ISSN 2706-655X Vol.6, Issue 5: 47-51, DOI: 10.25236/IJFET.2024.060507

Realization of elastic wave energy flow density based on MATLAB

Cheng Dao[1,a], Lu Shiwei[1,*], Zhang Maochen[2], Wang Siqin[1]

[1]School of Urban Construction, Yangtze University, Jingzhou, Hubei, China
[2]Northwest Water Resources and Hydropower Engineering Co., LTD., Xi'an, Shaanxi, China
[a]chengdao991010@163.com
[*]Corresponding author: lushiwei364@163.com

Abstract: *The dynamic response perturbation of structures due to elastic waves is typically associated with energy. Using MATLAB's powerful numerical simulation capabilities, we calculated the energy flux density for SH waves incident on a circular cavity and normalized the results. The findings indicate that the incident frequency significantly impacts the energy flux density. High-frequency waves, despite their rapid local variations, experience rapid energy attenuation, resulting in a more dispersed overall energy flux density distribution. In contrast, low-frequency waves exhibit a relatively smooth energy flux density distribution. These research outcomes are crucial for enhancing the safety and stability of underground structures.*

Keywords: *SH wave, energy flow density, MATLAB*

1. Introduction

Elastic waves are a type of wave phenomenon generated by external disturbances in materials, with their waveforms having widespread applications in engineering, geology, physics, and materials science. These applications include the dynamic response of structures to seismic waves [1][2] and the monitoring of material damage using ultrasonic waves. During the propagation of elastic waves, the transmission and distribution of the carried energy play critical roles in structural safety assessments and damage monitoring. Energy density describes the energy stored per unit volume, while energy flux density describes the energy passing through a unit area per unit time. Energy density serves as a key parameter for characterizing the energy transmission of elastic waves, directly reflecting the distribution and variation of energy within the medium.

Numerical simulations can overcome many limitations of experimental research and accurately predict wave behavior and energy transmission processes under complex conditions. MATLAB, as a powerful numerical computation and simulation tool, plays an indispensable role in the study of elastic waves. It offers a wealth of numerical computation functions and powerful data processing capabilities, enabling the rapid and accurate solution of complex partial differential equations and the simulation of elastic wave propagation in various media[3]. Furthermore, MATLAB's robust visualization capabilities allow for the intuitive presentation of elastic wave propagation processes and energy density distributions, providing convenient analytical tools.

2. Elastic wave theory

Elastic waves can be categorized into longitudinal waves (P-waves) and transverse waves (S-waves) based on the propagation direction relative to the direction of vibration. Transverse waves can further be divided into SH-waves and SV-waves. SH-waves are a type of transverse wave where the direction of vibration is perpendicular to the direction of propagation and vibrates horizontally. Due to their unique propagation characteristics, SH-waves have been widely applied in seismic wave analysis, ultrasonic testing, and materials science research. In isotropic, homogeneous, and purely elastic media, SH-waves satisfy the geometric equation, motion equation, and constitutive equation. Through the displacement method, the motion equation expressed by the displacement function can be derived. By substituting the geometric equation into the constitutive equation and performing spatial partial differentiation, the vector form of the motion equation can be obtained:

ISSN 2706-655X Vol.6, Issue 5: 47-51, DOI: 10.25236/IJFET.2024.060507

$$\rho \boldsymbol{u} = (\lambda + 2\mu)\nabla^2 \boldsymbol{u} - \mu \nabla \times (\nabla \times \boldsymbol{u}) + \boldsymbol{f} \tag{1}$$

Where $\boldsymbol{u}$ is the displacement potential function,∇ is the Laplace operator, ρ is the density of the medium, and λ and μ are Lamé constants.

