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Optimal Portfolio and Properties under Cumulative Prospect Theory

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Abstract: *This paper is done in a single period economy discussing the discrete-time behavioral portfolio choice under Cumulative Prospect Theory. A portfolio optimization model is set up with the goal of maximizing the distorted expected utility, and the analytic solutions for optimal allocation are obtained under the assumption that short-selling is forbidden. It is found that there is a unique optimal portfolio. Finally, the properties of the optimal portfolio solution* θ*, the CPT-ratio, and the parameter* λ *are discussed and proofs are provided.*

Keywords: *Cumulative Prospect Theory; CPT-ratio; Optimal Portfolio Choice; Value Function*

1. Introduction

The Expected Utility Function, developed in the 1950s by von Neumann and Morgenstern based on axiomatic assumptions, uses logic and mathematical tools to establish an analytical framework aimed at modeling the choices of a "rational person" under uncertainty. However, numerous psychological experiments, such as the Allais Paradox (Allais, M. (1953) [1]), Ellsberg Paradox(Ellsberg, D. (1961) [2]), Reflection Effect, Probability Weighting, and Isolation Effect, have demonstrated that people's decisions under uncertainty systematically violate the axiomatic assumptions of Expected Utility Theory. People exhibit different risk attitudes towards gains and losses: risk aversion in high-probability gain scenarios and risk-seeking in high-probability loss scenarios, as well as risk-seeking in low-probability gain scenarios and risk aversion in low-probability loss scenarios. Kahneman and Tversky (1979) [3] proposed "Prospect Theory" as an alternative to expected utility theory. Their main ideas include that people focus more on changes in wealth rather than final wealth levels; they tend to take risks when facing potential losses similar to the condition and prefer certain gains when facing potential profits similar to the condition. Kahneman and Tversky (1979) [3] describe individual choices and decision-making processes using two functions: a value function $V(x)$ and a decision weighting function $\pi(p)$. The value function replaces the utility function in traditional expected utility theory, while the decision weighting function transforms the probability p in the expected utility function into a decision weight $\pi(p)$.

In this paper, Prospect Theory serves as the theoretical framework to establish a single-period economy consisting of one risky asset and one risk-free asset. First, different utility functions and decision weighting functions are discussed. A new value function is then established to determine the optimal portfolio solution under certain conditions, specifying the amount of investment in risky assets that maximizes the investor's value. The paper also provides and proves related properties of the optimal portfolio solution, such as positive homogeneity. Finally, it studies the optimal allocation in risky assets assuming that the excess returns of the risky asset follow a normal distribution.

The rest of this paper is organized as follows: Section 2 sets up a portfolio optimization model aimed at maximizing expected utility and provides analytic solutions for the optimal portfolio in one case (i.e., $\alpha < \beta$); Section 3 discusses and proves the general properties of the optimal portfolio solution; Section 4, under the assumption that excess returns of the stock follow a normal distribution, examines the sensitivity of the optimal allocation to the CPT-ratio, the level of risk aversion on gains, and the level of risk propensity on losses; Section 5 concludes the paper.

2. Modeling

Consider a market consisting of one risk-free asset with initial price one and the interest rate $r \geq 0$,

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Based on a one period economy, let $X = R - r$ be the excess return on the risky asset over the risk-free rate r. Based on no-arbitrage, the investor's final wealth value can be expressed as:

$$\mathbf{W_T^{\theta}} = x_0(1 + r) + \theta X \quad (1)$$

where x_0 is the investor's initial capital at time 0. An amount θ is invested in the risky asset and the remaining wealth, $x_0 - \theta$, is invested in the risk-free asset.

Investors exhibit different risk attitudes when facing gains versus losses. Kahneman and Tversky (1992) [4] described this phenomenon using utility functions, suggesting that investors show risk-seeking when confronted with losses and risk-averse when dealing with gains. Three different widely used S-shaped utility functions are listed:

(1) Piecewise power utility function:

$$u_+(x) := x^\alpha,\ u_-(x) := -\lambda x^\beta \text{ for all } x \in [0, +\infty), \quad (2)$$

where $0 < \alpha \le 1$, $0 < \beta \le 1$ and $\lambda \ge 1$. Note that $\alpha, \beta > 0$ ensures that $u_\pm(\cdot)$ are strictly increasing, while $\alpha, \beta \le 1$ ensures that $u_\pm(\cdot)$ are concave.

