ISSN 2663-8169 Vol. 8, Issue 1: 1-7, DOI: 10.25236/IJNDE.2026.080101

Research on Teaching Strategies for Developing High School Students' Intuitive Imagination Literacy Based on GeoGebra—Taking "Conic Sections" as an Example

Jianhui Li[1,2,a], Lizi Yin[1,b*]

[1]School of Mathematical Science, University of Jinan, Jinan, China
[2]Zibo Qisheng Senior High School, Zibo, China
[a]cocojianhui@163.com, [b]ss_yinlz@ujn.edu.cn
**Corresponding author*

Abstract: *This paper aims to explore effective teaching strategies using the dynamic mathematics software GeoGebra to develop high school students' intuitive imagination literacy, with the content of conic sections in high school mathematics as the carrier. The research adopts literature research, experimental research, and case study methods. Firstly, a teaching model integrating the TPACK framework and constructivist learning theory was constructed. Then, a series of GeoGebra teaching cases for ellipse, hyperbola, and parabola were designed. Through a quasi-experimental study conducted in the second year of high school, self-developed tools such as the "High School Students' Intuitive Imagination Literacy Test", classroom observations, and interviews were used to collect data. The results show that the GeoGebra-based teaching strategy of "creating dynamic situations - guiding inquiry and discovery - promoting the connection between algebra and geometry - conducting variational exercises" can significantly improve students' ability to dynamically perceive graphics, imagine spatial relationships, and convert between algebraic and geometric representations; the experimental class performed significantly better than the control class in the post-test; students' classroom participation and interest in learning mathematics significantly increased. This study provides a referable path for the deep integration of information technology and mathematics teaching and offers an effective practical scheme for developing students' intuitive imagination literacy.*

Keywords: *GeoGebra; Intuitive Imagination Literacy; Teaching Strategies; Conic Sections; High School Mathematics*

1. Introduction

1.1 Research Background

With the rapid advancement of information technology, educational informatization has become a fundamental characteristic of modern education. The "General High School Mathematics Curriculum Standards (2017 Edition)" in China explicitly identifies intuitive imagination as one of the six core mathematical literacies, emphasizing the perception of the form and changes of mathematical objects through geometric intuition and spatial imagination [1]. This literacy encompasses the ability to use spatial forms, especially geometric figures, to understand and solve mathematical problems, to perceive and analyze the relationships between geometric elements, and to construct mathematical models of real-world objects. However, in actual teaching practice, the conic sections unit often presents significant learning difficulties for students due to its abstract concepts and complex geometric properties. GeoGebra, as dynamic mathematics software integrating geometry, algebra, probability, statistics, and calculus into a single, user-friendly package, offers new possibilities for addressing this teaching dilemma [2]. Its intuitive, dynamic, and interactive characteristics enable students to visually explore mathematical concepts, manipulate parameters in real-time, and observe corresponding changes in geometric figures. This capacity for dynamic representation aligns closely with the cognitive processes involved in developing intuitive imagination. Consequently, investigating how to effectively utilize GeoGebra's unique affordances to systematically foster students' intuitive imagination literacy has become an important and timely topic in contemporary mathematics education

ISSN 2663-8169 Vol. 8, Issue 1: 1-7, DOI: 10.25236/IJNDE.2026.080101

research.

1.2 Research Questions

This study primarily explores the following questions:

(1) How should teaching strategies based on GeoGebra for developing high school students' intuitive imagination literacy be designed?

(2) What is the effect of implementing GeoGebra teaching strategies on high school students' learning of conic sections?

(3) What difficulties and challenges are encountered during the implementation of GeoGebra teaching strategies?

1.3 Research Significance

Theoretical Significance: This research enriches the theoretical landscape concerning the deep integration of information technology and mathematics curriculum. It provides new perspectives and methodological approaches for cultivating intuitive imagination literacy, bridging theoretical constructs from TPACK [7], constructivism [8], and dual coding theory [6] with practical classroom applications.

Practical Significance: The study provides operable teaching strategies, detailed instructional sequences, and typical classroom cases for frontline mathematics teachers. It serves as a practical guide for transforming traditional teaching methods into more dynamic, student-centered approaches, ultimately aiming to improve teaching quality and student learning outcomes in a challenging mathematical domain.

