

Maximum output power design of wave energy based on particle swarm optimization algorithm

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Abstract: As human exploration of the ocean continues to deepen, wave energy as an important marine resource has entered people's view. In order to explore the basic relationship between the relevant variables in the wave energy conversion device, this paper starts from the force analysis of the system and establishes the mathematical model of the kinematic equations of float and oscillator to solve the corresponding problems. Not only the vertical swing of the float and oscillator, but also their longitudinal rocking, i.e., both translational and rotational, are considered. By numerical integration, the average output power is calculated for a certain time period. With the objective of maximizing the average output power, the particle swarm optimization algorithm is applied to obtain the maximum average output power with relatively smooth power.

Keywords: wave energy conversion device; floating body motion; particle swarm algorithm

1. Introduction

In recent years, as the contradiction between the depletion of fossil energy and the large amount of energy needed for economic development has become more and more prominent, as well as the excessive environmental degradation caused by excessive energy consumption, people have to turn to the marine sector to find new renewable energy sources [1]. According to the "Ocean Energy Development" published by UNESCO, the theoretical renewable power of various marine energy sources worldwide is about 7.66×10^{10} kW, among which the actual exploitable amount of wave energy is the largest in order of magnitude, 3×10^8 kW [2].

Wave energy comes from the wind generated by the sun heating the air, and its energy density is much greater than that of wind and solar energy [3]. At the same time, wave energy is consistent with the geographical distribution of seawater, so it can be used to power desalination plants to effectively alleviate the shortage of freshwater resources in coastal areas, and the conversion of wave energy can be done at a lower cost. According to the International Energy Agency (IEA), the global wave energy available will eventually provide more than 10% of the world's current electricity supply [4].

In this paper, the motion of floats and oscillators is modeled by considering the case of floats with only vertical oscillation and longitudinal rocking motion. A mathematical model is developed to solve for the maximum output power and the optimal damping coefficients of the linear and rotating dampers with constant damping coefficients for both the linear and rotating dampers.

2. Assumptions

Use the following assumptions.

- 1) Assume the mass and various friction of the center shaft, base, compartment and PTO can be neglected.
- 2) Assume the float is a rigid body and ignore its elastic deformation.
- 3) Assume that the wave force acting on the float is a regular wave, without considering the nonlinear effect.
- 4) Assume that the float does micro-amplitude oscillation in the regular wave, and the force is simplified to a linear problem.
- 5) When the longitudinal oscillation, only the vertical oscillation motion of the rotating axis is considered, and its horizontal motion is not considered.

3. Model construction and solving

3.1 Model building

The float and the oscillator are subjected to oscillating motion under the action of waves. Consider the vertical oscillation motion and the longitudinal rocking motion. Therefore, in this paper, we need to consider both x-axis and z-axis directions, and build a world coordinate system in the xOz plane with the point where the rotation axis is located at the initial time in the hydrostatic state as the origin of the coordinate system, in which the vertical upward direction is the positive direction of z-axis and the horizontal rightward direction is the positive direction of x-axis. Meanwhile, the float coordinate system x_f of z_f is constructed with the rotation axis as the origin and the center axis as the z-axis, which will move with the oscillating motion of the float[5].

In this paper, use the Newton-Euler equation (moment of moment theorem and Newton's translational equation of motion) to analyze the motion of the float and the spring, and use the pendular and angular displacements of the rotating axis, and the extension and contraction of the spring as the unknown. The kinetic equations are constructed using the pendular and angular displacements of the rotating shaft and the extension and angular displacements of the spring as unknown quantities.

Theorem of moment of momentum:

The theorem of moment of momentum is one of the general theorems of dynamics. It gives the relationship between the moment of momentum of the particle system and the impulse moment of the particle system under mechanical action. When a rigid body rotates around a certain point at an angular velocity Ω , its motion equation is,

$$\frac{dJ}{dt} = M \quad (1)$$

Where, M is the principal moment of each external force to the fixed point, and J is the moment of momentum when the rigid body rotates around the fixed point, which can be expressed as,

$$\begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \begin{bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{bmatrix} \quad (2)$$

Where, x, y, z are the three-dimensional coordinates of the coordinate points respectively, each component J_x , J_y , and J_z of the momentum moment vector J is a function of time, the three components Ω_x , Ω_y , and Ω_z of the angular velocity Ω are also functions of time, and I_{xx} , I_{yy} , and I_{zz} are inertia coefficients, which are all described. At the same time, since the rotation axis of this question is in the y-axis direction, the angular velocity in only one direction of this question is $\Omega_y = \dot{\theta}$, while the angular velocity in both the x-axis and z-axis directions is 0, that is, $\Omega_x = \Omega_z = 0$. Therefore, the momentum moment components J_x and J_z are both 0, then,

$$\begin{cases} J_x = -I_{xy} = 0 \\ J_y = I_{yy}\Omega_y \\ J_z = -I_{zy} = 0 \end{cases} \quad (3)$$

