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Sex Ratio of Seven Gill Eels Based on Lotka-Volterra Modelling and AHP Approach

Bufan Ying[1, †,*], Zhuo Xiao[1,†], Mingjie Zhou[1,†]

[1]*School of Basic Medicine, Naval Medical University, Shanghai, China*
[†]*These authors also contributed equally to this work.*
[*]*Corresponding author: 1543209472@qq.com*

Abstract: *In this study, we investigated the phenomenon of species-adapted sex ratio variation, focusing on the unique biological characteristics and sex ratio plasticity of the lampreys in aquatic ecosystems. First, the effects of different sex ratios on the ecosystem were simulated by integrating the lamprey's substitution model into a dynamic sex-induced ecosystem model. Then, based on the dynamic ecosystem model using AHP method, the lamprey's population was compared with the grass carp population in terms of sex ratio equilibrium. Next, an ecological network model was constructed to explore the effects of changes in the sex ratio of the lampreys on its predatory behavior, as well as on the stability of the ecosystem. From there, the findings were generalized to other species in the ecosystem.*

Keywords: *Lotka-Volterra Model, Ecological Network Model, AHP*

1. Introduction

The genetic code on chromosomes plays a pivotal role in biological inheritance and evolution, with genes and their expression products shaping the traits of organisms. Sex determination in mammals through chromosomes usually results in a sex ratio close to 1:1, in accordance with Mendel's laws. However, adaptive changes in sex ratios have been observed in nature, such as lampreys, in which the growth rate determines sex differentiation [1], with unique biology and plasticity of sex ratios. Not only does this enigmatic underwater inhabitant play a crucial role in lakes and oceans, but the variation in sex ratios within its populations is even more striking [2]. Our study aimed to analyze the strengths and weaknesses of the species' ability to alter its sex ratio based on resource availability. First, we embedded the lamprey's substitution model into our constructed dynamic sex-induced ecosystem model and interacted with other species in the ecosystem to simulate ecosystem impacts under different sex ratios. Second, based on the established ecosystem dynamics model, we selected three dimensions of stability, adaptability and ecological role. Then the AHP method was used to compare the sex ratio equilibrium between the lamprey's population and the grass carp population. Finally, an ecological network model was constructed to explore the effects of changes in the sex ratio of the lampreys on the stability of the marine ecosystem. Sex ratio is one of the key characteristics of a population, and changes in sex ratio can lead to fluctuations in fecundity, which in turn affects population size, food chain effects, ecosystem health, and impacts on human life. The results show that the model proposed in this paper has a clear structure, which helps to understand the impacts of sex ratio changes on the ecosystem and provides theoretical support for practical ecological management and conservation.

2. Dynamic gender induced ecosystem model

This research introduces a dynamic ecosystem model incorporating the Verhulst and Lotka-Volterra models to assess the impact of changing sex ratios in lamprey populations on broader ecosystem dynamics [3]. By employing a surrogate model to represent individual lampreys, encapsulating details such as age, sex, and reproductive status, the study simulates interactions within the ecosystem. This approach allows for a comprehensive analysis of ecological balance changes from the perspectives of food chain dynamics, reproductive capabilities, and habitat utilization, utilizing data fitting and computer simulations to visualize ecological impacts.

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2.1 Changes in sex ratios

For the lamprey, an animal whose sex ratio is subject to change, we focus on the dependence of its sex ratio SR on local conditions. Through Nick et al.'s study of the habits of the lamprey, we can assume that the rate of larval stage development affects the sex ratio. Therefore, we constructed a sex ratio model to simulate the effect of food availability on larval stage development, expressed as follows:

$$SR = \frac{P_{male}}{P_{female}} \tag{1}$$

$$P(t) = \frac{1}{1+e^{-r((t-t_0))}} \tag{2}$$

Where $P(t)$ represents the probability that a lamprey will develop into a female, which directly determines sex ratio change. t_0 is the time when the sex ratio began to change. Since there is no literature demonstrating that the sex ratio of lamprey populations is affected by environmental temperature, we did not introduce temperature as a regulatory parameter.

