \beta} \quad (15)$$

where  $\Xi_{11} = I_N \otimes \text{He}(PA + PBK) + I_N \otimes (\Lambda^\top \Lambda + \alpha P)$ ,  $\Sigma_{11} = I_N \otimes \text{He}(PA) - I_N \otimes \beta P$ ,  $\Xi_{22} = I_N \otimes \text{He}(PA + QG_1C) + I_N \otimes (\Lambda^\top \Lambda + \alpha Q)$ ,  $\Sigma_{22} = I_N \otimes \text{He}(PA) - I_N \otimes \beta Q$ ,  $\Xi_{33} = I_N \otimes \text{He}(SE_1 + SG_2BE_2) + I_N \otimes \alpha S$ ,  $\Sigma_{33} = I_N \otimes \text{He}(SE_1) - I_N \otimes \beta S$ .

Then the MASs (1) studied in this paper can achieve consensus.

**Proof** Construct the following Lyapunov functional:

$$V(t) = V_1(t) + V_2(t) + V_3(t) \quad (16)$$

where  $V_1(t) = \delta^\top(t) (I_N \otimes P) \delta(t)$ ,  $V_2(t) = e_x^\top(t) (I_N \otimes Q) e_x(t)$ , and  $V_3(t) = e_\xi^\top(t) (I_N \otimes S) e_\xi(t)$

Define  $x_e(t) = [\delta^\top(t) \quad e_x^\top(t) \quad e_\xi^\top(t)]^\top$ ,  $\eta(t) = [x_e^\top(t) \quad f^\top(\delta(t)) \quad f^\top(e_x(t))]^\top$ .

Within the control period  $\mathfrak{R}_1$ , that is  $t \in [T_k, S_k)$ . Taking the infinitesimal operator on  $V_1(t)$  and computing its expectation, we obtain the following:

$$\begin{aligned} \mathbb{E}\{\mathfrak{I}V_1(t)\} &= \delta^\top(t)[I_N \otimes He(PA) - 2cL \otimes PBK]\delta(t) + 2\delta^\top(t)(cL \otimes PBK)e_x(t) \\ &\quad + 2\delta^\top(t)(N \otimes PBE_2)e_\xi(t) + 2\delta^\top(t)(I_N \otimes PF)f(\delta(t)) \end{aligned} \tag{17}$$

Define  $\bar{\delta}(t) = (U^\top \otimes I_N)\delta(t)$ , where  $U^\top LU = \Lambda = \text{diag}\{0, \lambda_2, \dots, \lambda_N\}$  and  $U$  is the unitary matrix, then  $\bar{\delta}_i(t) = \sum_{i=1}^N \delta_i(t)$ , thus

$$\begin{aligned} -2c\delta^\top(t)(L \otimes PBK)\delta(t) &= -2c\bar{\delta}^\top(t)(\Lambda \otimes PBK)\bar{\delta}(t) \\ &\leq -2\sum_{i=2}^N \bar{\delta}_i^\top(t)c\lambda_2 PBK\bar{\delta}_i(t) \\ &\leq -2\sum_{i=2}^N \bar{\delta}_i^\top(t)PBK\bar{\delta}_i(t) = -2\bar{\delta}_i^\top(t)(I_N \otimes PBK)\bar{\delta}_i(t) \\ &= -2\bar{\delta}_i^\top(t)(I_N \otimes PBK)\delta_i(t) \end{aligned} \tag{18}$$

By the Lipschitz assumption (2) for nonlinear functions, further we have

$$\begin{aligned} \mathbb{E}\{\mathfrak{I}V_1(t)\} &\leq \eta^\top(t)\text{diag}\{\Xi_{11} - \alpha P, 0, 0, -I, 0\}\eta(t) + 2\delta^\top(t)(cL \otimes PBK)e_x(t) \\ &\quad + 2\delta^\top(t)(N \otimes PBE_2)e_\xi(t) + 2\delta^\top(t)(I_N \otimes PF)f(\delta(t)) \end{aligned} \tag{19}$$

$$\mathbb{E}\{\mathfrak{I}V_2(t)\} \leq \eta^\top(t)\text{diag}\{0, \Xi_{22} - \alpha Q, 0, 0, -I\}\eta(t) + 2e_x^\top(t)(I_N \otimes QF)f(e_x(t)) \tag{20}$$

$$\mathbb{E}\{\mathfrak{I}V_3(t)\} = \eta^\top(t)\text{diag}\{0, 0, \Xi_{33} - \alpha S, 0, 0\}\eta(t) \tag{21}$$