The parameter λ represents the level of loss aversions.

(2) Piecewise exponential utility function:

$$u_+(x) := 1 - e^{-\alpha x},\ u_-(x) := \lambda(1 - e^{-\beta x}) \ \textit{for all } x \in [0, +\infty), \quad (3)$$

where α, $\beta > 0$ and $\lambda > 0$. Note that α, $\beta > 0$ ensures that $u_\pm(\cdot)$ are strictly increasing and concave. The parameter also represents the level of loss aversions. Considering the Arrow-Pratt's measures of ARA by Föllmer and Schied (2016) [5], the piecewise exponential utility functions $u\pm(\cdot)$ exhibit constant absolute risk aversion (CARA) because

$$ARA_{u_+}(x) = -\frac{u_+''(x)}{u_+'(x)} = \alpha,$$

$$ARA_{u_-}(x) = -\frac{u_-''(x)}{u_-'(x)} = \beta.$$

The piecewise exponential utility functions $u_\pm(\cdot)$ is bounded since the preference value converges 1 as x tends to infinity (i.e., $\lim_{x\to+\infty} u_\pm(x) = 1 < +\infty$).

(3) Piecewise logarithmic utility function:

$$u_+(x) := log(1 + x),\ u_-(x) := \lambda log(1 + x) \ \textit{for all } x \in [0, +\infty) \quad (4)$$

where $\lambda > 0$ and λ represents the level of loss aversions. The relative risk aversion of the piecewise logarithmic utility converges to 1 as x tends to infinity; indeed,

$$RRA_{u_\pm}(x) = x(1 + x)^{-1} \xrightarrow[x\to+\infty]{} 1.$$

The piecewise logarithmic utility function $u_\pm(\cdot)$ is unbounded above (i.e., $\lim_{x\to+\infty} u_\pm(x) = +\infty$).

CPT investors do not weigh the outcomes according to objective probabilities. Kahneman and Tversky (1992) [4] think that the weight function is not a subjective probability, but a distortion of the given probability. From its form and shape, the weight function is non-linear. Its main characteristic is that people generally overestimate small probability events and underestimate large probability events. In this example, we list three different distorted probability functions.

(1) Tversky and Kahneman (1992) [4] use the following functional forms for weighted functions:

$$w_+(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}},$$

$$w_-(p) = \frac{p^\delta}{(p^\delta + (1-p)^\delta)^{1/\delta}}, \quad (5)$$

where $0.5 \le \gamma < 1, 0.5 \le \delta < 1$ and they estimated the parameter values: $\gamma = 0.61$, and $\delta = 0.69$.

(2) Prelec (1998) [6] presented the probability weighting functions:

$$w(p) = exp\{-\delta + (-logp)^\gamma\},$$

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where $0 < p \leq 1, \delta > 0, \gamma > 0$.

(3) A power distortion is of the form:

$$w(p) = p^{\gamma}, for\ all\ p \in [0,1],$$

with $\gamma > 0$. Note that $w(\cdot)$ are concave when $\gamma < 1$, while $w(\cdot)$ are convex when $\gamma > 1$.

Taking $\gamma = 1$ recovers the case where there is no distortion of probabilities (i.e., $w(p) = p$ is the identity function). Function $w(\cdot)$ are concave representing that investors overweight the actual probability p of events, while function $w(\cdot)$ are convex representing that investors underestimate the actual probability p of events.

