

The application of calculus thought in college physics teaching

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Abstract: College Physics is a compulsory course for science and engineering students, and it is also an important basic course for cultivating scientific thinking and innovation ability. However, due to its high requirements for mathematical knowledge, many mathematical ideas in the process of solving physical problems are easily ignored, which makes it difficult for students to learn. Based on the analysis of some knowledge points in college physics textbooks, use of calculus solving physics problems is the emphasis and difficulty in college physics teaching. Summarizes the methods and techniques of using calculus thought to analyze and solve problems, so that students can skillfully apply calculus thought to college physics learning, to achieve the purpose of improving teaching quality.

Keywords: university physics; calculus; differential element; limitation

1. Introduction

As a primary subject, college physics is an important part of higher education and plays a core supporting role in professional courses^[1]. This course can not only lead students to discover the universal laws of nature, but also cultivate students' comprehensive quality and ability. It is an important leading course for professional courses in science and engineering. It is of great significance in the development of new engineering education and the cultivation of new engineering talents^[2]. The idea of calculus is ubiquitous in the field of physics. As an important tool in mathematics, calculus provides a powerful mathematical framework for studying changes and movements. The application of calculus to solve relatively complex physical problems is simple, clear, scientific and accurate^[3,4]. Therefore, college physics courses are generally arranged after the study of 'higher mathematics'. The basic concepts of calculus, such as derivatives and integrals, provide important mathematical tools for understanding changes and motions in nature. Integrating the idea of calculus into physics teaching can truly describe the subtle physical process, which is conducive to students' in-depth understanding of the essence of physical phenomena, and then use the existing knowledge to solve practical physical problems. However, the current students' foundation of higher mathematics is not solid, and they do not really understand the meaning of calculus. It is often difficult for them to combine the knowledge of higher mathematics with college physics, so they lack confidence in learning college physics and even fear difficulties. At this time, teachers' thinking is often needed. However, in traditional physics teaching, teachers rarely link physical problems with calculus ideas, but directly present them as conclusions. This leads to the fact that students do not have enough opportunities to apply the idea of calculus to solve practical physical problems when learning the basic principles of physics. This separate teaching mode may limit students' understanding of the relationship between physics and calculus, thus affecting their in-depth mastery and application of physics.

To solve this problem, it is of great significance to introduce the idea of calculus into college physics teaching. By combining the concept of calculus with the basic principles of physics, students can better understand the mathematical principles behind physical phenomena and solve physical problems through mathematical methods. For example, in mechanics, the calculus method can be used to describe the motion and force of the object more accurately; in electromagnetism, calculus can be used to derive the equations of electric field and magnetic field, and to solve complex electromagnetic phenomena.

With the development of physics education and the evolution of teaching methods, more and more educators have begun realize the importance of introducing calculus into college physics teaching. This

paper will discuss how to effectively combine calculus with physics, and put forward some specific teaching methods and cases, so that students can skillfully apply the idea of calculus to analyze physical problems and improve the quality of learning. At the same time, it can help educators better integrate this concept into teaching practice. Through such efforts, we are expected to contribute cultivating more fully developed physics students and promote the innovation and progress of physics education.

2. The application of calculus in college physics

Physics and mathematics are interpenetrated disciplines. In physics, mathematics is regarded as an indispensable tool. The idea of calculus is an important part of mathematical thinking methods. It can transform complex physical processes into quantitative solvable processes^[5], which is of great help in understanding the relevant physical connotations. Its application is particularly important in college physics learning. It is the premise and foundation of learning college physics and is of great significance in practical teaching. Teachers should pay attention to students' understanding of calculus in the teaching process. Through teaching examples, students can have a clear understanding of calculus and can skillfully apply them to college physics learning. Next, we will discuss the application of calculus in college physics teaching.

2.1. The application of calculus to the curvilinear motion

When explaining this part of the curve motion, we can not simply list the expressions of tangential acceleration and normal acceleration in the form of conclusions. Still, we should use the idea of calculus to take students step by step to derive. In the process of explanation, the concept of curvature radius is first introduced. As shown in Figure 1, a particle makes the curve motion as shown in Figure 1, and its trajectory is like the blue curve in the figure. A tangent is drawn from the two adjacent points P_1 and P_2 on the curve. The angle between the two tangents is Δ , and the arc length between P_1 and P_2 is Δs . The curvature k of the point P_1 can be expressed as :

$$k = \lim_{\Delta s \rightarrow 0} \frac{\Delta \theta}{\Delta s} = \frac{d\theta}{ds} \quad (1)$$

