ISSN 2616-5902 Vol. 3, Issue 2: 74-85, DOI: 10.25236/AJBM.2021.030214

The Effects of Biased Technical Progress with Endogenous Growth on Factor Shares

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Abstract: *Based on the CES model by Arrow et al., this paper incorporates the hypothesis of induced innovation, takes household and firm behavior as subjects, and analyzes the effects of biased technical progress on factor shares with endogenous growth. The results show that, if substitution elasticity s more than 1, the factor-augmenting model generates endogenous growth; if saving rate is constant, and technical progress s Harrod neutral, there is a saddle path, capital share remains constant and technical progress is biased to capital; if savings rate is not a constant, nor technical progress Harrod neutral, then there is still a saddle path, but equilibrium point is half equilibrium point, and the bias turns towards labor. It is also found that when innovation possibility frontier is symmetric, capital share is less than 1/2, and long run factor shares are consistent with the theory.*

Keywords: *Factor-Augmenting Model, Endogenous Growth, Technical Progress Bias, Factor Shares*

1. Introduction

The distribution of factor shares is an issue worth noting in primary distribution, however in the past decades, it has been ignored by classical economists mainly because long run factor shares should remain constant [1], long run substitution elasticity between capital and labor equals 1 and factor shares don't change[2]. In reality, capital share tends to grow in developed countries with various degrees [3][4][5]; China as a developing country, its long run capital share also tends to grow [6][7][8][9]; this trend is especially obvious after 2000 for developing nations [10], and attracts attention from the academia. Many scholars represented by Acemoglu, after correcting neo-classical assumptions, pointing out that technical progress bias is a significant perspective for explaining factor shares change. Acemogluproved in many papers that if capital and labor are complementary (substitution elasticity is less than 1), capital-augmenting technical progress reduces capital utilization, but increases capital share, reduces labor share; while labor-augmenting progress leads to unchanging factor shares in the long term[11][12][13].

Regrettably, the current literature on the effects almost all uses neo-classical framework, ignoring two important problems, namely, the effects of technical progress on factor shares with endogenous growth and the long run dynamic changes in technical progress and factor shares. By using the CES model , Arrow et al. correct the definition of diminishing capital revenue, and assumed that capital stock approaches infinity and that marginal product of capital have a lower boundary (Inada condition fails), then this model generated endogenous growth (it is related with factor substitution elasticity)[14]. The induced innovation hypothesis by Kennedy was then introduced based on the factor-augmenting model, combined with conditions for endogenous growth, the relationship between capital and labor was fixed (substitutable, totally substitutable or complementary), then it can be explained that how technical bias effects the direction of factor shares[15]. Besides, the factor-augmenting model with the induced innovation hypothesis can be used to study the transfer dynamics between capital share and capital growth speed. Therefore, the model could analyze the effects of technical bias on factor shares well.

Next, this paper will use the factor-augmenting model with the induced innovation hypothesis to study the effects of technical progress on factor shares. The following is structured as: Part Two from household and firm behavior studies the conditions for endogenous growth, and analyzes the transfer dynamics between the average product of capital at market equilibrium and consumption-capital stock ratio; based on innovation possibility frontier assumption, Part Three examines the transfer dynamics

ISSN 2616-5902 Vol. 3, Issue 2: 74-85, DOI: 10.25236/AJBM.2021.030214

between capital share and capital growth speed; the final part is the conclusion

2. Technical bias and endogenous growth

Households and firms are the entities of economic activities. Firms provide products to serve the society, pay wage to labor and rent to capital; households offer labor for wage, take interests from assets, purchase products for consumption and accumulate assets for savings. Therefore, the endogenous growth model is constructed through studying household and firm behavior.

2.1 Household behavior analysis

Assuming the utility function of each household is:

$$U=\int_0^\infty (\frac{c^{1-\theta}-1}{1-\theta})e^{-(\rho-n)t} \tag{1}$$

in the equation, c is per capita consumption for adults, ρ time preference rate for utility, $\rho>0$, a positive ρ means that the later to obtain utility, the lower the value; $\theta>0$, its derivation was the substitution elasticity of any two consumption goods; n denotes the growth rate of the expanding household scale expected by current period adults. Certainly, household consumption was restrained by household income:

$$da/dt=(r-n)a+w-c \tag{2}$$

where, a is per capita assets, r interest rate, w wage rate. This function denotes that per capita assets increase with per capita income $(r-n)a+w$, and decreases with per capita consumption c. To prevent households from borrowing endlessly, households are restricted by the borrowing condition:

$$\lim_{t\to\infty}\{a(t)\cdot\exp[-\int_0^\infty[r(v)-n]dv]\}\geq 0 \tag{3}$$

Utility function optimization satisfies the following Euler equation: $\dot{c}/c=(1/\theta)\cdot(r-\rho)$ 。 And transversality condition is:

$$\lim_{t\to\infty}\{a(t)\cdot\exp[-\int_0^\infty[r(v)-n]dv]\}=0 \tag{4}$$

2.2 Firm behavior analysis

This paper adopts the production function with capital-augmenting and labor-augmenting technical progress:

$$Y=F(A_tK_t,B_tL_t) \tag{5}$$

Assuming the factor-augmenting CES model is:

$$Y=[a(AK)^\varepsilon+(1-a)(BL)^\varepsilon]^{\frac{1}{\varepsilon}} \tag{6}$$

among which, A is capital-augmenting technical progress index, B labor-augmenting technical progress index; K is capital, L labor, Y output; $a\in(0,1)$ denotes the distribution parameter of the importance of these two factors, $\varepsilon=[(\sigma-1)/\sigma]$ the substitution parameter of the inter-substitutability of these two factors, the substitution elasticity between capital and labor is $1/(1-\varepsilon)=\sigma$.

Let (11) be divided by BL, we obtain the following per capita expression (where, $k=K/(BL)$):

$$y=[a(Ak)^\varepsilon+(1-a)]^{1/\varepsilon} \tag{7}$$

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Marginal product and average product of capital are respectively:

$$f'(k) = aA^{\varepsilon}[aA^{\varepsilon} + (1-a)k^{-\varepsilon}]^{\frac{1-\varepsilon}{\varepsilon}} \tag{8}$$

$$f(k)/k = [aA^{\varepsilon} + (1-a)k^{-\varepsilon}]^{1/\varepsilon} \tag{9}$$

From (8)and (9), we get $f'(k)$, $f(k)/k$ are both positive, and all ε are decreasing with k.

This paper considers the case of $0<\varepsilon<1$($\sigma>1$), namely, there is a high substitutability between L and K. The limits of capital marginal product and average product are respectively:

$$\lim_{\hat{k}\to\infty} f'(k) = \lim_{\hat{k}\to\infty}[f(k)/k] = Aa^{1/\varepsilon} > 0 \tag{10}$$

$$\lim_{\hat{k}\to 0} f'(k) = \lim_{\hat{k}\to 0}[f(k)/k] = \infty \tag{11}$$

Therefore, when k approaches infinity, marginal product and average product both approach positive constants, not zero, the key Inada condition fails, this factor-augmenting CES model generates endogenous growth. But when $\varepsilon<0$ ($\sigma<1$), there is a low substitutability between L and K, capital marginal product and average product have the following limits

$$\lim_{\hat{k}\to\infty} f'(k) = \lim_{\hat{k}\to\infty}[f(k)/k] = 0 \text{ , } \lim_{\hat{k}\to 0} f'(k) = \lim_{\hat{k}\to 0}[f(k)/k] = Aa^{1/\varepsilon} < \infty$$

Now, the key Inada condition is met, this factor-augmenting CES model doesn't generate endogenous growth..

Therefore, the first key conclusion of this paper is that when factor substitution elasticity $\sigma>1$, the CES model has endogenous growth. With this, we move to the transfer dynamics analysis in market equilibrium study.

Profit maximization for firms requires capital marginal product equal to rent price $R=r+\delta$, here, marginal product is a constant $Aa^{1/\varepsilon}$, that is,

$$f'(k) = r + \delta = Aa^{1/\varepsilon} \tag{12}$$

Assume $A(t)=e^{x_1 t}$, $B(t)=e^{x_2 t}$ (where $x_1 \ge 0$, $x_2 \ge 0$, and both are constants). Because wage w equals labor marginal product, k value must meet (12):

$$[f'(k) - k \cdot f(k)]e^{(x_2-x_1)t} = w \tag{13}$$

The equation guarantees that whichever value BL takes, firms' profit is zero.