In prospect theory, it is posited that investors are concerned not with their final wealth value per se, but with the difference between the final wealth value and a reference point. If the wealth exceeds the reference point, the excess is considered a gain; if the wealth falls below the reference point, the shortfall is considered a loss. In equation (1), the risk-free asset $x_0(1 + r)$ serves as the reference point. The final wealth value $\boldsymbol{W_T^{\theta}}$ in excess of the risk-free asset is considered a gain (i.e., $\boldsymbol{\theta X > 0}$),while the shortfall is considered a loss (i.e., $\boldsymbol{\theta X < 0}$). Investors are concerned with how much quantity $\boldsymbol{\theta}$ to invest in risky assets to maximize their gains. Since different investors have different reference points, a value function is used to measure the value to the investor. Applying the Piecewise power utility function in equation (2), the utility for gains and losses is represented as $\boldsymbol{u_G(x) = (\theta X)^{\alpha}}$ and $\boldsymbol{u_L(x) = -\lambda(\theta X)^{\beta}}$ respectively. A value function in this case is defined as:

$$\boldsymbol{V(\theta X) := V_G\big(u_G(x)\big) - V_L\big(u_L(x)\big)}$$

$$= (\boldsymbol{C}) \int (\boldsymbol{\theta X})^{\boldsymbol{\alpha}} \boldsymbol{d}(\boldsymbol{w}_+ \circ \mathbb{p}) - (\boldsymbol{C}) \int \boldsymbol{\lambda}(\boldsymbol{\theta X})^{\boldsymbol{\beta}} \boldsymbol{d}(\boldsymbol{w}_- \circ \mathbb{p})^1$$

$$= \boldsymbol{\theta}^{\boldsymbol{\alpha}} \int_{\boldsymbol{0}}^{+\infty} \boldsymbol{X}^{\boldsymbol{\alpha}} \boldsymbol{d}(\boldsymbol{w}_+ \circ \mathbb{p}) - \boldsymbol{\lambda}\boldsymbol{\theta}^{\boldsymbol{\beta}} \int_{\boldsymbol{0}}^{+\infty} \boldsymbol{X}^{\boldsymbol{\beta}} \boldsymbol{d}(\boldsymbol{w}_- \circ \mathbb{p})$$

$$=: \boldsymbol{\theta}^{\boldsymbol{\alpha}} \mathbb{G}(\boldsymbol{X}) - \boldsymbol{\lambda}\boldsymbol{\theta}^{\boldsymbol{\beta}} \mathbb{L}(\boldsymbol{X}) \tag{6}$$

The function $\mathbb{G}(\mathbf{X}) = \int_{\mathbf{0}}^{+\infty} \boldsymbol{X}^{\boldsymbol{\alpha}} \boldsymbol{d}(\boldsymbol{w}_+ \circ \mathbb{p})$ measures the ability of an asset to generate gains, while $\mathbb{L}(\mathbf{X}) = \int_{\mathbf{0}}^{+\infty} \boldsymbol{X}^{\boldsymbol{\beta}} \boldsymbol{d}(\boldsymbol{w}_- \circ \mathbb{p})$ measures the ability of an asset to generate losses. To find the optimal quantity of $\boldsymbol{\theta}$, the first derivative of the value function $\boldsymbol{V(\theta X)}$ with respect to $\boldsymbol{\theta}$ is taken. The parameters that affect the optimal solution are $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{\theta}$. Here $\mathbb{G}(\mathbf{X})$ and $\mathbb{L}(\mathbf{X})$ do not depend on $\boldsymbol{\theta}$. The relationship between $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ determines the nature of the optimal solution. Under the assumption of short-selling and the ability to borrow funds, in the case of $\boldsymbol{\alpha < \beta}$, the quantity of capital to be invested in risky assets should be:

$$\theta = \left(\frac{\alpha}{\lambda\beta}\right)^{\frac{1}{\beta-\alpha}} \Omega(X)^{\frac{1}{\beta-\alpha}} \tag{7}$$

where $\Omega(X) = \frac{\mathbb{G}(\boldsymbol{X})}{\mathbb{L}(\boldsymbol{X})}$ is the CPT-ratio defined by Bernard and Ghossoub (2010) [7].