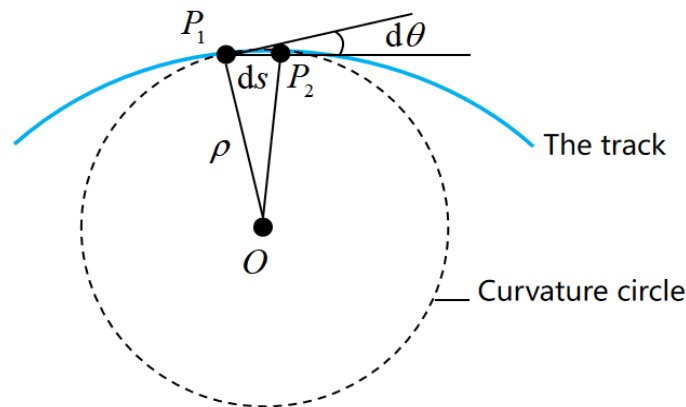


Figure 1: Radius of curvature

If the curvature of the circle is equal to the curvature of the curve at the point, it is called the curvature circle of the point, and its radius M is called the radius of curvature, which can be expressed as Equation (2).

$$\rho = \frac{1}{k} = \frac{ds}{d\theta} \quad (2)$$

On the premise of introducing the radius of curvature, we solve the tangential acceleration and normal acceleration. The trajectory of the particle is shown as the black dotted line in Figure 2. To calculate the acceleration of a point P_1 in the curve motion, we take two points P_1 and P_2 on the

curve, and the time interval between the two points is infinitely small as Δt . The velocity of the P_1 point is \vec{v}_1 , and the velocity of the P_2 point is \vec{v}_2 . The two velocities are vectors, the angle is $\Delta\theta$, and the distance of the particle is Δs . We use the right figure in Figure 2 to represent the two velocity vectors. The initial position velocity is \vec{v}_1 , and the final position

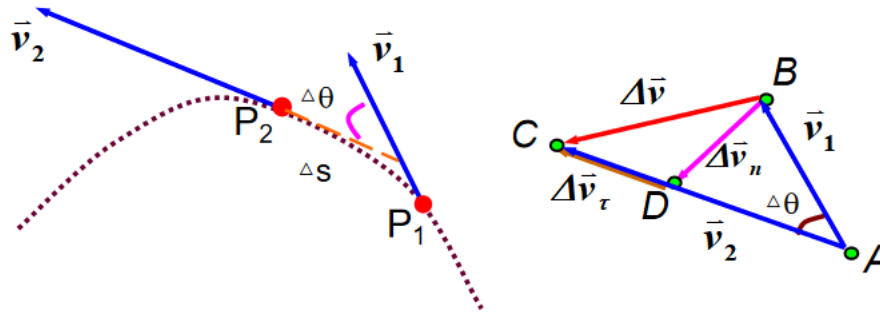


Figure 2: Acceleration decomposition diagram

velocity is \vec{v}_2 . We draw a circle with the length of the AB line segment centered on point A as the radius, and the intersection of the circle and the AC line segment is D. Then the length of the AD line segment is equal to the length of the AB line segment. \vec{DC} is represented by $\vec{\Delta v}_\tau$, the size of $\vec{\Delta v}_\tau$ is represented by Δv , \vec{BC} is represented by $\vec{\Delta v}$, and \vec{BD} is represented by $\vec{\Delta v}_n$. According to the definition of acceleration, we can calculate the acceleration as shown in Equation (3).

$$\vec{a} = \lim_{\Delta t \rightarrow 0} \frac{\vec{\Delta v}}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{\vec{BC}}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{\vec{\Delta v}_n + \vec{\Delta v}_\tau}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{\vec{\Delta v}_n}{\Delta t} + \lim_{\Delta t \rightarrow 0} \frac{\vec{\Delta v}_\tau}{\Delta t} \tag{3}$$

According to the limit thought in calculus because $\Delta t \rightarrow 0$, then $\Delta\theta \rightarrow 0$, we can know that $\sin \Delta\theta \approx \Delta\theta$ then $\left| \vec{\Delta v}_n \right| = \left| \vec{v}_1 \right| \cdot \sin \Delta\theta = v \cdot \Delta\theta$, where $\left| \vec{v}_1 \right| = v$. The Equation (3) can be transformed into Equation (4), where \vec{n}_0 and $\vec{\tau}_0$ are unit vectors in the directions of $\vec{\Delta v}_n$ and $\vec{\Delta v}_\tau$, respectively.

$$\vec{a} = \lim_{\Delta t \rightarrow 0} \frac{v \cdot \Delta\theta}{\Delta t} \vec{n}_0 + \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} \vec{\tau}_0 = v \frac{d\theta}{dt} \vec{n}_0 + \frac{dv}{dt} \vec{\tau}_0 \tag{4}$$

According to the concept of curvature radius introduced above and using the method of inserting variables $\frac{d\theta}{dt} = \frac{d\theta}{ds} \cdot \frac{ds}{dt}$, we can further transform Equation (4) to obtain Equation (5). So far we can get the corresponding tangential acceleration and normal acceleration formula.

$$\vec{a} = \frac{v^2}{\rho} \vec{n}_0 + \frac{dv}{dt} \vec{\tau}_0 \tag{5}$$

In the whole process, we use the idea of approximation and limit step by step to derive the formulas of tangential acceleration and normal acceleration in curve motion, rather than simply presenting them to students in the form of conclusions, which can deepen understanding, impress, and gradually form the idea of 'derivative'. Students learn to use calculus to solve problems, not only can be used to solve other physical problems, but also can be applied to other fields, innovative solutions to unknown problems, improve students' ability to solve problems, so that students benefit for life.

