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Construction of evaluation index system of inquiry teaching in high school mathematics based on analytic hierarchy process

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Abstract: *From the perspective of the characteristics of mathematics inquiry learning and experiential learning, it provides innovative ideas for the construction of the evaluation index system of mathematics inquiry teaching in high school. There are three goals in this system: teacher's teaching development, student behavior performance and process factors. And 11 first-level indicators and 31 second-level indicators are constructed. Through the questionnaire survey of experts, the consistency check using the analytic hierarchy process (AHP), it determines the weight of indicators, enriches the development of mathematics inquiry teaching theory, and accumulates experience for the research of mathematics inquiry teaching evaluation in high school.*

Keywords: *mathematics inquiry teaching; experiential learning; evaluation index system; analytic hierarchy process*

1. Research background

In the ordinary high school mathematics curriculum standards (2017 Edition 2020 Revision), As one of the main lines of high school mathematics learning, mathematical inquiry activities run through the whole process. Mathematical inquiry learning is the main way of Practice-based Integrate Curriculum. High school is an important period for students' thinking pattern transiting from the concrete to abstract, mathematical inquiry learning as a “scaffolding”, can effectively achieve the development of thinking.

In recent years, the teaching of mathematical inquiry is in full swing, and the evaluation of mathematical inquiry teaching has also emerged, but there are still problems such as superficial and inadequate implementation, and the evaluation research is mainly tend towards theoretical analysis. What's more, there are fewer innovation ideas in the evaluation dimension, and there is a lack of practical research on the index system. Starting from the characteristics of mathematical inquiry learning and experiential learning, this paper constructs an evaluation index system, and determines the weight of each index, with the help of AHP method, so as to provide direction and ideas for mathematical inquiry teaching.

2. Construction of mathematics inquiry teaching evaluation system

Mathematics inquiry teaching is a collaborative and interactive process between teachers and students, as well as an experiential learning process to harvest process knowledge [1]. In previous studies, scholars usually build the evaluation index system of inquiry teaching from teachers' teaching, students' performance and inquiry teaching effect [2] [3]. However, the depth of experiential learning in the inquiry still needs to be carried out. This paper continues to study preening achievements and experiences, and from the perspective of mathematical inquiry learning characteristics and experiential learning, it pays attention to the thinking process of students, reflects on actions, and constantly push on lifelong learning.

2.1. Teacher's teaching

Experiential learning is a full process of fitting to the world [4]. Accordingly, mathematics inquiry

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teaching should also be a full interactive process. The development of teacher teaching is composed of 5 first-level indicators and 16 second-level indicators. The specific indicator design in Table 1.

Table 1: Evaluation index of teaching development

Target layer	First-level	Second-level
Teacher's teaching (A1)	Theme selection (B1)	Openness, flexibility and dynamic generation(C1)
		Zone of Proximal Development(C2)
	Situation setting (B2)	Closely linked with teaching objectives(C3)
		The characteristics of key situations and concepts of key disciplines(C4)
		Strength motivation theory(C5)
		Higher cognitive needs(C6)
	Activity-guided (B3)	Teachers' active participation(C7)
		Lead to deep thinking problems and formulation(C8)
		Intelligence autonomy(C9)
		“rediscovery” and “re-creation”(C10)
		Interpretative listening and explanative listening(C11)
		visual aids(C12)
	Procedure summary (B4)	Mastery goals(C13)
		Summary of process(C14)
	Content extension (B5)	Obedient and assimilative(C15)
		Variant and application(C16)

Theme selection: In the high school mathematics teaching stage, not all contents are applicable to inquiry teaching, so the theme selection of inquiry teaching should be open, flexible and dynamic, allowing students to have space for thinking and exploration, in line with the "Zone of Proximal Development ".

Situation setting: Situation must be closely connected with teaching aims and key disciplines concepts, and always maintain the high cognitive needs of the task. In addition, the situation itself should strengthen students' motivation and pave the way for deepening students' thinking.