When $\alpha > \beta$, there is no optimal portfolio. When $\alpha = \beta$, the optimal portfolio depends on the relationships between CPT-ratio $\Omega(X)$ and parameter λ. When $\Omega(X) < \lambda$, no capital should be invested in risky assets. When $\Omega(X) = \lambda$, the optimal portfolio is infinite, meaning that investing more and more in risky assets is preferable since borrowing is allowed. When $\Omega(X) > \lambda$, there is no optimal portfolio.

3. CPT Optimal Portfolio and Properties

This section primarily discusses the properties of the optimal portfolio in the case where short-selling is not allowed and $\boldsymbol{\alpha < \beta}$.

CPT-ratio $\boldsymbol{\Omega(X)}$ is an extension of Gain-Loss Ratio put forward by Bernard and Ledoit (2000) [8] as well as of Omega measure proposed by Keating and Shadwick (2002) [9]. But CPT-ratio has made some changes on the basis of these two classic performance measurements. To see this, we first defined Gain-Loss Ratio and Omega Measure below.

1 $(\boldsymbol{C})$ implies Choquet integral approach.

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Definition 3.1. [**Gain-loss ratio**] Recall that the random variable $\boldsymbol{X}$ is the excess return on the risky assets, the Gain-loss ratio is defined as:

$$\boldsymbol{GL}(\boldsymbol{X}) = \frac{\mathbb{E}_{\mathbb{p}^*}[X^+]}{\mathbb{E}_{\mathbb{p}^*}[X^-]},$$

where $\mathbb{E}_{\mathbb{p}^*}[\cdot]$ donate the expectations under actual probability.

The random variable $\boldsymbol{X}^+$ represents a positive return on stocks and $\boldsymbol{X}^-$ represents a negative return on stocks.

Definition 3.2. [Omega Measure] Let $\overline{\boldsymbol{X}}$ be a given return random variable, the Omega measures defined as:

$$\overline{\boldsymbol{\Omega}}(\boldsymbol{X}) = \frac{\mathbb{E}_{\mathbb{P}}[\boldsymbol{u}(\boldsymbol{X}-\overline{\boldsymbol{X}})]}{\mathbb{E}_{\mathbb{P}}[\boldsymbol{u}(\overline{\boldsymbol{X}}-\boldsymbol{X})]} > \boldsymbol{0}.$$

The random variable $\overline{\mathbf{X}}$ represents investors' reference level of stocks return. The part of return X beyond $\overline{\mathbf{X}}$ can be seen as a positive return, or satisfactory stocks return, where the part of return X below $\overline{\boldsymbol{X}}$ can be seen as a negative return, or a disappointing stocks return. Both Gain-Loss Ratio and Omega measure are performance measurement ratios, which measure how attractive risk assets are to investors. The higher the value of GL, the more attractive the investment. For a given reference level of stocks return, the higher the value of $\overline{\boldsymbol{\Omega}}$, the more attractive the investment. The difference between them is that Omega measure considers the benchmark of return of investors, while Gain-Loss Ratio does not. The CPT-ratio is also a measure of the attractiveness of invest- ment, but the difference lies in that the CPT-ratio is a subjective performance measurement, which varies with different investors. Because it depends not only on the distribution of stock returns, but also on the subjective probability weight generated by investors' preference.

Proposition 3.3. [**Positive homogeneity**] The CPT-ratio is positively homogeneous of degree $\boldsymbol{p} = \boldsymbol{\alpha} - \boldsymbol{\beta}$, that is

$$\boldsymbol{\Omega}(\boldsymbol{mX}) = \boldsymbol{m}^{\alpha-\beta}\boldsymbol{\Omega}(\boldsymbol{X}).$$