2.2. Solve the calculation problem of gravity work with the idea of calculus

When explaining the work of gravity, we should not only give a simple formula, but also take the students to analyze step by step, and integrate the idea of calculus to analyze and finally get the expression of the work of gravity. This will help students understand the process of gravity work, deepen their impression, and facilitate them to draw inferences. Next, we explain in detail the typical examples of gravity work : a particle makes a flat throwing motion, the trajectory is shown in the red and orange dotted lines in Figure 3, and the work of the particle moving from

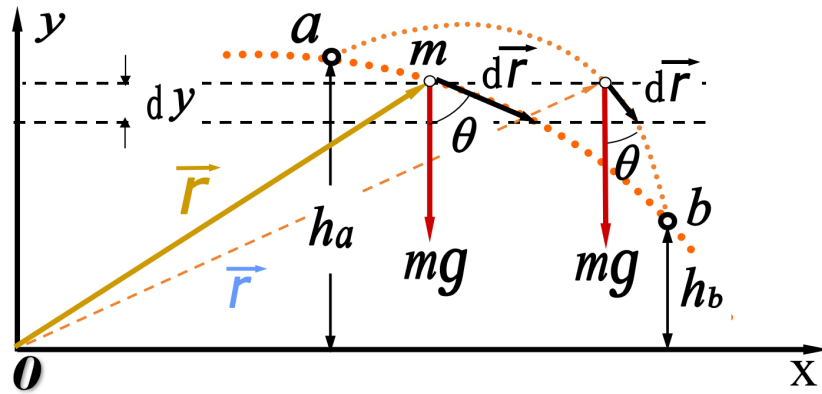


Figure 3: Gravity work decomposition diagram

Point a to point b is calculated. We first establish a rectangular coordinate system. The height of the particle at point a and point b from the x-axis is h_a and h_b , respectively. Here we need to pay attention to the fact that the particle can take many paths from point a to point b, that is, the distance of the particle is a variable quantity. Because the particle is doing a curvilinear motion, the direction of motion is always changing, we can divide the displacement of ab into many infinitesimal elements, any of which is $d\vec{r}$. According to the definition of constant force work, the corresponding element work is calculated, and the sum of all element work can be obtained. The work done by the corresponding

gravity, as shown in Equation (6). Here we can regard $|d\vec{r}| \cos \theta$ as a whole, which is the projection of displacement in the direction of force. Then the Equation (6) can be replaced by the Equation (7), and the upper and lower limits of the integral can be easily determined. It can be seen that no matter which path the particle takes, the corresponding gravity work Equation can be written as Equation (7). Therefore, we say that gravity as a conservative force, the magnitude of its work has nothing to do with the path, only with the beginning and end position.

$$dW = m\vec{g} \cdot d\vec{r} = mg * |d\vec{r}| * \cos \theta$$

$$W = \int dW = \int_a^b mg * |d\vec{r}| * \cos \theta \tag{6}$$

$$W = \int_a^b mg * |d\vec{r}| * \cos \theta = \int_{h_a}^{h_b} -mg dy = mgh_a - mgh_b \tag{7}$$

In the process of calculating the work done by gravity, we first simply give the infinitesimal of displacement, and then calculate the work done by gravity, and then calculate its integral to solve the problem. Using the idea of calculus to solve the problem of gravity work can greatly simplify the calculation process, so that students can think that the problem is very simple in subjective consciousness, to eliminate the fear of difficulty. By mastering the idea of calculus, students' significance is far greater than remembering physical conclusions. Because with the passage of time, the knowledge learned may be forgotten, but the methods to solve physical problems will be unconsciously applied in work and life to solve various problems.

