

# Performance Analysis of Three Attitude Algorithms for SINS

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**Abstract:** The attitude algorithms can enormously affect the navigation accuracy of SINS and Runge-kutta algorithm, Peano-Baker approximation algorithm and rotation vector algorithm are the common algorithms in engineering application. Analysis and comparison are done to the above three algorithms in this paper, the three algorithms are simulated and verified under coning motion and the result show that under the condition of low angle vibration, fourth-order Runge-kutta algorithm has the best comprehensive performance.

**Keywords:** Runge-kutta algorithm, Rotation vector algorithm, Peano-Baker approximation algorithm, Conical motion

## 1. Introduction

The SINS (strapdown inertial navigation system) is fixed on the carrier, This template, Gyroscopes and accelerometers are respectively used to measure the angular motion parameters and the line motion parameters. The onboard computer can work out the attitude and position information of the carrier in real time. The attitude transfer matrix can be used to calculate the attitude angles and also used to convert specific force form the ballistic coordinate system to the navigation coordinate system, and then the navigation information such as position and speed in navigation coordinate system can be obtained by integral operation. Hence, the attitude transfer matrix is an important factor which affects the navigation accuracy. Quaternion method has become one of the most commonly used methods in engineering application because of its many advantages such as simplicity and the less calculation time<sup>[1-3]</sup>. Three kinds of high accuracy algorithm method based on quaternion: fourth-order Runge-kutta algorithm, fourth-order Peano-Baker approximation algorithm and four-sample rotation vector algorithm have been analyzed in this paper. In the mode of typical coning motion input, the three algorithms are simulated and verified and the error of yaw, pitch and roll angle are used as the outputs. The result show that the fourth-order Runge-kutta algorithm has the best performance when the angular vibration is low.

## 2. Mathematical Model to Quaternion Method

Quaternion algebra can be regarded as an extension of complex number in four dimensional space and consist of a real number and three imaginary numbers. This can be expressed as:

$$Q = q_0 + q_1i + q_2j + q_3k \quad (1)$$

Where  $q_0, q_1, q_2, q_3 \in \mathbb{R}$ ,  $i^2 = j^2 = k^2 = ijk = -1$ . If  $\|Q\|_2=1$ , then  $Q$  is unit quaternion and can be expressed as:

$$Q = \cos\left(\frac{\theta}{2}\right) + \xi \sin\left(\frac{\theta}{2}\right) \quad (2)$$

Unit quaternion can be used to described a rotation of rigid body around instantaneous axis( $\xi$ ) in three dimensional space, and the rotation angle is  $\theta$ .

The attitude matrix in quaternion form can be expressed as:

$$C_b^n = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 - q_0q_3) & 2(q_1q_3 + q_0q_2) \\ 2(q_1q_2 + q_0q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 - q_0q_1) \\ 2(q_1q_3 - q_0q_2) & 2(q_2q_3 + q_0q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix} \quad (3)$$

Eq.(3) is the attitude transfer matrices between the body coordinates and navigation coordinates.

### 3. Analysis of Three Quaternion Methods to Attitude Algorithms

#### 3.1. Updating of Quaternion by Runge-kutta Algorithm

In fourth-order Runge-kutta algorithm and fourth-order Peano-Baker approximation algorithm, the updating of quaternion based on the following differential equation:

$$\dot{Q} = \frac{1}{2} Q \otimes P_{nb}^b \quad (4)$$

Where  $P_{nb}^b$  is the quaternion expanded by  $[\omega_x \ \omega_y \ \omega_z]$ , which denote the rotation angular velocity of navigation coordinate system relative to inertial system.  $\otimes$  is the multiplication operator in quaternion. Eq. (4) is transformed into the Eq. (5) as followed:

$$\dot{Q} = \begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \frac{1}{2} \Omega_{nb}^b Q = \frac{1}{2} \begin{bmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} \quad (5)$$

Eq.(5) can be solved by fourth-order Runge-kutta algorithm as followed:

$$\begin{aligned} K_1 &= \frac{1}{2} \Omega_{nb}^b(t)Q(t) \\ K_2 &= \frac{1}{2} \Omega_{nb}^b(t + \frac{h}{2}) \left[ Q(t) + \frac{K_1 h}{2} \right] \\ K_3 &= \frac{1}{2} \Omega_{nb}^b(t + \frac{h}{2}) \left[ Q(t) + \frac{K_2 h}{2} \right] \\ K_4 &= \frac{1}{2} \Omega_{nb}^b(t + h) \left[ Q(t) + K_3 h \right] \\ Q_n &= Q_{n-1} + \frac{h}{6} (K_1 + 2K_2 + 2K_3 + K_4) \end{aligned} \quad (6)$$

Where  $h$  is the update period of quaternion. Fourth-order Runge-kutta algorithm is simple and has high calculation accuracy, and it requires the data has better smoothness.