Synthesizing the above three formulas and combining them with (16),  $\mathbb{E}\{\mathfrak{I}V(t)\} \leq -\alpha V(t)$  is obtained. Integrate over  $T_k$  to  $t \in [T_k, S_k)$ , we have

$$\mathbb{E}\{V(t)\} \leq e^{-\alpha(t-T_k)} \cdot V(T_k) \tag{22}$$

Within the rest period  $\mathfrak{R}_2$ , that is  $t \in [S_k, T_{k+1})$ , similar to  $t \in [T_k, S_k)$ , by simple calculation, we have  $\mathbb{E}\{\mathfrak{I}V(t)\} \leq \beta V(t)$ , for  $\mathfrak{I}V(t)$ , integrate over  $S_k$  to  $t \in [S_k, T_{k+1})$ , one has

$$\mathbb{E}\{V(t)\} \leq e^{\beta(t-S_k)} \cdot V(S_k) \tag{23}$$

According to (22) and (23), one has

$$0 \leq V(T_0) = V(0) = x_e^\top(0)\text{diag}\{P, Q, S\}x_e(0)$$

$$0 \leq V(S_0) \leq e^{-\alpha(S_0-T_0)}V(T_0)$$

$$0 \leq V(T_1) \leq e^{\beta(T_1-S_0)}V(S_0) \leq e^{-\alpha p_0 + \beta q_0}V(T_0)$$

$$0 \leq V(S_1) \leq e^{-\alpha(S_1-T_1)}V(T_1) \leq e^{-\alpha(p_0+p_1) + \beta q_0}V(T_0)$$

By induction, when  $k > 1$

$$0 \leq V(S_k) \leq e^{-\alpha(S_k-T_k)}V(T_k) = e^{-\alpha\sum_{i=0}^k p_i + \beta\sum_{i=0}^{k-1} q_i}V(0) \tag{24}$$

$$0 \leq V(T_{k+1}) \leq e^{\beta(T_{k+1}-S_k)}V(S_k) = e^{-\alpha\sum_{i=0}^k p_i + \beta\sum_{i=0}^k q_i}V(0) \tag{25}$$

According to the definition of  $\bar{\gamma}$  and  $\bar{\gamma} < \alpha(\alpha + \beta)^{-1}$ , for any  $\mathcal{G} \in (0, \alpha(\alpha + \beta)^{-1} - \bar{\gamma})$ ,  $\exists \kappa_\epsilon$ , such that for any  $k > \kappa_\epsilon$ ,  $\gamma_k < \bar{\gamma} + \mathcal{G}$  holds. Further, for any  $k > \kappa_\epsilon$ ,  $q_k < (\bar{\gamma} + \mathcal{G})(p_k + q_k)$ ,  $p_k > (1 - \bar{\gamma} - \mathcal{G})(p_k + q_k)$  always holds. This facilitates the following derivation:

$$\begin{aligned}
 -\alpha \sum_{i=0}^k p_i + \beta \sum_{i=0}^{k-1} q_i &\leq -\alpha \sum_{i=0}^{k-1} p_i + \beta \sum_{i=0}^{k-1} q_i - \alpha \bar{v} \\
 &\leq -\alpha \sum_{i=0}^{\kappa_\varepsilon} p_i + \beta \sum_{i=0}^{\kappa_\varepsilon} q_i - \alpha \bar{v} \\
 &\quad -\alpha \sum_{i=\kappa_\varepsilon+1}^{k-1} (p_i + q_i)(1 - \bar{\gamma} - \vartheta) + \beta \sum_{i=\kappa_\varepsilon+1}^{k-1} (p_i + q_i)(\bar{\gamma} + \vartheta) \\
 &= -\alpha \sum_{i=0}^{\kappa_\varepsilon} p_i + \beta \sum_{i=0}^{\kappa_\varepsilon} q_i - \alpha \bar{v} + [(\bar{\gamma} + \vartheta)(\alpha + \beta) - \alpha](T_k - T_{\kappa_\varepsilon+1}) \\
 &\triangleq M_1 - M_2(T_k - T_{\kappa_\varepsilon+1})
 \end{aligned} \tag{26}$$

where  $M_1 = -\alpha \sum_{i=0}^{\kappa_\varepsilon} p_i + \beta \sum_{i=0}^{\kappa_\varepsilon} q_i - \alpha \bar{v}$ ,  $M_2 = \alpha - (\bar{\gamma} + \vartheta)(\alpha + \beta) > 0$ .