Activity-guided: Putting forward questions to guide students' thinking, deal with mathematical problems into formulation, screen renewable and unpredictable problems, guide students to intellectual autonomy[5], and promote the occurrence of "rediscovery" and "re-creation". What's more, students should also learn to listen interpretatively and explanatively, and truly communicate with students on the same channel. In this process, teachers' knowledge structure will also change and develop towards lifelong learning. Finally, we should use visual aids and procedural expressions to help students to construct their knowledge.

Process summary: Formative evaluation and summative evaluation can provide valuable experience for the subsequent teaching, and pay attention to the all-round development of students from the aspects of target mastery degree and process introspection. In addition, learners' self-reflection and generalization is an important method to refine acquired skills, experience and integrate knowledge structure. It is also one of the important indicators to evaluate learners' participation in the process of mathematical inquiry.

Content extension: Mathematics inquiry teaching is not only to teach students to do exercise, but more importantly to learn to “gaining new insights through restudying old material”, build a bridge with the previous knowledge, and through the topic variation and relevant applications to extend the content, truly sublimate the role of learning, not limited to the examination and admission.

2.2. Student behavior

Student behavior is composed of 3 first-level indicators and 9 second-level indicators, and the specific indicators are designed in Table 2. From a deeper perspective, the measurement of the experience in the learning process and the expectation after learning can more effectively evaluate the mathematics inquiry teaching.

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Table 2: Evaluation index of student behavior

Target layer	First-level	Second-level
Student behavior (A2)	Experiential learning (B6)	Depth of experience(C17)
		Deliberate learning(C18)
		Formation of learning style(C19)
	Sharing and communication(B7)	Conversion between words, symbols, and graphics(C20)
		Cooperation with teachers and peers(C21)
		Judge, question and explanation(C22)
	Reflective expectation(B8)	The cultivation of mathematical ability chain(C23)
		Acquisition and development of tacit knowledge(C24)
		The development of mathematics core literacy(C25)

Experiential learning: Experience is the source of learning. In mathematical inquiry, learners consciously guide self-learning through deliberate attention, from concept abstraction to concept formation, by focusing to gather inner feelings, change the way of thinking, and find solutions to problems [6]. Deliberate learning is naturally formed on the basis of deep experience learning. It seeks and understands its own learning style through experience, and controls behavior through conscious guidance [5]. After long years of study, you may have formed your own learning style, but how to develop to a learning style that is more conducive to stimulating your own potential is the focus of direction.

Sharing and communication: The sharing and interaction with teachers and peers is the process of clarifying mind, bursting new ideas in the communication, assimilating and accommodating to the cognitive structure. In addition, the formation of a class discussion centered on students' mathematical ideas will be help them find thinking loopholes, so that students will strive to find a reliable explanation to prove that their solutions are correct, and the process of judgment, questioning and explanation will stimulate the motivation to explore mathematical knowledge, and develop mathematical literacy to express the world in mathematical language.

Reflection expectation: Five mathematical ability chains (program fluency ability chain, concept understanding ability chain, strategy ability chain, adaptive reasoning ability chain, efficient processing ability chain) [5], from simple to deep, step by step, starting from the understanding of procedural knowledge to the key concepts, and finally, the belief of intellectual autonomy as the standard of the highest ability chain. Tacit knowledge is the existence of special knowledge in individuals, it is not a calculation formula, but engraved in the brain of mathematical thinking, mathematical spirit, and it has lifelong benefit for learners.

2.3. The efficacy of process elements

The efficacy of process elements is composed of 3 first-level indicators and 6 second-level indicators, and the specific indicators are designed in Table 3. Process knowledge is the experiential knowledge acquired by learners during their active participation in mathematical inquiry, which can monitor the learning process and promote the development of non-intellectual elements.