The calculation formula of inertial coefficient is,

$$\begin{cases} I_{xx} = \int (y^2 + z^2) dm \\ I_{yy} = \int (z^2 + x^2) dm \\ I_{zz} = \int (x^2 + y^2) dm \\ I_{yz} = I_{zy} = \int yz dm \\ I_{zx} = I_{xz} = \int zx dm \\ I_{xy} = I_{yx} = \int xy dm \end{cases} \quad (4)$$

Where dm is the mass element. Since the float consists of a cylindrical shell with uniform mass distribution, it is $dm = \sigma_1 dS \left(\sigma_1 = \frac{m_1}{S_1} \right)$ for the float, and S_1 is the surface area of the float. If the

vibrator is a cylinder with uniform mass distribution, it has $dm = \sigma_2 dV \left(\sigma_2 = \frac{m_2}{V_2} \right)$, and V_2 is the volume of the vibrator.

In the two-dimensional plane xOz , assume that the pendulum displacement of the rotating axis is x_3 , the angular displacement of the float is θ_1 , the expansion length of the oscillator is η , and the angular displacement of the oscillator is θ_2 . The angular velocity is expressed in terms of the first-order derivatives of the angular displacement θ_1 and θ_2 , and the angular acceleration is expressed in terms of the second-order derivatives of the angular displacement θ_1 and θ_2 . In this problem, the oscillatory motion of the float and the oscillator is considered as the rotation of the two-dimensional plane by the fixed point, which is the axis of rotation. Also, a single angle θ is defined as the angular displacement of the float and oscillator, and "positive angle means counterclockwise rotation". Assuming that a point $P(x, z)$ in the base coordinate system xOz is known, and the coordinates of this point are rotated by θ from the origin O , the value of the point P in the new coordinate system $x'O'z'$ is (x', z') .

The coordinates of the two coordinate systems are converted as follows,

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \mathbf{R} \begin{bmatrix} x \\ y \end{bmatrix} \quad (5)$$

Where, \mathbf{R} is the coordinate rotation matrix, and its inverse matrix is,

$$\mathbf{R}^{-1} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \quad (6)$$

The plane motion of a rigid body can be determined by the base point velocity \vec{v}_c and the angular velocity $\vec{\Omega}$ of rotation around the base point, so the velocity \vec{v} of the center of mass can be expressed as,

$$\vec{v} = \vec{v}_c + \vec{\Omega} \times \vec{r} \quad (7)$$

Where, \vec{r} is the position vector from the center of mass to the base point, then the acceleration of the center of mass \vec{a} is,

$$\vec{a} = \vec{a}_c + \frac{d\vec{\Omega}}{dt} \times \vec{r} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) \quad (8)$$

Where, \vec{a}_c is the shaft acceleration. Since the angular velocity Ω in this question is only the

component in the y-axis direction, and $\vec{\Omega} \times (\vec{\Omega} \times \vec{r}) = -\Omega^2 \vec{r}$, at the same time, the rotation axis acceleration \vec{a}_c is only the component in the middle axis direction, so the component of the acceleration \vec{a} is,

$$a_m = a_c + \left(\frac{d\vec{\Omega}}{dt} \times \vec{r} - \Omega^2 \vec{r} \right) \cdot \vec{e}_z \quad (9)$$

Where, \vec{e}_z is the unit direction vector in the middle axis direction. In consideration of the fact that the swing angle of the float will not be too large, so the change of the angular velocity will not be large, that is, the angular acceleration $\frac{d\vec{\Omega}}{dt} \times \vec{r}$ can be ignored, and the above equation can be simplified as,

$$a_m = a_c - \Omega^2 r \quad (10)$$

The float is subjected to gravitational force G1, hydrostatic recovery force F1, additional inertia force F2, spring force F3, linear damper force F4, pendulum wave damping force F5, and wave excitation force F6. In addition to the above seven forces in the vertical direction, the float is also subjected to longitudinal rocking excitation moment M1, hydrostatic recovery moment M2, longitudinal wave damping moment M3, torsional spring torque M4

Rotation damper torque M5, weight moment M6, buoyancy moment M7, additional longitudinal moment of inertia M8.