According to Helmholtz's theorem, any vector field $\boldsymbol{u}$ can be expressed as the sum of the gradient of a scalar field φ and the curl of a vector field $\boldsymbol{\psi}$:

$$\boldsymbol{u} = \boldsymbol{u}_p + \boldsymbol{u}_s = \nabla \varphi + \nabla \times \boldsymbol{\psi}, \quad \nabla \bullet \boldsymbol{\psi} = 0 \tag{2}$$

Substituting equation (2) into equation (1) and further analyzing, we can derive:

$$\begin{aligned} &\frac{\partial^2 \varphi}{\partial t^2} - c_p{}^2 \nabla^2 \varphi = C \\ &\rho \frac{\partial^2 \boldsymbol{\psi}}{\partial t^2} - c_s{}^2 \nabla^2 \boldsymbol{\psi} = -C \\ &\nabla \bullet \boldsymbol{\psi} = 0; \quad c_p{}^2 = \frac{\lambda + 2\mu}{\rho}; c_s{}^2 = \frac{\mu}{\rho}; \quad C \text{ be arbitrary constant} \end{aligned} \tag{3}$$

Where c_p and c_s are the wave speeds of P-waves and S-waves, respectively, with the speed of P-waves being greater than that of S-waves. The simple harmonic wave displacement expression for SH-waves vibrating with time is generally:

$$u = A \cos k(x - ct) \tag{4}$$

In the above equation, the wave number $k=\omega/c$, where ω is the angular frequency; A is the amplitude, $\boldsymbol{x}$ represents the projection of a spatial point in the direction n f wave propagation; c is the wave speed, specifically c_s; *and t is the time.*

3. Elastic wave energy flux density

When elastic waves propagate into an elastic medium, they cause deformation of the medium's microscopic units, resulting in elastic potential energy. Simultaneously, each microscopic unit oscillates around a certain equilibrium position, generating kinetic energy. In the process of elastic wave propagation, there is both energy inflow and outflow for the elastic medium, meaning wave propagation is accompanied by energy transfer. The elastic potential energy per unit volume in an elastic medium is referred to as the elastic potential energy density w_u, and the kinetic energy per unit volume is called the kinetic energy density w_k, Where e is the strain, the expression is:

$$\begin{aligned} w_u &= \frac{1}{2}\sigma_{ji} e_{ji} = \frac{\lambda}{2}\left(e_{11} + e_{22} + e_{33}\right)^2 + \mu\left[\left(e_{11}{}^2 + e_{22}{}^2 + e_{33}{}^2\right) + 2\left(e_{12}{}^2 + e_{23}{}^2 + e_{31}{}^2\right)\right] \\ w_k &= \frac{1}{2}\rho\left(\frac{\partial u}{\partial t}\right)^2 \end{aligned} \tag{5}$$

The total energy density w is the sum of the elastic potential energy density w_u and the kinetic energy density w_k. For an elastic wave incident on a uniform elastic medium, analyzing the time rate of change of energy flow is crucial. Therefore, we calculate the time rate of change for both the elastic potential energy density w_u and the kinetic energy density w_k respectively. The time rate of change of the elastic potential energy density w_u is:

$$\begin{aligned} \frac{\partial w_u}{\partial t} &= \sigma_{11}\frac{\partial^2 u_1}{\partial t \partial x_1} + \sigma_{22}\frac{\partial^2 u_2}{\partial t \partial x_2} + \sigma_{33}\frac{\partial^2 u_3}{\partial t \partial x_3} + \\ &\sigma_{12}\left(\frac{\partial^2 u_1}{\partial t \partial x_2} + \frac{\partial^2 u_2}{\partial t \partial x_1}\right) + \sigma_{22}\left(\frac{\partial^2 u_3}{\partial t \partial x_2} + \frac{\partial^2 u_2}{\partial t \partial x_3}\right) + \sigma_{13}\left(\frac{\partial^2 u_1}{\partial t \partial x_3} + \frac{\partial^2 u_3}{\partial t \partial x_1}\right) \end{aligned} \tag{6}$$

The time rate of change of the elastic potential energy density w_k is:

ISSN 2706-655X Vol.6, Issue 5: 47-51, DOI: 10.25236/IJFET.2024.060507

$$\frac{\partial w_k}{\partial t} = \rho \frac{\partial u}{\partial t} \cdot \frac{\partial^2 u_2}{\partial t^2} \tag{7}$$