We model the changing relationship between a gender-affirming procedure and food availability by setting the initial male ratio at 22% under low food availability conditions and 56% under high food availability conditions, based on the data from Nick et al. On this basis, the changes in the proportion of males are simulated and adjust according to food availability. Through a certain number of iterations, we get the results displayed in Figure 1.

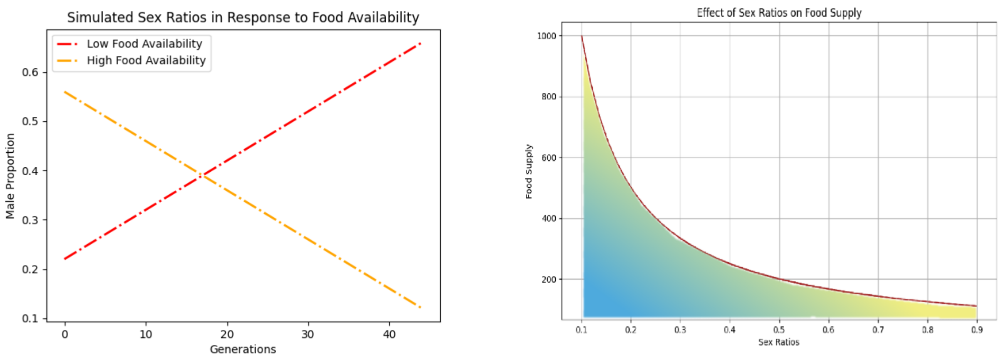

Figure 1: Sex ratios in response to food availability

Based on the results, we analyze that as the living environment becomes harsh, food availability decreases, and growing is restricted, the sex of the lamprey population is skewed toward males. When food is extremely scarce, severe sex differentiation may occur, and the presence of a large number of males may be beneficial to the acquisition of food, resulting in the survival of the fittest. When the sex ratio reaches balanced, the proportion of females rises, and there is an adequate supply of food for the lamprey, the chances of males lacking mates within the population are reduced. Females can utilize the abundant supply of energy for spawning.

2.2 Changes in population dynamics

In the natural state, resources are limited, and the rate of population growth will be influenced by the capacity of the environment. By reviewing the results of ecological research, we use the Verhulst model to describe the changes in the population of the lamprey [4], expressed as follows:

$$N = P_{male} + P_{female} \tag{3}$$

$$\frac{dN}{dt} = \gamma N\left(1 - \frac{N}{K}\right) - \alpha N^2 \tag{4}$$

Where K is a parameter in ecology that represents the environmental capacity, and its magnitude reflects the environmental situation. Compared with the traditional population dynamics model, we add

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α as a saturation parameter for population growth to fully take into account the limitations of resource allocation.

We adjusted the values of K and α to simulate the population dynamics of lampreys under two scenarios, which are sufficient resources and only half of the remaining resources. The results are shown in Figure 2. The population growth can be divided into three stages namely Initial Stage, Rapid Growth Stage as well as Stationary Stage. When the environmental capacity decreases, food availability decreases and the competition within the population increases. Although the growing condition in the initial stage is the same as that in the condition of high food availability, the population growth rate decreases significantly after a period. And the population development will be faster. However, after that, the population growth rate decreases significantly, and the population entered the Stationary Stage earlier, and the final population size is only about half of the high food availability condition.