Since the spring constrains the oscillator and the float, i.e., the acceleration of the spring should be the relative acceleration of the float and the oscillator, the force analysis of the oscillator is also required to find the acceleration of the oscillator. In this problem, the oscillator is subjected to its own gravity G2, the spring force F7 and the linear damper force F8. Since the motion of the oscillator is along the central axis, it is necessary to decompose the various forces on the oscillator into the direction along the central axis and the direction perpendicular to the central axis. The component in the vertical neutral axis direction does not contribute to the axial motion, so only the above forces along the neutral axis need to be considered here direction.

The float and the oscillator are subjected to different degrees of vertical and vertical oscillations, i.e., translational and rotational, under the action of forces and moments, so the equations of motion for the float and the oscillator can be constructed using the Newton-Euler equations for translational motion and rotation, respectively.

The equations of motion of the float are as follows

$$\ddot{z} = \frac{1}{m_1} \vec{F}_z - \left(\frac{d\vec{\Omega}}{dt} \times \vec{r} - \Omega^2 \vec{r} \right) \cdot \vec{e}_z = \frac{1}{m_1} \vec{F}_z - \Omega^2 r \quad (11)$$

The kinematic equation of the spring is as follows

$$\begin{cases} \ddot{\theta}_1 = \sum_{i=1}^8 M_i / I_{yy}; \\ \ddot{\eta} = \ddot{z} \cos \theta_2 - \frac{1}{m_2} \sum \vec{F}_\eta; \\ \ddot{\theta}_2 = -(M_4 - M_5 + M_6) / I'_{yy} \end{cases} \quad (12)$$

3.2 Model solving

The float and the oscillator are analyzed separately, and the dip displacement and velocity and the longitudinal rocking displacement and angular velocity of the float and the oscillator are solved by using

the Newton-Euler equation for the first 40 wave cycles with a time interval of 0.2s.

The plumbing displacement, plumbing velocity, longitudinal rocking angle, longitudinal rocking angular velocity and time variation of the float and oscillator are shown in Figure 1, 2, 3 and 4, respectively. It can be seen that the amplitude of the pendular displacement stabilizes after 150s; the pendular velocity stabilizes after 140s; the amplitude of the longitudinal rocking angle stabilizes after 140s; the amplitude of the longitudinal rocking velocity stabilizes after 120s. The amplitude of float pendular displacement gradually tends to 0.4192m, the amplitude of pendular velocity gradually tends to 0.6888m/s, the amplitude of longitudinal rocking angle displacement gradually tends to 22.67, the amplitude of longitudinal rocking angle velocity gradually tends to 0.2288rad/s; the amplitude of oscillator pendular displacement gradually tends to 0.0531m, the amplitude of pendular velocity gradually tends to 0.09029m/s, the amplitude of longitudinal rocking angle displacement gradually tends to 0.09029m/s. The amplitude of the angular displacement gradually tends to 23.57° and the amplitude of the angular velocity gradually tends to 0.3484rad/s. As shown in Table 1 and Table 2.