#### 3.2. Updating of Quaternion by Runge-kutta Algorithm

The outputs of gyroscope in SINS are the angle increment in sampling interval. Runge-kutta algorithm solves the quaternion differential equation by using angle increment in sampling interval and so it can suppress the noise enlargement. Eq. (4) can be transformed into the following equation:

$$Q(t) = e^{\frac{1}{2} \int_{t_0}^t \omega(\tau) d\tau} Q(t_0) \quad (7)$$

Let

$$\left\{ \begin{aligned} [\Delta\theta] &= \int_{t_0}^t \omega(\tau) d\tau = \begin{bmatrix} 0 & -\Delta\theta_x & -\Delta\theta_y & \Delta\theta_z \\ \Delta\theta_x & 0 & \Delta\theta_z & -\Delta\theta_y \\ \Delta\theta_y & -\Delta\theta_z & 0 & \Delta\theta_x \\ \Delta\theta_z & -\Delta\theta_y & -\Delta\theta_x & 0 \end{bmatrix} \\ \Delta\theta_0 &= \sqrt{\Delta\theta_x^2 + \Delta\theta_y^2 + \Delta\theta_z^2} \end{aligned} \right. \quad (8)$$

Then Eq.(7) can be written as:

$$q(t) = e^{\frac{1}{2}[\Delta\theta]} q(t_0) = \cos\left(\frac{\Delta\theta_0}{2}\right) I + \frac{\sin\left(\frac{\Delta\theta_0}{2}\right)}{\frac{\Delta\theta_0}{2}} [\Delta\theta] q(t_0) \quad (9)$$

With the fourth-order approximate form of the trigonometric functions, Eq.(9) can be written as:

$$q(t) = \left( I \left( 1 - \frac{\Delta\theta_0^2}{8} + \frac{\Delta\theta_0^4}{384} \right) + \left( \frac{1}{2} - \frac{\Delta\theta_0^2}{48} [\Delta\theta] \right) \right) q(t_0) \quad (10)$$

Eq.(10) is the fourth-order Peano-Baker approximation algorithm of the quaternion differential equation. The problem with fourth-order Runge-kutta algorithm and the fourth-order Peano-Baker approximation algorithm is that they can't effectively correct non-exchangeable errors, and so they are only applicable to attitude algorithm for lower dynamic carrier.

### 3.3. Updating of Quaternion by Four-sample Rotation Vector Algorithm

In order to reduce non-commutative errors, Bortz proposed the rotation vector algorithm[4,5], rotation vector can be used to represent the rotation of coordinate system and we have

$$\Phi = \cos \frac{\theta}{2} + u \sin \frac{\theta}{2} \quad (11)$$

Where  $u$  is the unit rotation vector and  $\theta$  is the rotation angle.

In order to update quaternion we need solve the following differential equation

$$\dot{\Phi} = \omega + \frac{1}{2} \Phi \times \omega + \frac{1}{\phi^2} \left( 1 - \frac{\phi \sin \phi}{2(1 - \cos \phi)} \right) \quad (12)$$

By solving Eq.(12) , we can obtained

$$\begin{aligned} \Delta\Phi &= \Delta\theta_1 + \Delta\theta_2 + \Delta\theta_3 + \Delta\theta_4 \\ &+ \frac{736}{945} (\Delta\theta_1 \times \Delta\theta_2 + \Delta\theta_3 \times \Delta\theta_4) \\ &+ \frac{334}{945} (\Delta\theta_1 \times \Delta\theta_3 + \Delta\theta_2 \times \Delta\theta_4) \\ &+ \frac{526}{945} (\Delta\theta_1 \times \Delta\theta_4) + \frac{654}{945} (\Delta\theta_2 \times \Delta\theta_3) \end{aligned} \quad (13)$$

Where  $\Delta\theta_1, \Delta\theta_2, \Delta\theta_3, \Delta\theta_4$  are the angular integral of the appropriate quarter of update interval. Finally, using  $q(t+h) = q(t) \otimes q(h)$ , we can update quaternion.