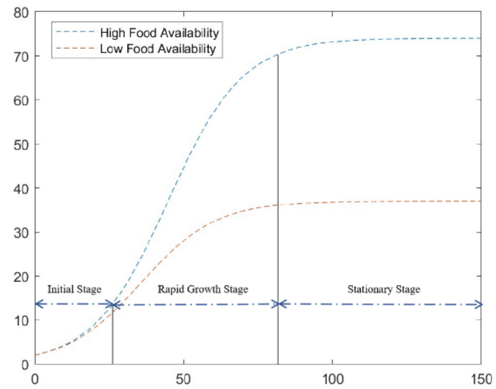

Figure 2: Population dynamics under different conditions

Based on the surrogate model constructed, we then simulate the interactions between the lamprey and other biological populations, with extra attention to their position in the food chain. By understanding the food chain associated with the lamprey, we find that due to its parasitic and blood-sucking nature, the number of natural enemies of the lamprey is very low in nature, except for anthropogenic fishing. Therefore, for the interactions between sea lampreys and other populations, we modify the respected Lotka-Volterra model, which is widely used to describe the dynamics of biological systems in which multiple populations interact, in which the lamprey serves as a predator and views the fish it feeds on as prey. We constructed the prey-predator model with a sex structure and consider that different sexes of predators with different predatory abilities on the predator sex ratio, expressed as follows:

$$\begin{cases} \dfrac{dP_{Male}}{dt} = b_1 \cdot P_{Female} - d_1 \cdot P_{Male} + k \cdot c_1 \cdot P_{Prey} \cdot P_{Male} \\ \dfrac{dP_{Female}}{dt} = b_2 \cdot P_{Female} - d_2 \cdot P_{Female} + k \cdot c_2 \cdot P_{Prey} \cdot P_{Female} \\ \dfrac{dP_{Prey}}{dt} = b_3 \cdot P_{Prey} - d_3 \cdot P_{Prey} - \varphi \cdot {P_{Prey}}^2 - c_1 \cdot P_{Prey} \cdot P_{Male} - c_2 \cdot P_{Prey} \cdot P_{Female} \end{cases} \tag{5}$$

Where $b_i(i=1,2,3)$ represents the birth rate of male predators, female predators and prey respectively, $d_i(i=1,2,3)$ represents the mortality rate of the three, respectively ($b_i > d_i$), and c_1、c_2 represents the predation capacity of male predators and female predators. In this model, we also consider the coefficient of competition within the prey population φ and the coefficient of conversion of the prey population into the predator population K. Its significance is to quantify how much of the prey can be efficiently utilized by the predators in the ecosystem, and thus sustain the survival and growth of the predators. It is worth noting that we specify the mortality rate in order to better apply the model to the lamprey population as:

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$$d_i = d_{nature} + d_{fishing} \tag{6}$$

Where d_{nature} represents natural mortality and $d_{fishing}$ represents mortality from anthropogenic fishing. All of the above parameters are non-negative constants.

To solve for the prey-predator system equilibrium point, we construct the following system of equations:

$$\begin{cases} b_1 \cdot P_{Female} - d_1 \cdot P_{Male} + k \cdot c \cdot P_{Prey} \cdot P_{Male} = 0 \\ b_2 \cdot P_{Female} - d_2 \cdot P_{Female} + k \cdot c \cdot P_{Prey} \cdot P_{Female} = 0 \\ b_3 \cdot P_{Prey} - d_3 \cdot P_{Prey} - \varphi \cdot {P_{Prey}}^2 - c \cdot P_{Prey} \cdot P_{Male} - c \cdot P_{Prey} \cdot P_{Female} = 0 \end{cases} \tag{7}$$

From this system of equations it is easy to solve that there exists a boundary equilibrium point $O(0,0,0)$, $E1\left(\frac{b_3 - d_3}{\varphi}, 0, 0\right)$, when $b_2 < d_2 < b_2 + d_1$ and $kc(b_3 - d_3) > \max\{d_1 \cdot \varphi, (d_2 - b_2) \cdot \varphi\}$,the system has two more equilibrium points $E_2\left(\frac{d_1}{kc}, \frac{kc(b_3 - d_3) - d_1\varphi}{kc^2}, 0\right)$, $E_3\left(\frac{d_2 - b_2}{kc}, \frac{kcb_1(b_3 - d_3) - b_1\varphi(d_2 - b_2)}{kc^2(b_1 + b_2 + d_1 - d_2)}, \frac{[kc(b_3 - d_3) - \varphi(d_2 - b_2)](d_1 + b_2 - d_2)}{kc^2(b_1 + b_2 + d_1 - d_2)}\right)$.