Table 3: Evaluation index of the efficacy of process elements

Target layer	First-level	Second-level
The efficacy of process elements(A3)	Process knowledge (B9)	Transformation ways: generation, construction and expansion(C26)
		A plateau of translation(C27)
	Reasoning activity (B10)	Treatment of quasi-empirical reasoning(C28)
		The balance between empirical reasoning and deductive reasoning(C29)
	Monitoring process (B11)	Teachers should be “data collectors”, not “problem solvers”(C30)
		The deal of situations that are wrong or not in the direction of the teaching objective(C31)

Process knowledge: Process knowledge is often existed in the process of transforming external objects into internal thinking. In mathematical inquiry activities, learners will transform knowledge into inner knowledge, and then the process of finding solutions of problems is structural transformation, further promoting and applying knowledge and experience gained from inquiry activities. In the

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process of transformation, there will be a "plateau period". Once break through its "bottleneck", learners will get the joy of "hundreds and thousands of times I searched for her in the crowd, suddenly I turned, and there she stood, in the dim light looking for him, but suddenly look back, the man is in the dim lights".

Reasoning activities: quasi-empiricism holds that mathematics is not absolute but revisable, and that new knowledge is generated and developed in the process of refuting old knowledge. This refuting process is similar to the process of mathematical reasoning, and mathematical reasoning activities are always accompanied by mathematical inquiry teaching. In the teaching of mathematical inquiry, reasonable reasoning and deductive reasoning complement each other and jointly assist the development of teaching activities. How to balance the relationship between reasonable reasoning and deductive reasoning is a question which is worth thoroughly pondering.

Monitoring process: In the process of mathematical inquiry teaching, learners should not be afraid of making mistakes or being corrected, and teachers should also regard themselves as "data collectors" rather than "problem solvers". In this way, some methods leading to learners' mistakes can be mined and some "questions with unknown answers" can be put forward [5]. Raising "questions with unknown answers" can not only help teachers judge whether learners' reasoning methods are reasonable, but also promote the development of learners' intellectual autonomy.

3. Construction of evaluation index system of mathematical inquiry teaching based on AHP

3.1. Construct judgment matrix

In this study, the evaluation of high school mathematics inquiry teaching is divided into 3 target levels, 11 first-level indicators and 31 second-level indicators. According to T.L. Saaty's 1-9 scale method [7], questionnaires were distributed to first-line high school mathematics teachers (63.89% of whom had taught for more than 10 years), and a total of 36 questionnaires were collected. Part of the questionnaire is shown in Figure 1.

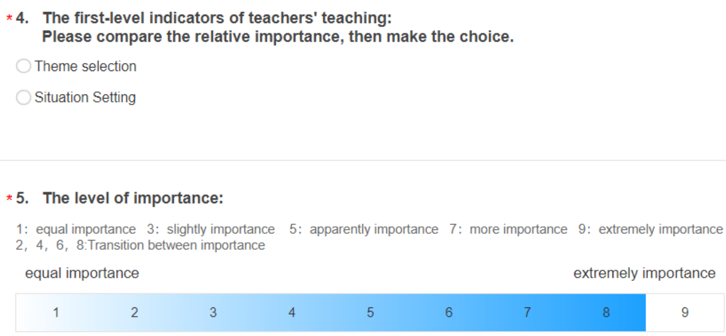

Figure 1: Part of the first-level index questionnaire

The comparison matrix obtained after data handling is:

First order index comparison matrix:

$$B_1=\begin{bmatrix}1 & 3 & \frac{1}{3} & 3 & 1\\ \frac{1}{3} & 1 & \frac{1}{4} & 1 & \frac{1}{3}\\ 3 & 4 & 1 & 4 & 3\\ \frac{1}{3} & 1 & \frac{1}{4} & 1 & \frac{1}{3}\\ 1 & 3 & \frac{1}{3} & 3 & 1\end{bmatrix},\quad B_2=\begin{bmatrix}1 & 4 & 3\\ \frac{1}{4} & 1 & \frac{1}{2}\\ \frac{1}{3} & 2 & 1\end{bmatrix},\quad B_3=\begin{bmatrix}1 & \frac{1}{4} & 1\\ 4 & 1 & 4\\ 1 & \frac{1}{4} & 1\end{bmatrix}$$