### 4. Algorithm Simulation

In order to show the performance of the above algorithms, the simulation is carried out in this section. Assuming that there is a relative rotation between body coordinate system and relative to navigation coordinate system,  $OL$  and  $\alpha$  are the rotation vector and the rotation angle respectively and  $OL$  rotate at a constant speed  $\omega_0$  in  $YOZ$  plant of navigation coordinate system. We can denote  $OL$  by

$\Phi$  , then

$$\Phi = \begin{bmatrix} 0 & \alpha \cos \omega_0 t & \alpha \sin \omega_0 t \end{bmatrix}^T \quad (14)$$

Where  $t$  is the time of coning motion. The corresponding quaternion of  $\Phi$  is

$$Q = \begin{bmatrix} \cos \frac{\alpha}{2} & 0 & \sin \frac{\alpha}{2} \cos \omega_0 t & \sin \frac{\alpha}{2} \sin \omega_0 t \end{bmatrix}^T \quad (15)$$

The rotation angle between body coordinate system and navigation coordinate system is

$$\omega_{nb}^b = \omega_0 \begin{bmatrix} \cos \alpha - 1 & -\sin \alpha \sin \omega_0 t & \sin \alpha \cos \omega_0 t \end{bmatrix}^T \quad (16)$$

The angular velocity is the input of simulation which can be obtained by calculating Eq.(17). The specific simulation conditions are as follows:  $\alpha = 10^\circ$ ,  $\omega_0 = 2\pi(\text{rad} / \text{s})$ , the update interval  $h$  is equal to 0.01 second and the sampling interval  $T$  is equal to 0.0025 second. Simulation time  $t$  is 10 seconds. The simulation results are shown in Figure1-3.

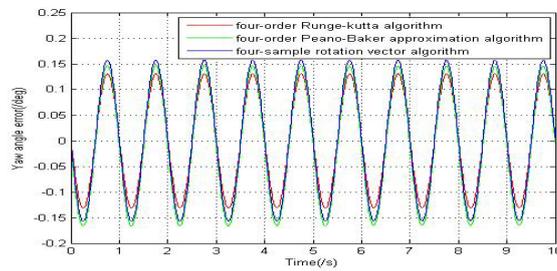


Figure 1: Comparison of yaw angle errors of three algorithms.

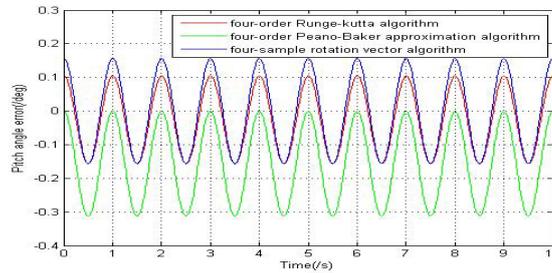


Figure 2: Comparison of pitch angle errors of three algorithms.

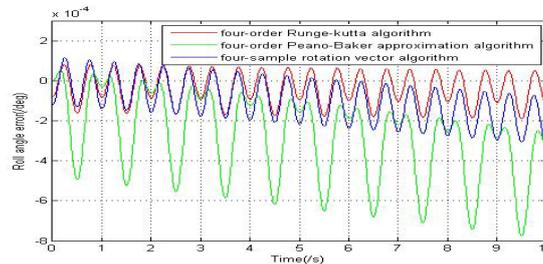


Figure 3: Comparison of roll angle errors of three algorithms

## 5. Conclusions

We can obtain that the fourth-Order Runge-kutta algorithm is more ideal in the attitude calculation of the carrier under the condition of low angular vibration. Runge-kutta algorithm is used in the low-cost integrated navigation system of an unmanned supersonic vehicle. The steady-state error value of pitch angle is about  $0.03^\circ$ , and the steady-state error value of yaw angle convergence is about  $0.01^\circ$ , which meets the requirements of accuracy index and achieves good results.

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