By collecting and analyzing the fishery observation data, we set the parameters of the model (birth rate and death rate of lamprey, etc.) reasonably. According to the established differential equations, the dynamics between populations are plotted as shown in Figure 3.

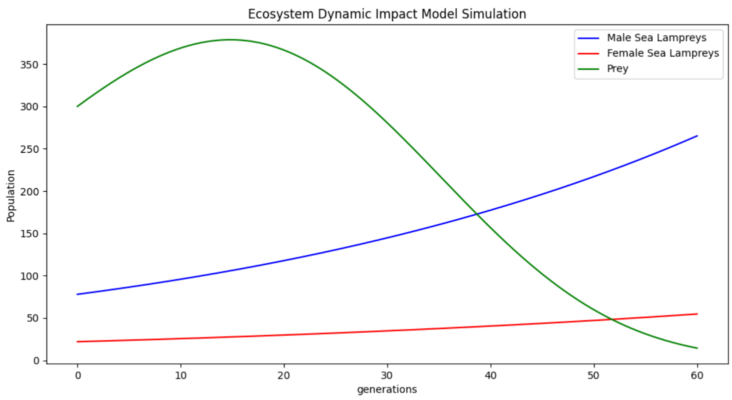

Figure 3: Ecosystem Dynamic Impact Model simulation

During the first 15 generations, when the sex ratio grows to approximately 3:1, the number of prey-predators undergoes a brief period of growth, and a peak appears. As the difference between the male and female ratios within the lamprey population continued to increase, the density of bait-feeding populations showed a long-term and rapid decline. And as the number of males in the population became more and more dominant, the prey tended to die out. Meanwhile, we find that the magnitude of change of female lampreys is significantly smaller than that of male lampreys, suggesting that male lampreys are more affected by the availability of resources.

Finally, in order to objectively larger ecosystems will be affected, we combined the above ecosystem indicators such as biodiversity, food chain stability, and the number and distribution of other species, and used a weighted score to assess ecosystem impacts with the following formula:

$$CI = \lambda_1 \cdot (P_{prey})^2 \cdot e^{-P_{prey}} + \lambda_2 \cdot RS + \lambda_3 \cdot \frac{1}{AP_{habitat}} \tag{8}$$

Where λ_1 represents the bait-feeding population density factor, λ_2 is the reproductive success

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factor, and the resource impact factor is λ_3. We determine the weighting matrix by asking experts and reviewing information:

$$\lambda_i = (0.35 \quad 0.3 \quad 0.35) \tag{9}$$

Based on Eq. (8) and Eq. (9), we plot the changes in ecosystem composite scores under the three scenarios of gender balance, male dominance, and female dominance by means of computer simulation, as shown in Figure 4.