2.3. Solve the problem of calculating the electric field intensity by using the idea of calculus

Find the field strength of the points on the axis of the uniformly charged ring in Fig.4. When explaining this problem, teachers should consciously guide students how to use calculus to solve it in teaching. Firstly, the coordinate system is established, and then the differential operation is carried out. The differential is 'infinite subdivision'. We select any infinitesimal dq on the uniformly charged ring, and the corresponding line element is dl . Here, the electric field intensity generated by dq at point P is $d\vec{E}$, the distance is r , and the angle is θ . Here, we can calculate the electric field intensity generated by the infinitesimal dq at point P as Equation (8). $d\vec{E}$ can be decomposed into x-axis direction and perpendicular to the x-axis direction, which are $d\vec{E}_x$ and $d\vec{E}_\perp$, respectively. As shown in Figure 5, for the electric field strength $d\vec{E}_\perp$ perpendicular to the x-axis direction, the electric field strength of the corresponding line element

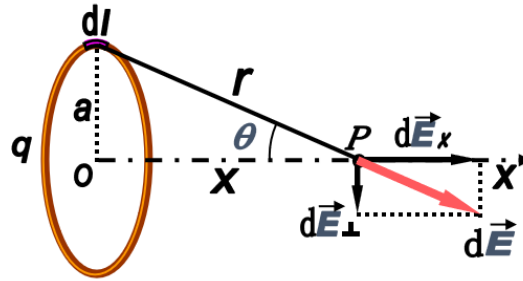


Figure 4: Uniformly charged ring

at the P point can always be found to be decomposed into the direction perpendicular to the x-axis, and is equal to the size of the $d\vec{E}_\perp$, in the opposite direction., so the electric field intensity of the entire charged ring perpendicular to the x-axis direction at the P point is 0. Therefore, the $d\vec{E}$ size is equal to $d\vec{E}_x$, such as Equation (10).

$$d\vec{E} = \frac{dq}{4\pi\epsilon_0 r^2} \vec{r}_0 \tag{8}$$

$$d\vec{E} = d\vec{E}_x + d\vec{E}_\perp \tag{9}$$

$$dE_x = dE \cos\theta = \frac{dq}{4\pi\epsilon_0 r^2} \frac{x}{r} \tag{10}$$

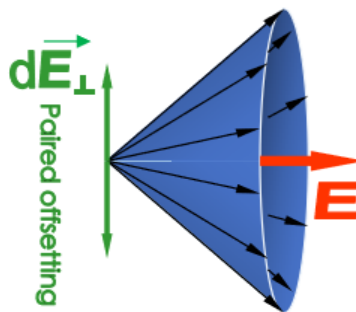


Figure 5: Electric field intensity decomposition in the direction perpendicular to the x-axis

The calculus includes differential and integral. The integral is 'infinite summation'. The differential operation has been carried out before. Here we carry out the integral. The uniformly charged ring can be split into infinite dq , and the electric field strength of each dq at P point has been calculated. Then we can calculate the size of the field strength generated by the uniformly charged ring at point P through the form of infinite summation, that is, integration, as shown in Equation (11). At this time, as long as the upper and lower limits of the integral are determined, the final result can be calculated.

$$E = \int \frac{x dq}{4\pi\epsilon_0 r^3} \quad (11)$$

The idea of substitution is used to determine the upper and lower limits of the integral. There are three kinds of continuous charged bodies in the electrostatic field: charged body, charged wire and charged surface. The charged ring here can be equivalent to charged wire. The uniform charged wire can be regarded as composed of countless point charges. It is divided into countless blocks by the idea of 'differential'. Each region has a charged amount dq . The total amount of electricity throughout the charged wire is the integral of the micro-element dq . We can use the idea of substitution in calculus to calculate this integral: for the uniform charged wire, its charged wire density is expressed by λ , then $dq = \lambda dl$. After this substitution operation, we can easily determine the upper limit line of the integral, then $q = \int_0^l \lambda dl = \lambda l$. The integral upper and lower limits of formula (11) can be determined, and the field strength of P point on the axis of uniformly charged ring can be calculated by integral operation.

$$E = \int_0^{2\pi a} \frac{x dl}{4\pi\epsilon_0 r^3} \quad (12)$$

Through the explanation of the above problems, we can see that the whole process is through the idea of infinite division and summation, which can make students understand the physical knowledge such as the calculation of electric field strength and superposition principle more deeply, simplify the complex problems, and solve the physical problems accurately and easily. This teaching method can better cultivate students' inquiry spirit and innovation ability, enable them to understand the basic concepts and laws of physics more deeply, and improve their interest in physics learning.

3. Conclusion

In college physics learning, because of its high requirements for mathematical knowledge, and many mathematical ideas in the process of solving physical problems are easily ignored, it is difficult for students to learn. We need to fully understand its importance and necessity in teaching and learning, and constantly improve the quality of teaching and students' quality. In this paper, through the analysis and explanation of several examples, we integrate the idea of calculus into the analysis of physical problems. It can be seen that the solution methods of these physical problems are all around the idea of segmentation, approximation, limit, and then differential or integral. With the passage of time, the knowledge learned may be forgotten, but the methods to solve physical problems will be unconsciously applied in work and life to solve various problems. This teaching method can better cultivate students' inquiry spirit and innovation ability, enable them to understand the basic concepts and laws of physics more deeply, and improve their interest in physics learning.

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