2. Literature Review and Theoretical Foundation

2.1 Definition of Core Concepts

Intuitive Imagination Literacy: This refers to the cognitive capacity to perceive the form, structure, and dynamic changes of mathematical objects through geometric intuition and spatial imagination, and to proficiently use graphical representations to understand, reason about, and solve mathematical problems [1]. Within the specific context of this study, intuitive imagination literacy is operationalized and assessed through three distinct but interrelated dimensions:

(1) Graphical Dynamic Cognition: The ability to perceive, mentally represent, and predict the dynamic behavior and transformations of geometric figures [4,9].

(2) Spatial Relationship Imagination: The capacity to visualize, mentally manipulate, and comprehend the spatial relationships and configurations between geometric elements in two and three dimensions [4,9].

(3) Algebraic-Geometric Conversion: The skill to fluently translate between algebraic representations (equations, parameters) and geometric representations (graphs, properties) of mathematical objects, and to use insights from one domain to inform understanding in the other [2,10].

GeoGebra: GeoGebra is an open-source, cross-platform dynamic mathematics software that seamlessly integrates geometry, algebra, spreadsheets, graphing, statistics, and calculus into a single, coherent environment [2]. Its key features—including dynamic manipulation, real-time linked multiple representations, and interactive exploration—make it particularly suitable for inquiry-based mathematics teaching and learning [2,3]. It serves not merely as a demonstration tool but as a cognitive scaffold that can mediate students' mathematical thinking and understanding.

2.2 Theoretical Basis

TPACK Framework: The Technological Pedagogical Content Knowledge (TPACK) framework emphasizes the complex interplay and essential integration of three primary knowledge forms: technological knowledge (TK), pedagogical knowledge (PK), and content knowledge (CK) [7]. In this study, TPACK manifests as the deliberate and thoughtful integration of GeoGebra technology (TK), specialized knowledge for teaching conic sections (CK), and pedagogical methods specifically suited

ISSN 2663-8169 Vol. 8, Issue 1: 1-7, DOI: 10.25236/IJNDE.2026.080101

for cultivating intuitive imagination (PK). This framework guides the design of lessons where technology use is not an add-on but is intrinsically woven into the learning process to address specific content-related learning challenges. Learners build new understandings upon their existing knowledge and experiences through social negotiation and interaction with their environment [5,8].

Dual Coding Theory: Paivio's Dual Coding Theory suggests that visual and verbal information are processed through distinct but interconnected cognitive systems [6]. The simultaneous presentation of mathematical concepts through visual-geometric representations and symbolic-algebraic representations in GeoGebra potentially facilitates deeper and more flexible learning by engaging both processing channels and creating multiple retrieval paths for mathematical knowledge.

3. Research Design and Methods

This study operates within the pragmatic paradigm, employing a mixed-methods approach to comprehensively investigate the development of intuitive imagination literacy through GeoGebra-based instruction. The sequential explanatory design was specifically adopted, wherein quantitative data collection through quasi-experimental methods precedes and informs the subsequent qualitative data collection through observations and interviews. This design allows for a thorough understanding of both the "what" and the "why" behind the research phenomena—not only determining the effectiveness of the intervention but also elucidating the underlying mechanisms and contextual factors that influence its implementation and outcomes.

Preparatory Phase (Week 0): During this preliminary stage, the researcher developed all instructional materials, including GeoGebra applets, lesson plans, and student worksheets for the experimental class, along with parallel traditional teaching materials for the control class.

Intervention Phase (Weeks 1-5): This core phase constituted the implementation of the pedagogical intervention. The experimental class received instruction incorporating GeoGebra-based activities following the "four-step" teaching strategy model, with approximately 70% of instructional time dedicated to dynamic geometric exploration and connected mathematical discourse.

Evaluation Phase (Week 6): The final week was dedicated to post-intervention data collection. The "High School Students' Intuitive Imagination Literacy Test" was administered as a post-test to both classes under standardized conditions. Following the quantitative assessment, semi-structured interviews were conducted with purposively selected students from the experimental class (n=12) representing high, medium, and low performance levels based on their post-test scores, as well as with both participating teachers (n=2).