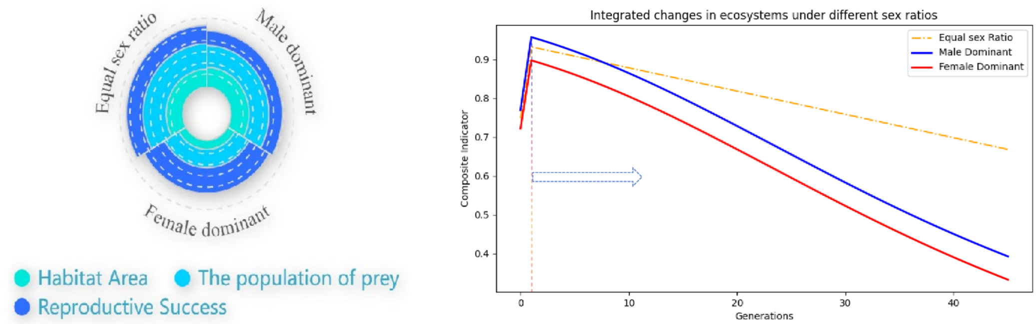

Figure 4: Integrated changes in ecosystem under different sex ratios

We select the simulation data of lampreys breeding to the 20th generation and plot Figure 4. When the population is balanced in terms of sex, the reproductive success rate will be dominant, and the habitat resources occupied will be more appropriate, which is more conducive to the survival of the population and has a positive significance for the maintenance of the stability of the food chain. As the lamprey population continues to reproduce, we find that the gender-balanced population has a higher overall impact score on the ecosystem, which means that it has fewer negative impacts on the ecosystem. For the lamprey population, the ecosystem impact of female dominance is the highest, and the ecosystem score is only half of that of the gender-balanced population, while the ecosystem impact of male dominance was small. It can be hypothesized that since female lampreys are tasked with spawning, they not only consume more food for their own functioning during the reproductive period, but also occupy the resources and eco-logical niches of other populations, which increases the factors that adversely affect the eco-system by influencing the upper level, as well as the sibling consumers and lower-level producers.

2.3 Discussion on population situation

For a population to be considered dominant within its ecosystem, it must demonstrate resilience to environmental fluctuations and external pressures, maintaining a stable composite index across varying conditions. This study delves into three secondary indicators—genetic diversity, environmental quality, and population size—to enhance the understanding of population stability. Dominant populations are characterized by their ability to adapt to diverse environmental settings, sustain high reproductive success with balanced sex ratios, and employ effective migratory strategies that contribute to overall population fitness.

Within the broader ecological context, each population occupies a distinct niche, contributing to the food chain and the maintenance of ecological balance and stability. This research utilizes the constructed ecosystem dynamics model to juxtapose lampreys against other species lacking the capability to adjust their sex ratios. This comparison focuses on aspects such as population stability, adaptability, and ecological roles, employing the Analytic Hierarchy Process (AHP) to objectively assess the strengths and weaknesses of the lamprey population.

AHP analysis involves several steps, starting with pairwise comparisons of key factors by a panel of experts to ascertain their relative importance. This is followed by the construction of a comparison matrix based on the established relationships among the factors. Finally, a consistency test is conducted using the matrix's eigenvalues and eigenvectors to ensure the reliability and validity of the comparisons made [5], allowing for a comprehensive evaluation of the lamprey population's ecological impact.

The weight calculation of AHP method (based on the square root method) shows that the weight of stability is 22.554%, the weight of ecological role is 67.381%, and the weight of adaptability is 10.065%, in which the largest characteristic root is 3.086. The corresponding RI value is 0.525 according to the

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RI table. The calculation yielded $CR = 0.082 \leq 0.1$, which passed the test of consistency.

We chose grass carp with a balanced sex ratio in nature as the comparative object, and based on the determined indicators and corresponding weights, combined with the collected number and distribution of the two species, we used computer simulation to iterate and attain the results as shown in Figure 5.