When  $k > \kappa_\varepsilon$ ,  $0 \leq e^{-\alpha \sum_{i=0}^k p_i + \beta \sum_{i=0}^{k-1} q_i} V(0) \leq e^{M_1} e^{-M_2(T_k - T_{\kappa_\varepsilon+1})} V(0)$ ,  
 $\lim_{k \rightarrow \infty} \exp\{-\alpha \sum_{i=0}^k p_i + \beta \sum_{i=0}^{k-1} q_i\} V(0) = 0$ , and further  $\lim_{k \rightarrow \infty} V(S_k) = 0$ ,  $\lim_{k \rightarrow \infty} V(T_{k+1}) = 0$  are available  
 according to (24) and (25). Using (22) and (23) again,  $\lim_{k \rightarrow \infty} V(t) = 0$ , the proof is completed.

### 3.2. Controller Design

**Theorem 2** Given communication topology graph  $\mathcal{G}$ , given positive scalars  $\alpha, \beta$ . If there exists positive definite matrices  $P, Q, S$ , exists matrix  $\bar{K}$ , satisfying:

$$\begin{bmatrix}
 \bar{\Xi}_{11} & cL \otimes B\bar{K} & N \otimes PBE_2 & I_N \otimes PF & 0 \\
 * & \bar{\Xi}_{22} & 0 & 0 & I_N \otimes QF \\
 * & * & \bar{\Xi}_{33} & 0 & 0 \\
 * & * & * & -I & 0 \\
 * & * & * & * & -I
 \end{bmatrix} < 0 \tag{27}$$

$$\begin{bmatrix}
 \Sigma_{11} & 0 & 0 & I_N \otimes PF & 0 \\
 * & \Sigma_{22} & 0 & 0 & I_N \otimes QF \\
 * & * & \Sigma_{33} & 0 & 0 \\
 * & * & * & -I & 0 \\
 * & * & * & * & -I
 \end{bmatrix} < 0 \tag{28}$$

$$\bar{\gamma} < \frac{\alpha}{\alpha + \beta} \tag{29}$$

where  $\bar{\Xi}_{11} = I_N \otimes He(PA + B\bar{K}) + I_N \otimes (\Lambda^\top \Lambda + \alpha P)$ ,  
 $\bar{\Xi}_{22} = I_N \otimes He(QA) + I_N \otimes (\Lambda^\top \Lambda + \alpha Q - 2C^\top C)$ ,  $\bar{\Xi}_{33} = I_N \otimes He(SE_1) + I_N \otimes (\alpha S - 2E_2^\top B^\top BE_2)$ . The rest of the notations have the same meaning as in Theorem 1.

Given  $G_1 = -Q^{-1}C^\top$ ,  $G_2 = S^{-1}E_2^\top B^\top$ , and control gain can be solved as  $K = \bar{P}^{-1}\bar{K}$ .

**Proof:** Substitute the observer gain  $G_1 = -Q^{-1}C^\top$ ,  $G_2 = S^{-1}E_2^\top B^\top$  into  $\bar{\Xi}_{22}$  and  $\bar{\Xi}_{33}$  in theorem 1,  $\bar{\Xi}_{22}$  and  $\bar{\Xi}_{33}$  can be obtained. According to lemma 1, for  $P = Udiag\{P_1, P_2\}U^\top$ , exists  $\bar{P} = VB_1^{-1}P_1B_1V^\top$ , satisfying  $PB = B\bar{P}$ . Replacing  $PBK$  in (13) with  $B\bar{P}K$  and denoting  $\bar{P}K$  as  $\bar{K}$ , (27) is obtained. Then  $P, Q, S, \bar{K}$  can be solved separately, and the control gain can be obtained according to the solution  $K = \bar{P}^{-1}\bar{K}$ . The proof is completed.

### 4. Simulation

The parameters of the nonlinear continuous MASs are set as:

$$A = \begin{bmatrix} -0.11 & 0.43 \\ -0.19 & 0.09 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, C = [2 \quad 1], D_1 = \begin{bmatrix} 0 & 0.02 \\ 0 & 0 \end{bmatrix}, D_2 = [0.01 \quad 0.01]$$

$$E_1 = -\begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}, E_2 = \begin{bmatrix} 0.7 & -0.12 \\ 0.1 & 0.7 \end{bmatrix}, F = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \Lambda = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}$$