The self-developed "High School Students' Intuitive Imagination Literacy Test" served as the primary quantitative instrument for assessing students' competencies across the three defined dimensions of intuitive imagination. The test development process followed a rigorous, multi-stage validation procedure:

4. Teaching Strategy Design and Implementation

4.1 Teaching Strategy Framework

The instructional framework consists of four interconnected phases, each with specific teaching objectives, key activities, and intended cognitive outcomes:

Step 1: Creating Dynamic Situations to Transform Abstraction into Intuition

The teacher introduces mathematical situations using GeoGebra's dynamic capabilities to make abstract concepts visually accessible and intuitively comprehensible. This builds vivid mental imagery that serves as an intuitive foundation for formalization [4,9]. For example, rather than simply stating the geometric definition of an ellipse, the teacher demonstrates the locus formation process through the "dragging" feature, allowing students to observe the invariant property (sum of distances to foci) across countless specific instances. Key instructional activities in this phase include.

Step 2: Guiding Inquiry and Discovery, Moving from Observation to Conjecture

Building upon the foundational visual experiences, this phase engages students as active investigators who explore mathematical relationships through hands-on manipulation of GeoGebra

ISSN 2663-8169 Vol. 8, Issue 1: 1-7, DOI: 10.25236/IJNDE.2026.080101

applets. Essential elements of this phase include: (1) providing structured exploration guides; (2) encouraging systematic variation and pattern recognition; (3) supporting conjecture formulation through sentence frames; and (4) creating opportunities for preliminary hypothesis testing. The cognitive focus shifts from perceptual intuition to operative thinking, as students begin to internalize operations and develop anticipatory schemas for mathematical relationships.

Step 3: Promoting the Connection between Algebra and Geometry to Construct Mathematical Essence

Instructional strategies in this phase include: (1) facilitating discourse that connects visual evidence to mathematical formalisms; (2) employing multiple linked representations simultaneously; (3) guiding the process of mathematical derivation and justification; and (4) highlighting correspondences between geometric features and algebraic parameters. This phase aims to develop students' capacity for representational fluency—the ability to flexibly translate between different mathematical representation systems.

Step 4: Conducting Variational Exercises to Deepen Conceptual Understanding

The final phase consolidates and extends students' understanding through carefully designed variational exercises that promote mathematical reasoning and transfer. Variation theory informs the design of these exercises, which might include: (1) varying non-critical attributes to highlight essential features; (2) presenting problems in novel contexts to promote transfer; (3) using reverse problems to encourage reversible thinking; and (4) introducing open-ended problems to develop creative application. For example, after learning the standard equation of an ellipse centered at the origin, students might encounter problems with shifted centers, rotated axes, or applied contexts. This phase aims to develop robust, transferable knowledge structures that support mathematical problem-solving beyond the immediate learning context.

4.2 Teaching Case Design

Phase 1: Creating Dynamic Situations (10 minutes)

The lesson began with a contextualized problem: "How would you describe the orbit of Mars around the Sun, and what mathematical shape might it approximate?" After brief discussion, the teacher introduced the formal geometric definition of an ellipse using a pre-constructed GeoGebra applet titled "The Gardener's Method." This applet visually simulated the classic string construction of an ellipse, showing how maintaining a constant sum of distances to two fixed points generates an elliptical locus. Students observed as the teacher dynamically manipulated the foci positions and string length, with the ellipse transforming in real-time.

Phase 2: Guiding Inquiry and Discovery (15 minutes)

Students then transitioned to hands-on exploration using individual devices with a guided inquiry worksheet. The GeoGebra applet for this phase included interactive sliders controlling focal distance (2c) and constant sum (2a), allowing students to discover relationships between these parameters and the resulting ellipse. Specific investigation prompts included: "What happens when the constant sum equals the distance between foci? What happens when the constant sum is only slightly greater than the focal distance? How does eccentricity (c/a) relate to the 'flatness' of the ellipse?" Students worked in pairs to systematically explore these relationships, recording observations and formulating conjectures. The teacher circulated to prompt deeper thinking through questions like "What pattern are you noticing? How could you test your hypothesis? Does this remind you of any other mathematical relationships?"