Proof: Recall the CPT-ratios $\boldsymbol{\Omega}(\boldsymbol{X}) = \frac{\mathbb{G}(X)}{\mathbb{L}(X)} = \frac{\int_0^{+\infty} X^\alpha d(w_+ \circ \mathbb{p})}{\int_0^{+\infty} X^\beta d(w_- \circ \mathbb{p})}$. Therefore, for any $\boldsymbol{m} > \boldsymbol{0}$,

$$\begin{aligned}
\boldsymbol{\Omega}(\boldsymbol{mX}) &= \frac{(\boldsymbol{C})\int_0^{+\infty}(\boldsymbol{mX})^\alpha \boldsymbol{d}(\boldsymbol{w}_+ \circ \mathbb{p})}{(\boldsymbol{C})\int_0^{+\infty}(\boldsymbol{mX})^\beta \boldsymbol{d}(\boldsymbol{w}_- \circ \mathbb{p})} \\
&= \frac{\boldsymbol{m}^\alpha \int_0^{+\infty} \boldsymbol{X}^\alpha \boldsymbol{d}(\boldsymbol{w}_+ \circ \mathbb{p})}{\boldsymbol{m}^\beta \int_0^{+\infty} \boldsymbol{X}^\beta \boldsymbol{d}(\boldsymbol{w}_- \circ \mathbb{p})} \\
&= \boldsymbol{m}^{\alpha-\beta}\boldsymbol{\Omega}(\boldsymbol{X}).
\end{aligned}$$

Proposition 3.3 indicates that when CPT investors' risk aversion on gains dominates the risk propensity on losses, i.e., $\boldsymbol{\alpha} - \boldsymbol{\beta} < \boldsymbol{0}$, the excess return on stocks increases by $\boldsymbol{m} > \boldsymbol{0}$ proportionately, the CPT-ratio will decrease by $\boldsymbol{m}^{\beta-\alpha}$ proportionately.

Proposition 3.4. For any random variables $\boldsymbol{X_1}$, $\boldsymbol{X_2}$, if

$$\boldsymbol{\Omega}(\boldsymbol{X_1}) \le \boldsymbol{\Omega}(\boldsymbol{X_2}),$$

then

$$\boldsymbol{\theta}(\boldsymbol{X_1}) \le \boldsymbol{\theta}(\boldsymbol{X_2}),$$

where $\boldsymbol{\theta}(\boldsymbol{X_i}) := (\frac{\alpha}{\lambda\beta})^{\frac{1}{\beta-\alpha}}\boldsymbol{\Omega}(\boldsymbol{X_i})^{\frac{1}{\beta-\alpha}}$ for $\boldsymbol{i} \in \{\boldsymbol{1}, \boldsymbol{2}\}$.

Proof: Let $\boldsymbol{X_1}$ and $\boldsymbol{X_2}$ such that $\boldsymbol{\Omega}(\boldsymbol{X_1}) \le \boldsymbol{\Omega}(\boldsymbol{X_2})$. Denote by the CPT optimal investment in the risk asset with excess return, $\boldsymbol{\theta}(\boldsymbol{X_i})$, since $\boldsymbol{\beta} - \boldsymbol{\alpha} > \boldsymbol{0}$, then

$$(\boldsymbol{\Omega}(\boldsymbol{X_1}))^{\frac{1}{\beta-\alpha}} \le (\boldsymbol{\Omega}(\boldsymbol{X_2}))^{\frac{1}{\beta-\alpha}},$$

Note that $(\frac{\alpha}{\lambda\beta})^{\frac{1}{\beta-\alpha}} > \boldsymbol{0}.$ Therefore

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$$(X_1) \leq \theta(X_2).$$

Proposition 3.4 shows that optimal investment is increasing in the CPT-ratio. The higher CPT-ratio, the more willing CPT investors are to allocate more capitals in risky assets.

Proposition 3.5. Let $\lambda_1,\ \lambda_2 \in [1, +\infty)$, if $\lambda_1 < \lambda_2$, then

$$\theta(\lambda_1) > \theta(\lambda_2),$$

where $\theta(\lambda_i) := (\frac{\alpha}{\lambda_i \beta})^{\frac{1}{\beta-\alpha}} \Omega(X)^{\frac{1}{\beta-\alpha}}$ for $i \in \{1, 2\}$.