The comparison matrix of secondary indicators of teachers' teaching:

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$$C_1=\begin{bmatrix}1 & \frac{1}{3}\\ 3 & 1\end{bmatrix},\ C_2=\begin{bmatrix}1 & 1 & 3 & 4\\ 1 & 1 & 3 & 4\\ \frac{1}{3} & \frac{1}{3} & 1 & 2\\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & 1\end{bmatrix},\ C_3=\begin{bmatrix}1 & 1 & \frac{1}{2} & 2 & 3 & 3\\ 1 & 1 & \frac{1}{2} & 2 & 3 & 3\\ 2 & 2 & 1 & 3 & 4 & 4\\ \frac{1}{2} & \frac{1}{2} & \frac{1}{3} & 1 & 2 & 2\\ \frac{1}{3} & \frac{1}{3} & \frac{1}{4} & \frac{1}{2} & 1 & 1\\ \frac{1}{3} & \frac{1}{3} & \frac{1}{4} & \frac{1}{2} & 1 & 1\end{bmatrix},\ C_4=\begin{bmatrix}1 & 2\\ \frac{1}{2} & 1\end{bmatrix} C_5=\begin{bmatrix}1 & \frac{1}{3}\\ 3 & 1\end{bmatrix}$$

Secondary index comparison matrix of student behavior performance:

$$C_6=\begin{bmatrix}1 & 4 & 4\\ \frac{1}{4} & 1 & 1\\ \frac{1}{4} & 1 & 1\end{bmatrix},\ C_7=\begin{bmatrix}1 & \frac{1}{2} & \frac{1}{4}\\ 2 & 1 & \frac{1}{3}\\ 4 & 3 & 1\end{bmatrix},\ C_8=\begin{bmatrix}1 & 3 & \frac{1}{3}\\ \frac{1}{3} & 1 & \frac{1}{4}\\ 3 & 4 & 1\end{bmatrix}$$

Secondary index comparison matrix of process element efficacy:

$$C_9=\begin{bmatrix}1 & 4\\ \frac{1}{4} & 1\end{bmatrix},\ C_{10}=\begin{bmatrix}1 & \frac{1}{3}\\ 3 & 1\end{bmatrix},\ C_{11}=\begin{bmatrix}1 & 5\\ \frac{1}{5} & 1\end{bmatrix}$$

3.2. Consistency

(1)The maximum eigenvalue of the judgment matrix λ_{max};

It can be calculated by the instruction "[VD]= eig(A)" in MATLAB, where A is the judgment matrix.

(2)Calculate the consistency index;

Consistency index is $CI=\frac{\lambda_{max}-n}{n-1}$, when the dimension of the matrix increase, with the aid of CI to test will reduce accuracy, and therefore combines the mean consistency index RI to correction index test, $CR=\frac{CI}{RI}$.

When $CR<0.1$, consistency inspection passing, when $CR\geq 0.1$, the judgment matrix need to be corrected [7]. The consistency test of specific indicators at all levels in table 4.

Table 4: Consistency test results of indicators at all levels

	First-level			**Second-level**										
CR	0.0237	0.0158	0	0	0.0076	0.0066	0	0	0	0.0176	0.0707	0	0	0

3.3. Index weight calculation

Normalizing processing for the matrix, the weights of indicators at all levels are obtained, as shown in Table 5.