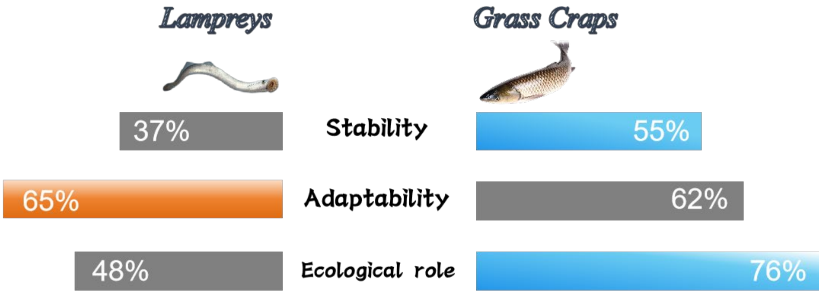

Figure 5: Comprehensive evaluation results of two species

The ability of lampreys to dynamically adjust their sex ratios in response to resource availability enhances their adaptability, allowing them to outcompete species like grass carp by approximately 3% in terms of environmental resilience. This mechanism not only facilitates their survival in adverse conditions but also plays a crucial role in controlling the population sizes of other species, thereby contributing to the maintenance of ecosystem stability. However, the benefits come with drawbacks; the variability in sex ratios can hinder genetic diversity by limiting mating opportunities for certain individuals, leading to reduced gene flow, and affecting the transfer of advantageous traits. This contributes to the lampreys' lower population stability compared to more genetically diverse species such as grass carp. The lampreys' dependency on external environmental conditions also heightens their vulnerability to habitat and climate changes, increasing their survival pressures. Additionally, the rapid shifts in lamprey populations can disrupt the adaptability of coexisting species, potentially destabilizing the ecosystem. The interaction between lampreys and other organisms introduces unpredictability, altering the food web dynamics and creating a disparity in ecological roles when compared to species with stable sex ratios like grass carp, thus underscoring the complex implications of lamprey population dynamics on ecological balance.

3. Ecological network model

3.1 Health status of marine ecosystem

There is a close relationship between the lampreys and the health of marine ecosystems, but this relationship is complex and multifaceted, involving the sea lamprey at different stages of its life cycle, its impact on other species, and the effects of environmental change. We believe that the health of marine ecosystems can be measured in terms of changes in parasite populations, dispersal capacity and ecosystem resistance to internal parasites and external pathogens.

(1) Parasites and disease transmission:

Changes in population size and reproductive success of lampreys, which serve as hosts for parasites, directly affect the means of transmission of parasites and diseases. We hypothesize the following relationship between changes in parasite abundance and population density of lampreys:

$$\frac{dP_{parasite}}{dt} = \beta N - \varepsilon \tag{10}$$

Where β represents the parasitism coefficient and ε is the environmental deterrent factor.

(2) Ecosystem immunity:

The spread of parasites and diseases may affect other organisms through the lamprey as an intermediary. This may include other fish, aquatic insects, etc. that share the same waters and resources as the lamprey.

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$$\rho_{immunity} = -kP_{parasite} + \theta \tag{11}$$

We construct an equation for the change in ecosystem immunity, where $\rho_{immunity}$ represents ecosystem immunity, k denotes the transmission coefficient of the parasite in the ecosystem, and θ summarizes the effects of other factors in the ecosystem.

By simulating changes in the marine ecosystem, the results are shown in Figure 6. We suggest that the proportion of females within the lamprey population is appropriately elevated and ecosystem immunity increases. Analogous to the human immune system, it can be hypothesized that the presence of lampreys may alter the host selection and transmission range of the parasite, and that their over- or under-population may lead to a weakened or over-activated immune system, which is detrimental to the system as a whole.

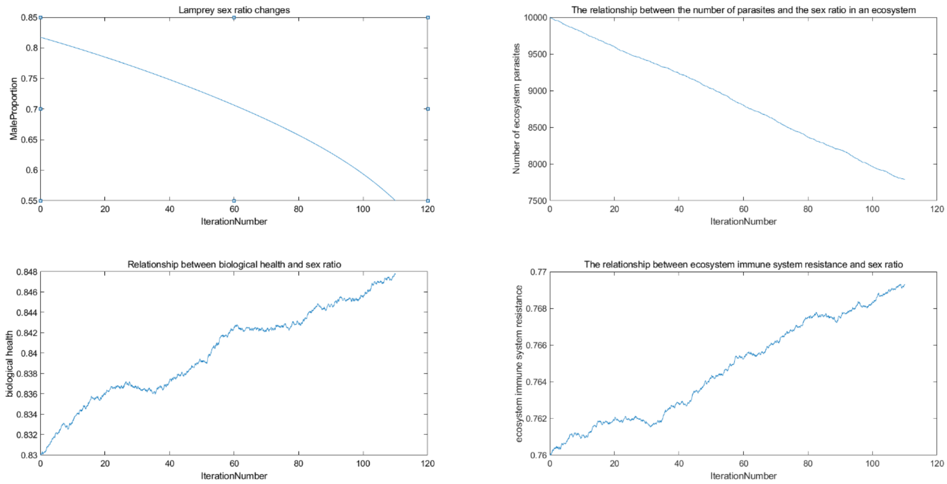

Figure 6: Relationship between ecosystem health status and sex ratio changes

3.2 The impact on human

Lampreys are parasitic in some stages, where they parasitize other fish to feed on their blood. This may have a negative impact on some economically important fishery target fish by reducing their abundance and quality. Adult lampreys may compete with other fishery target fish for food resources when feeding, leading to increased competition for these resources [6], which can affect the productivity and sustainability of fisheries.