Proof: *Let* λ_1 and λ_2 such that $\lambda_1 < \lambda_2$. Because $\beta - \alpha > 0$,

$$(\frac{\alpha}{\lambda_1 \beta})^{\frac{1}{\beta-\alpha}} > (\frac{\alpha}{\lambda_2 \beta})^{\frac{1}{\beta-\alpha}}.$$

Note that $\frac{\alpha}{\beta} > 0$. Therefore

$$\Omega(X)^{\frac{1}{\beta-\alpha}} \left(\frac{\alpha}{\lambda_1 \beta}\right)^{\frac{1}{\beta-\alpha}} > \Omega(X)^{\frac{1}{\beta-\alpha}} \left(\frac{\alpha}{\lambda_2 \beta}\right)^{\frac{1}{\beta-\alpha}}$$

$$\theta(\lambda_1) > \theta(\lambda_2).$$

It shows that optimal investment is decreasing in loss aversion. The higher the degree of loss aversion, the less willing CPT investors are to invest in risky assets.

Proposition 3.6. In the case of $\alpha < \beta$, given $m > 0$, the optimal risky assets holding has the homogeneous of degree $p = -1$, that is

$$\theta(mX) = m^{-1}\theta(X).$$

Proof: Let $m > 0$. Denote by

$$\theta(mX) := \Omega(mX)^{\frac{1}{\beta-\alpha}} \left(\frac{\alpha}{\lambda \beta}\right)^{\frac{1}{\beta-\alpha}} = m^{-1}\theta(X),$$

where $\Omega(mX) = m^{\alpha-\beta}\Omega(X)$ is known from Proposition *3.3.*

Proposition 3.6 shows that for every m times increase in excess return on stocks, the optimal allocation in stocks reduce by m times.

4. Example: optimal allocation in normal distribution

This section studies the optimal allocating in risk assets where assuming the risky asset has a normally distributed excess return.

Assume that the excess return on risk asset follows a univariate normal distribution under $\mathbb{P}$ with mean μ and variance σ^2, written as $X \sim N(\mu,\ \sigma^2)$.

We noticed that the optimal holding is dependent on α and β, but analytically the sensitivity cannot be determined, so here provide some graphs to show how α and β affect the optimal portfolio. Before that, recall Proposition 3.4 that the CPT-ratio $\Omega(X)$ plays an important role in CPT investors' optimal allocation in the risky asset, and also affected by α and β. According to the numerical results by Bernard and Ghossoub (2010) [7], the CPT-ratio is increasing with respect to $1 - \alpha$ and is decreasing with respect to $1 - \beta$. They applied the piecewise power utility and distorted probability proposed by Tversky and Kahneman (1992) [4]. They also concluded that the behavior of CPT investors is extremely sensitive to changes in CPT-ratio $\Omega(X)$, especially when when the values of α and β are very close. For the set of λ = 2.25, a $\alpha = 0.8$, $\beta = 0.88$, γ = 0.61, and δ = 0.69, Bernard and Ghossoub (2010) [7] shows that CPT investors tend to not invest almost any capital in risk assets when $\Omega(X)$ = 0.8, whereas CPT investors tend to invest almost all capitals in risk assets when $\Omega(X)$ = 1.2. The Figure 1 shows the optimal portfolio as a function of $1 - \alpha$ (while fix β = 0.88) and as a function of $1 - \beta$ (while fix α = 0.8) when CPT-ratio $\Omega(X)$ is 0.8, 1 and 1.2 respectively, referring directly to Bernard and Ghossoub (2010) [7]'s graphs. Note that in their paper, both short-selling constraint and borrowing constraint are imposed in discussion of a given distributed excess return, while we impose that short-selling is forbidden and borrowing is allowed, and that is the reason why, in two panel of Figure 1, there are two upper limits for the optimal allocation when $\boldsymbol{\Omega}(\boldsymbol{X})$ = 1.2. The upper limit is exactly the initial capital $\boldsymbol{x_0}$ = 2. One questionable thing is that the CPT-ratio depends on $\boldsymbol{\alpha}$, therefore the value of the former changes as the latter varies. For this reason, if we keep the CPT-ratio constant at a given value and then plot the mapping $(\mathbf{1}-\boldsymbol{\alpha}) \mapsto \left(\frac{\alpha}{\lambda\beta}\right)^{\frac{1}{\beta-\alpha}} \boldsymbol{\Omega}(\boldsymbol{X})^{\frac{1}{\beta-\alpha}}$, what we obtain is not necessarily the plot of the CPT optimal portfolio as a function of $\mathbf{1}-\boldsymbol{\alpha}$.