The agent communication topology is shown as fig.2. The eigenvalues of the corresponding Laplacian matrix are 0, 0.5, 0.5, 1.5, 1.5, 2, respectively, and  $c = 2$  is taken. And given that  $\alpha = 0.1$ ,  $\beta = 0.4$ . A series of feasible solutions are obtained:

$$P = \begin{bmatrix} 0.9215 & -0.3375 \\ -0.3375 & 0.7865 \end{bmatrix}, Q = \begin{bmatrix} 7.2414 & -3.0800 \\ -3.0800 & 6.4640 \end{bmatrix}, S = \begin{bmatrix} 15.8082 & -2.5581 \\ -2.5581 & 18.1904 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} -0.4289 \\ -0.3591 \end{bmatrix}, G_2 = \begin{bmatrix} 0.0442 & 0.0128 \\ -0.0004 & 0.0403 \end{bmatrix}, K = \begin{bmatrix} 0.9262 & 0.4171 \\ 0.6968 & 0.4592 \end{bmatrix}.$$

The other simulation parameters are set as  $\bar{v} = 400$ ,  $\bar{v} = 100$ ,  $\bar{\gamma} < 1/5$ . The specific steps of the control period and interval period are randomly generated according to the above constraints. The simulation results are shown in the figure.3 – figure.6 (Only the first dimension values are displayed).

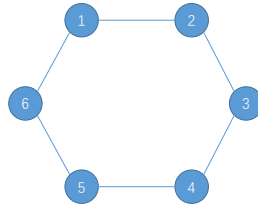


Figure 2: System topology

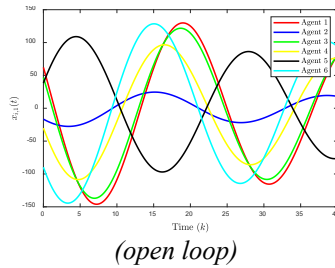


Figure 3: State trajectories (open loop)

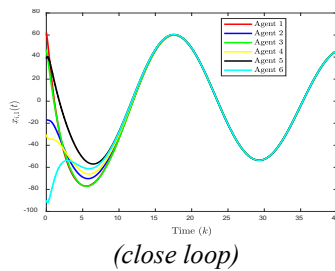


Figure 4: State trajectories (close loop)

The running trajectories of MASs in open and closed loop states are depicted in Figures 3 and 4, respectively. It is evident that agents can not achieve consensus in the open-loop situation. However, MASs shows good consensus performance under closed loop control. Figure 5 provides a detailed view of MASs consensus error under closed-loop control conditions. It can be observed from Figure 5 that the consensus error presents a decreasing trend, which indicates that the states of each agent in the system are gradually converging to a common value, that is, the average state of the MASs. Figure 6 and 7 respectively show the state estimation error and fault estimation error of MASs under closed-loop control, both of which converge rapidly and are kept within a small range, indicating that the designed observer and estimator are effective and can effectively track the real state of the system and estimate the possible

faults of the system during operation

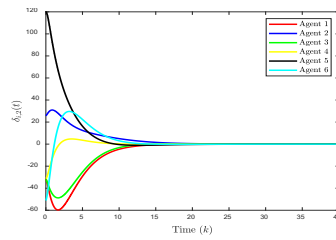


Figure 5: Consensus error

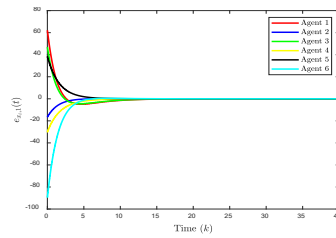


Figure 6: State observation error

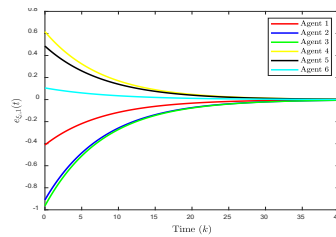


Figure 7: Fault estimation error

The above simulation results show that the system state observer and the transition variable-based fault estimator designed in this paper can effectively estimate the state and possible faults of the system during operation, and verify the effectiveness of the proposed compensation control strategy and the reliability of intermittent control, and finally achieve internal consistency of the system.

## 5. Conclusion

In this paper, a continuous multiagent system model under complex constraints is established by considering the internal structure of the multiagent system, fully considering the system's actuator fault, disturbance, and nonlinear dynamic term. At the same time, intermittent control is considered, and a sufficient condition is derived to make the multiagent system meet the consensus. This research mainly provides two contributions: first, it uses the fault estimation method to compensate the controller so as to make the control more secure and effective; second, it guarantees system stability and enhances actuator service life under intermittent control.

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