Adding equations (6) and (7) yields the time rate of change of the mechanical energy density. The energy flux density vector field ***I*** is defined as:

$$I_j = -\sigma_{ji} \frac{\partial u_i}{\partial t} \tag{8}$$

Applying the time rate of change of the mechanical energy density to equation (8), we get:

$$\frac{\partial w}{\partial t} + \nabla \bullet I = 0 \tag{9}$$

4. MATLAB example implementation

For a cylindrical cavity with a radius a, an SH wave incident from the positive xxx-direction, the model diagram is shown in Figure 1:

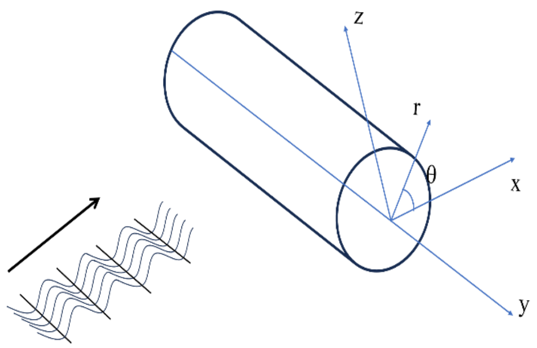

Figure 1: Model incidence diagram

In this scenario, it is necessary to establish a cylindrical coordinate system and transform the incident wave equation (4) into the function U_r in cylindrical coordinates. Using the wave function expansion method [4] and the integral definition of the Bessel function, U contains the Bessel function. Due to diffraction phenomena when waves encounter different media, scattered waves are generated[5]. The scattered wave is also an SH wave, and its functional expressions U_r and U_s are:

$$U_r = A_0 \sum_{n=0}^{\infty} \varepsilon_n i^n J_n(kr) \cos(n\theta) e^{-i\omega t}$$
$$U_s = A_0 \sum_{n=0}^{\infty} A_n H_n^{(1)}(kr) \cos(n\theta) e^{-i\omega t} \tag{10}$$

Where A_0 is the constant amplitude, r is the cavity radius, $H_n^{(1)}(kr)$ is the n-th order Hankel function of the second kind, and $J_n^{(1)}(kr)$ is the first kind Bessel function. Here, $\varepsilon_n=2(n\neq0)$;$e^{-i\omega t}$ is the time factor.

Adding the two equations in (10) yields the total wave field U, from which the corresponding strain and stress can be derived. Additionally, the boundary condition, i.e., no stress on the cavity surface ($\sigma_{rz}=0$), is applied to the stress obtained from the total wave field to solve for A_n.For computational convenience, let $A_0=1$,$a=3.5$, with incident wave frequencies $f=50$ Hz ,100Hz,150 Hz, SH wave speed $c_s=3500 m/s$.

According to equation (8), it is necessary to determine the stress and the corresponding displacement. For this example, the cavity only exhibits displacement u_3. Using MATLAB, the radial and angular partial derivatives of the displacement U on the cavity surface are calculated. Combining with equation (8), the energy flux density I_u on the cavity surface can be obtained:

ISSN 2706-655X Vol.6, Issue 5: 47-51, DOI: 10.25236/IJFET.2024.060507

$$I_u = \sqrt{\left(\sigma_{13}\frac{\partial u_3}{\partial t}\right)^2 + \left(\sigma_{23}\frac{\partial u_3}{\partial t}\right)^2} = \mu e^{-i\omega t}\sqrt{\left|-i\omega U\frac{\partial U}{\partial r}\right|^2 + \left|-i\omega U\frac{1}{r}\frac{\partial U}{\partial \theta}\right|^2} \quad (11)$$

It is evident that the final energy flux density is positively correlated with the shear modulus μ, which may result in larger numerical values. Therefore, normalization is performed based on these results, and the normalized results are compared with the energy flux density of the incident wave on the cavity surface, denoted as I_f. This method effectively compares the impact of the cavity on the incident wave, eliminating absolute value deviations due to parameter selection, making the results more intuitive and comparable. The final calculation is shown in Figure 2:

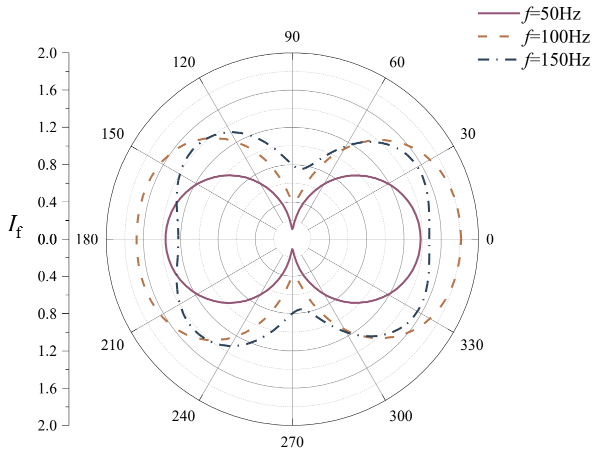

Figure 2: If distribution of relative energy flux density

It is clear that as the frequency increases, the local variation in relative energy flux density becomes more pronounced. This is due to stronger scattering and interference effects at the cavity edge for high-frequency waves, leading to significant local peaks and valleys in relative energy flux density. Low-frequency waves, due to their longer wavelengths, exhibit weaker scattering effects around the cavity, resulting in a smoother distribution of relative energy flux density and a more uniform energy distribution. The relative energy flux density distribution on the cavity surface shows different characteristics under incident waves of different frequencies. The figure shows that the energy flux density at certain locations for mid-frequency waves (100 Hz) is significantly higher than that for high-frequency waves (150 Hz), possibly because the mid-frequency wave has a moderate wavelength, which neither distributes smoothly like the low-frequency wave nor attenuates rapidly and fluctuates sharply like the high-frequency wave, thus forming higher energy flux density at certain locations. This is closely related to wave attenuation, scattering effects, and interference patterns. Although high-frequency waves have stronger scattering and interference effects, their energy may also attenuate faster, resulting in lower energy flux density at some locations compared to mid-frequency waves. Low-frequency waves, due to their longer wavelengths, have a more uniform energy flux density distribution, with less pronounced local maxima and minima.

Furthermore, at $\theta=\pi/2$ and $3\pi/2$, the energy flux density reaches its minimum. This is because destructive interference between the incident and scattered waves may occur at these locations, leading to a reduction in local relative energy flux density. Additionally, due to the symmetry of the circular cavity, these angles may be symmetrical points of energy distribution during wave propagation and scattering, resulting in minimum relative energy flux density distribution.

5. Conclusion

This study investigates the relative energy flux density distribution of SH waves of different frequencies incident on a deeply buried circular cavity, with numerical simulations conducted using MATLAB. The results reveal the relative energy flux distribution at various angles and the influence of frequency on this distribution. The findings indicate that selecting an appropriate frequency can effectively control and optimize the energy flux density distribution in practical engineering applications. This has significant implications for the design of underground structures and seismic wave protection.

ISSN 2706-655X Vol.6, Issue 5: 47-51, DOI: 10.25236/IJFET.2024.060507

References

[1] Sun Jinshan, Zuo Changqun, Zhou Chuanbo, et al. Dynamic disturbance characteristics of blasting stress waves on adjacent circular tunnels [J]. Journal of Vibration and Shock, 2015, 34(18): 7-12+18.
[2] Lu Shiwei, Zhou Chuanbo, Liu Hongyu, et al. Law of blasting vibration in rock masses with a single layer interface structure [J]. Engineering Blasting, 2021, 27(02): 29-34.
[3] Zhang Maochen, Lu Shiwei, Zhou Chuanbo, et al. Analysis of the propagation characteristics of cylindrical SH waves in soil-rock strata [J]. Engineering Blasting, 2023, 29(04): 35-42.
[4] Mao C C, Pao Y H. The diffraction of elastic waves and dynamic stress concentrations [M]. New York: Crane, Russak &Company Inc.,1972
[5] Liu Zhongxian, Liang Jianwen, Zhang He. Scattering of plane P-waves and SV-waves by a lined cavity in an elastic half-space (I)—Method [J]. Journal of Natural Disasters, 2010, 19(4): 71-76.