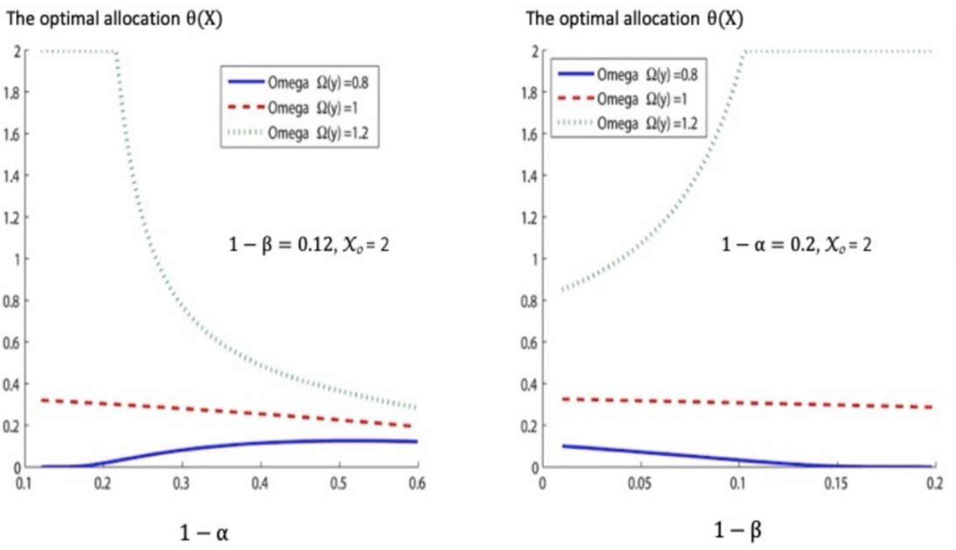

Figure 1: Optimal allocation with respect to $\mathbf{1}-\boldsymbol{\alpha}$ *and* $\mathbf{1}-\boldsymbol{\beta}$.

However, as Bernard and Ghossoub (2010) [7] report, Figure 1 shows that the sensitivity of the optimal allocation to $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ is ambiguous. See the left panel in Figure 1, when $\boldsymbol{\Omega}(\boldsymbol{X})$ = 1, the optimal allocation slightly decreases with the increase of $\mathbf{1}-\boldsymbol{\alpha}$, when $\boldsymbol{\Omega}(\boldsymbol{X})$ = 0.8, the optimal allocation slightly increases with the increase of $\mathbf{1}-\boldsymbol{\alpha}$; see the right panel in Figure 1, when $\boldsymbol{\Omega}(\boldsymbol{X})$ = 1, the optimal allocation does not seem to have changed much, when $\boldsymbol{\Omega}(\boldsymbol{X})$ = 0.8, the optimal allocation slightly decreases with the increase of $\mathbf{1}-\boldsymbol{\beta}$. Therefore, under different CPT-ratio, the CPT-ratio allocation could be either $\boldsymbol{\alpha}$ increasing function or a decreasing function of parameter $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$. We may not be able to draw a conclusion about the the sensitivity of the optimal allocation to $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$.

5. Conclusions

This paper employs Prospect Theory to analyze portfolio optimization in a single-period economy with both risky and risk-free assets. By integrating a novel value function and decision weighting function, we derive the optimal investment strategy that maximizes investor value under one condition. The paper confirms the properties of the optimal portfolio solution and explores its behavior with normally distributed excess returns. The findings highlight how the CPT-ratio, risk aversion, and risk propensity impact investment decisions. These insights extend the understanding of investor behavior beyond traditional Expected Utility Theory, offering a more nuanced approach to portfolio management under uncertainty.

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