Phase 3: Promoting Algebraic-Geometric Connection (12 minutes)

Building on students' exploratory discoveries, the teacher facilitated a whole-class derivation of the standard equation. Beginning with the geometric definition $|PF_1| + |PF_2| = 2a$, the teacher guided students through the algebraic process of simplifying this statement using coordinates, with $F_1(-c, 0)$ and $F_2(c, 0)$. Strategic questions supported the reasoning process: "How can we represent the distance $|PF_1|$ using coordinates? What algebraic technique might help us eliminate the radicals? How can we introduce parameter b to simplify the equation?" The simultaneous display of the geometric construction and the evolving algebraic equation in GeoGebra helped students maintain connection between the formal derivation and its geometric meaning. Students recognized the emergence of the relationship $b^2 = a^2 - c^2$ not as an arbitrary definition but as a necessary simplification that also corresponds to a meaningful geometric measurement.

ISSN 2663-8169 Vol. 8, Issue 1: 1-7, DOI: 10.25236/IJNDE.2026.080101

Phase 4: Conducting Variational Exercises (8 minutes)

The lesson concluded with a series of carefully designed variational problems that progressively increased in complexity: (1) Given different sets of a and b values, students sketched the corresponding ellipses and positioned the foci; (2) Given an ellipse equation with centered not at the origin but at (2, -1), students adapted their understanding to the new context; (3) Given a real-world problem about a whispering gallery (ellipse-shaped room), students applied their knowledge to determine where speakers and listeners should be positioned. These problems reinforced the connections between geometric features and algebraic representations while promoting flexible application of knowledge.

5. Results Analysis and Discussion

5.1 Quantitative Results Analysis

Following the six-week intervention period, post-test scores revealed substantial differences between the experimental and control groups. An independent samples t-test on the post-test total scores showed a statistically significant advantage for the experimental group ($t = 3.26$, $p = 0.002 < 0.01$), with the experimental group ($M = 82.4$, $SD = 8.7$) outperforming the control group ($M = 74.1$, $SD = 10.2$). The effect size, calculated using Cohen's d, was 0.89, indicating a large practical significance according to conventional benchmarks.

To provide a more precise estimate of the intervention effect while controlling for pre-test differences, an analysis of covariance (ANCOVA) was conducted with post-test total score as the dependent variable, group membership as the fixed factor, and pre-test total score as the covariate. The ANCOVA revealed a significant effect of group membership after controlling for pre-test scores ($F(1, 99) = 9.84$, $p = 0.002$, partial $\eta^2 = 0.09$), confirming the superior performance of the experimental group beyond what would be predicted based on initial ability levels alone.

Subgroup analyses were conducted to examine whether the intervention effects varied across students with different initial achievement levels (categorized as high, medium, and low based on pre-test scores). Interestingly, while all subgroups in the experimental group showed significant gains, the medium-achieving students demonstrated the largest effect size ($d = 1.02$), followed by low-achieving ($d = 0.85$) and high-achieving students ($d = 0.71$). This pattern suggests that the GeoGebra-based approach may be particularly beneficial for students who traditionally struggle with abstract mathematical concepts but possess adequate foundational knowledge to benefit from visual scaffolding.

5.2 Qualitative Results Analysis

Analysis of the classroom observation data revealed distinctive patterns of student engagement and mathematical behavior between the experimental and control classrooms:

Patterns of Engagement: In the experimental classroom, observers documented significantly higher levels of sustained behavioral engagement (85% of students consistently on task vs. 62% in control classroom) and more frequent episodes of collaborative problem-solving. Visualization as Scaffolding: Students reported that dynamic visuals helped them "see" relationships, leading to integrated mental models. "Before, the ellipse equation was just symbols. Now I picture the GeoGebra screen," one student noted [6,10].Observed Student Growth: Teachers noted dramatic improvements in students' abilities to articulate geometric reasoning and mentally manipulate figures [1,4,9].The qualitative records noted students' spontaneous expressions of curiosity and excitement, particularly during dynamic exploration phases, with utterances such as "What if we try..." and "Look what happens when..." being three times more frequent than in control classrooms.