Table 5 Weights of indicators at all levels

Target layer	First-level	Second-level
Teacher's teaching	Theme selection (0.2019)	C1(0.2859) C2(0.7141)
	Situation setting (0.0791)	C3(0.3854) C4(0.3854) C5(0.1421) C6(0.0871)
	Activity-guided (0.4380)	C7(0.2031) C8(0.2031) C9(0.3383) C10(0.1173) C11(0.0691) C12(0.0691)
	Procedure summary (0.0791)	C13(0.6667) C14(0.3333)
	Content extension (0.2019)	C15(0.2500) C16(0.7500)
Student behavior	Experiential learning (0.6250)	C17(0.6666) C18(0.1667) C19(0.1667)
	Sharing and communication (0.1365)	C20(0.1365) C21(0.2385) C22(0.6250)
	Reflective expectation (0.2385)	C23(0.2684) C24(0.1172) C25(0.6144)
The efficacy of process elements	Process knowledge (0.1667)	C26(0.800) C27(0.200)
	Reasoning activity (0.6666)	C28(0.2500) C29(0.7500)
	Monitoring process (0.1667)	C30(0.8333) C31(0.1667)

4. Result analysis

4.1. Analysis of teachers' teaching index weight

Among the first-level indicators of teachers' teaching, the weight of “activity guidance” occupies the primacy. As the core step of mathematics inquiry teaching, it is reasonable to get this weight. In addition, it is worth noting that teachers should pay more attention to theme selection and content extension, choose content that conforms to the characteristics of inquiry teaching and students' cognition level. In addition, context creation should be “pragmatic”, do not follow the trend. Appropriate and inspiring context creation will yield twice the result with half the effort in inquiry teaching.

Among the secondary indicators of teachers' teaching development, as mentioned above, it emphasizes the connection between situation creation, teaching objectives and key subject concepts. In the index of activity guidance, the weight of “students' intellectual autonomy” is 0.3383, ranking the first place. In teaching, students often follow teachers' ideas, and believe that teachers are better at mathematical calculation and reasoning, and they should follow their ideas. This will have a bad effect on students' active thinking. In mathematics inquiry more need the “intellectual autonomy”, encourage students to think spontaneously.

4.2. Analysis of student behaviour performance index weight

Among the first-level indicators of student behavior performance, the weight of experiential learning is 0.6250. “Learners learn and develop in inquiry activities” is the inherent value concept of inquiry teaching. For example, when learning the *Infinite convergent series*, there is always exist a problem, $0.\dot{3}\times 3=1$? The question that seems to contradict the conclusion of the facts can quickly stimulate students' motivation. This problem can be explored by the following teaching scheme: Let

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students think actively $2+1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+......\to ?$ And the 2 by 2 square have been cut for the original $\frac{1}{2}$, you can directly say that the infinite geometric series summation gets as close to 4. This will guide students to think about $0.\dot{3}$ as a geometric series summation, then ponder the question from the limits of thinking, the final inquiry results are obtained. This teaching process is a process of deep experience, which is in line with the goal of inquiry teaching [8].

4.3. Analysis of the efficacy of process elements index weight

In the first-level index of the efficacy of process elements, the weight of reasoning activities is 0.6666, more than half. There is a two-way coupling relationship between mathematical reasoning and mathematical knowledge. The accumulation of knowledge promotes the smooth progress of reasoning, and the "procedural knowledge" will also promote the positive learning experience of individuals.

In the secondary index of the efficacy of process elements, the balance between rational reasoning and deductive reasoning is recognized by most experts. In mathematical inquiry class, students can find ideas in rational reasoning and demonstrate ideas in deductive reasoning. Reasonable reasoning has the function of predicting and explaining information, and should be paid as much attention as deductive reasoning.

5. Summary

It is an innovative attempt to construct an evaluation index of high school mathematics inquiry teaching from the perspective of the characteristics of mathematical inquiry learning and experiential learning, which has enriched the research and development of inquiry teaching. In addition, the process of determining the weight of the index is also scientific. However, it is undeniable that the establishment of indicators still needs to be further refined, and the linkage between indicators needs to be strengthened. The AHP method can scientifically obtain the weight of each index, but the questionnaire survey of experts has some subjectivity, so the evaluation index system of mathematics inquiry teaching in high school needs to be optimized in the subsequent teaching practice.

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