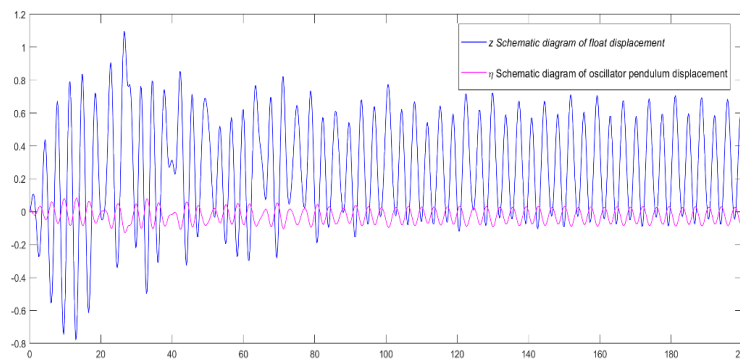


Figure 1: Schematic diagram of float and oscillator pendulum displacement

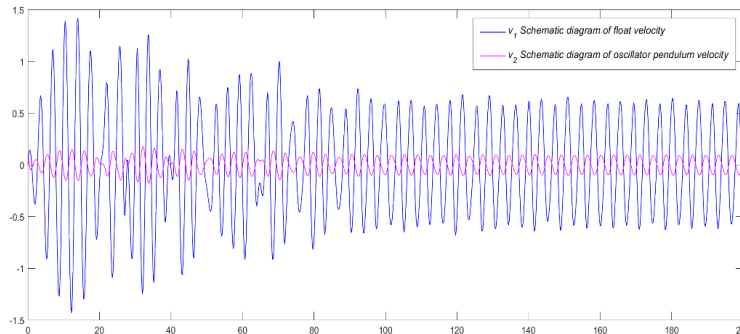


Figure 2: Schematic diagram of float and oscillator pendulum velocity

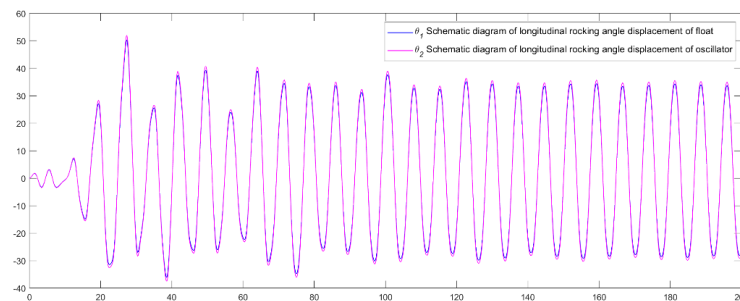


Figure 3: Schematic diagram of longitudinal rocking angle displacement of float and oscillator

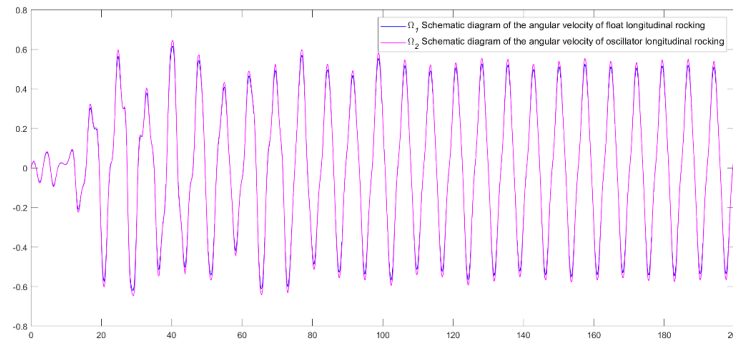


Figure 4: Schematic diagram of the angular velocity of float and oscillator longitudinal rocking

Table 1: Displacement and Velocity Results of Float Oscillation

Time (s)	Heave displacement (m)	Heave velocity (m/s)	Pitch angular displacement	Pitch angular velocity (s ⁻¹)
10	-0.526577219	0.968970402	-0.011419922	0.018002182
20	-0.021581087	-0.19712065	0.372440555	-0.349539779
40	0.312596782	-0.025245365	-0.084709219	0.605374133
60	0.620356944	-0.169581041	-0.376771873	-0.056137328
100	0.594383562	0.528150633	0.578934446	0.246903436

Table 2: Table of Displacement and Velocity Results of Oscillator Oscillation

Time (s)	Heave displacement (m)	Heave velocity (m/s)	Pitch angular displacement	Pitch angular velocity (s ⁻¹)
10	0.070260567	-0.068349296	-0.012099974	0.018767137
20	-0.001087856	0.022430691	0.391767627	-0.36403567
40	-0.01599492	0.011273509	-0.089129748	0.634357215
60	-0.079267647	-0.024266279	-0.39067653	-0.059035185
100	-0.05310617	-0.093561692	0.602147846	0.243702996

The displacements x_1 and x_2 of the float and oscillator are known with respect to time t and the angular displacements θ_1 , θ_2 of the rotating shaft and spring with respect to time t . Then the average power equation is:

$$P = \frac{\int_0^T F_5 \cdot |x_1 - x_2| dt + \int_0^T M_5 \cdot \dot{\theta}_1 - \dot{\theta}_2 \cdot dt}{T} \quad (13)$$

In order to maximize the average output power of the PTO system, the objective function is set as:

$$\max P = \max \frac{\int_0^T F_5 \cdot |x_1 - x_2| dt + \int_0^T M_5 \cdot \dot{\theta}_1 - \dot{\theta}_2 \cdot dt}{T} \quad (14)$$

Using the particle swarm optimization algorithm, the maximum average output power with relatively smooth power in [150s,200s] is calculated as 61.8706w, and the optimal damping coefficient is 36391 for the linear damper and 95992 for the rotary damper.

4. Conclusion

The motion model of float and oscillator is established by considering the case where the float only does vertical oscillation and longitudinal rocking motion. In the case that the damping coefficients of both linear and rotating dampers are constant, a mathematical model is established to solve for the maximum output power and the optimal damping coefficients of both linear and rotating dampers.

In this paper, the physical definitions and practical situations are fully integrated in the discrimination of each acting force and force direction. In modeling, different scenarios of the model are fully considered in this paper so that the floating body can give better prediction results in the case of dangling motion

and swinging motion.

References

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