5.3 Integrated Discussion

The integrated analysis of quantitative and qualitative data reveals a coherent picture of how GeoGebra-based instruction influenced students' intuitive imagination development. The statistically significant gains in test scores across all three dimensions of intuitive imagination are substantiated and elucidated by the rich qualitative evidence of changes in students' mathematical behaviors, discourse patterns, and problem-solving approaches.

Professional Development Needs: The teacher's reported challenges highlight the need for

ISSN 2663-8169 Vol. 8, Issue 1: 1-7, DOI: 10.25236/IJNDE.2026.080101

comprehensive professional development that addresses not only technical skills but also the pedagogical shifts required for inquiry-based learning with technology.

Curriculum Design Considerations: The positive outcomes suggest that mathematics curriculum materials should incorporate opportunities for guided exploration and multiple representation connections, potentially reorganizing content sequences to leverage visual intuition before formal symbolic treatment.

6. Conclusion and Reflection

Based on the comprehensive analysis of both quantitative and qualitative data, this study reaches several firm conclusions regarding the development of intuitive imagination literacy through GeoGebra-based instruction:

First, the GeoGebra-based "four-step" teaching strategy demonstrates significant effectiveness in developing high school students' intuitive imagination literacy, with particularly strong effects on spatial relationship imagination and graphical dynamic cognition. The consistent patterns across different data sources provide compelling evidence that the intervention enhanced students' abilities to mentally manipulate geometric figures, anticipate dynamic transformations, and flexibly translate between algebraic and geometric representations.

Second, the dynamic visualization capabilities of GeoGebra serve as powerful cognitive tools that help overcome the abstract nature of conic sections concepts. The technology enables students to build vivid mental imagery that serves as foundation for formal mathematical understanding, effectively bridging the gap between concrete experience and abstract reasoning that often impedes mathematics learning.

Third, successful implementation requires teachers to develop integrated technological, pedagogical, and content knowledge that enables them to effectively scaffold student exploration, facilitate mathematical discourse, and connect visual discoveries to formal mathematics. The pedagogical shifts involved are non-trivial and require substantial support and professional development.

Longitudinal Studies: Research tracking the development of intuitive imagination over extended periods could provide insights into the long-term benefits and potential trajectories of spatial reasoning development.

Curriculum Design Research: Development and testing of comprehensive curriculum materials that systematically incorporate dynamic mathematics approaches across multiple mathematical topics could significantly impact classroom practice.

In conclusion, this study demonstrates that thoughtfully designed technology integration, grounded in sound theoretical principles and implemented through structured pedagogical frameworks, can significantly enhance students' intuitive imagination literacy. The findings contribute to both theory and practice in mathematics education, offering valuable insights for educators, curriculum developers, and researchers seeking to enhance mathematics learning through appropriate technology integration.

References

[1] Ministry of Education of the People's Republic of China. (2018). General high school mathematics curriculum standards (2017 edition). Beijing: People's Education Press.
[2] Hohenwarter, M., & Jones, K. (2007). Ways of linking geometry and algebra: the case of GeoGebra. Proceedings of the British Society for Research into Learning Mathematics, 27(3), 126-131.
[3] Zhang, J. Z., & Peng, X. C. (2010). Dynamic Geometry Tutorial. Beijing: Science Press.
[4] Piaget, J., & Inhelder, B. (1967). The child's conception of space. New York: W. W. Norton.
[5] Vygotsky, L. S. (1978). Mind in society: The development of higher psychological processes. Cambridge, MA: Harvard University Press.
[6] Paivio, A. (1990). Mental representations: A dual coding approach. New York: Oxford University Press.
[7] Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. Teachers College Record, 108(6), 1017-1054.
[8] Bruner, J. S. (1966). Toward a theory of instruction. Cambridge, MA: Harvard University Press.
[9] Kozhevnikov, M., Motes, M. A., & Hegarty, M. (2007). Spatial visualization in physics problem

ISSN 2663-8169 Vol. 8, Issue 1: 1-7, DOI: 10.25236/IJNDE.2026.080101

solving. Cognitive Science, 31(4), 549-579.
[10] Arcavi, A. (2003). The role of visual representations in the learning of mathematics. Educational Studies in Mathematics, 52(3), 215-24