At the same time, lampreys are a source of food in some parts of the world, and because of their high protein and vitamin content, they are not only avidly consumed by ancient European aristocrats, but also fished by indigenous peoples in some seas nowadays.

In order to calculate the impact of sea lampreys on humans, we consider the economic output and get the following formula:

$$Output = \eta\left(N_{(t)} \cdot Q_{1(t)} - \Delta P_{prey} \cdot Q_{2(t)}\right) - Cost \tag{12}$$

We consider the economic benefit of lampreys to be the income earned from catching lampreys minus the economic loss due to their predation on other income-generating stocks, where η is the human fishing intensity coefficient per unit area, $Q_{1(t)}$ is the market price of lampreys, $Q_{2(t)}$ is the market price of prey, and ΔP_{prey} is the amount of prey that lampreys consume, and we have also taken into account the cost of fishing at sea.

Combining fishery data and simulation results in Figure 7, we find that when the male ratio de-creased, the economic income of humans increased. Comparing with previous analysis, we hypothesize that this phenomenon occurs because female lampreys are a key factor in population reproduction. And when population reproduction increases, the probability of humans catching lampreys per unit area increases.

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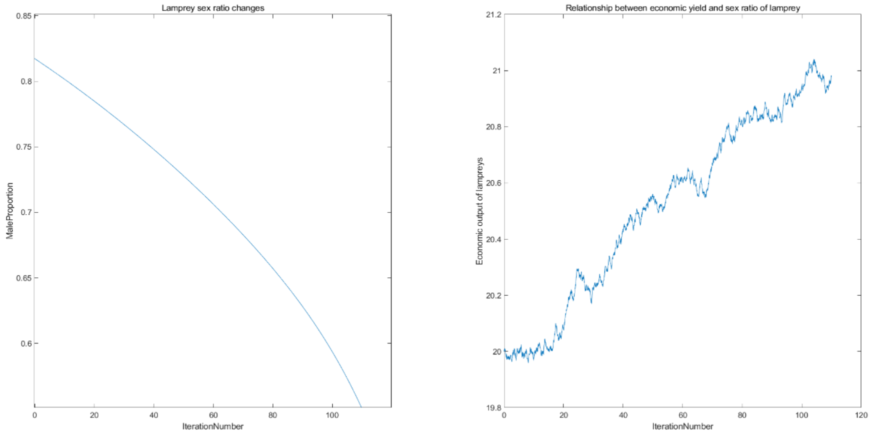

Figure 7: The impact of sex ratio changes on human

4. Conclusions

In this study, we present a dynamic model incorporating species diversity, food availability, growth rate, sex ratio, and ecosystem stability, which provides a comprehensive framework for understanding the effects of changing sex ratios on ecosystems. First, we constructed a dynamic sex-induced ecosystem model, and found that when the sex ratio of lampreys s changes, they take up additional resources and increase competition. Through the food chain, the adverse effects transmitted to the ecosystem will increase. Then, based on the established ecosystem dynamics model, the AHP method was used to compare the sex ratio equilibrium of the lamprey's population with the grass carp population. It was found that the positive regulation mechanism of sex ratio favors population reproduction. Finally, an ecological network model was constructed to explore the effects of changes in the sex ratio of lampreys s on the stability of marine ecosystems. We found that too many or too few of their populations may lead to a weakened or over-activated immune system, which is detrimental to the whole ecosystem. The study in this paper provides theoretical support for practical ecological management and conservation, and avenues for future research and development of ecological modeling and management strategies.

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