2.3 Market equilibrium

Assuming the economy was a closed one, then there was $a=k_0=K/L$ .Inserting $a=k_0$, $k=k_0 e^{(x_2-x_1)t}$ and (12) (13)into (2), we get:

$$\dot{k}/k = f(k)/k - c/k - (x_2 - x_1 + n + \delta) \tag{14}$$

$$\dot{c}/c = 1/\theta[f'(k) - \delta - \rho - \theta(x_2 - x_1)] \tag{15}$$

among which, transversality condition becomes:

$$\lim_{t\to\infty}\{k \cdot \exp(-\int_0^t [f'(k) - \delta - (x_2 - x_1) - n]dv)\} = 0 \tag{16}$$

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When $k \to \infty$, the limit of $f'(\hat{k})$ is $Aa^{1/\varepsilon}$, so, model (6) has endogenous growth, steady state growth rates of c, k, y are determined by:

$$\gamma^* = 1/\theta[Aa^{1/\varepsilon} - \delta - \rho - \theta(x_2 - x_1)] \tag{17}$$

When $Aa^{\frac{1}{\varepsilon}} > \rho + \delta + \theta(x_2 - x_1)$, $\gamma^* > 0$ (assuming $\rho > n$, it means $Aa^{\frac{1}{\varepsilon}} > \rho + n + \theta(x_2 - x_1)$, if not, when $\hat{c}$ remains constant, the utility would be boundless).

2.4 Transfer dynamics analysis

To seek the transfer dynamics of model (6) through establishing a phase diagram of (k,c) space is futile, because when $\gamma^* > 0$, k and c will always grow. In order to use phase diagram, we need variables constant in the steady state by transformation. Examine the evolution of average product of capital (marked as $z = f(k)/k$) and consumption-capital stock ratio (marked as $\chi = c/k$). Note, z is a state variable, χ is a control variable. Different from k and c, z and χ in the steady state approach constants. Using (14) and (15), taking the derivation of z and χ, after some algebra treatment, we get the dynamic equations of z and χ, expressed as:

$$\dot{z}/z = (z/D)^{-\varepsilon}[z - \chi - (x_2 - x_1) - n - \delta] \tag{18}$$

$$\dot{\chi}/\chi = (D/\theta)[(z/D)^{-\varepsilon} - 1] - (z - D) + (\chi - \psi) \tag{19}$$

where, $D = Aa^{1/\varepsilon}$, $\psi = (D-\delta)(\theta-1)/\theta + \rho/\theta - n$. Because $f(k)/k$ can't be lower than D, analysis here is fit for $z \geq D$. Steady state values are $z^* = D$, $\chi^* = \psi$.

where, $D = Aa^{1/\varepsilon}$, $\psi = (D-\delta)(\theta-1)/\theta + \rho/\theta - n$. Because $f(k)/k$ can't be lower than D, analysis here is fit for $z \geq D$. Steady state values are $z^* = D$, $\chi^* = \psi$.

To analyze the dynamic paths of model (6), we construct the phase diagram of space (z, χ), illustrated in Figure 1, equations $\dot{z} = 0$ and $\dot{\chi} = 0$ divide the first quadrant into four areas, marked as I, II, III and IV. If $\theta > 1 - \varepsilon$, in areas I and IV, average product of capital z is less than z^*. $z = z^* = D$ is corresponding to the condition $\dot{z} = 0$, therefore, when $z < D$ and $\chi < D - (x_2 - x_1) - n - \delta$, $\dot{z} > 0$. Thereby, in areas I and IV, average product of capital increases. In contrast, in areas II and III, average product of capital decreases. In areas III and IV, consumption-capital ratio χ is larger than that at the curve $\dot{\chi} = 0$, therefore, the ratio decreases. But in areas I and II, the ratio increases.

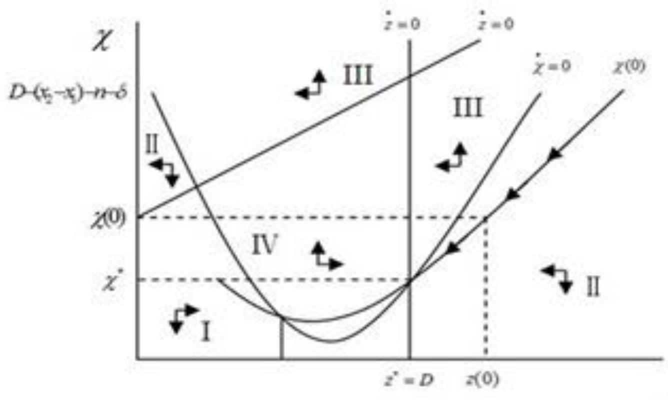

Figure. 1 The transfer dynamics in the CES model ($0 < \varepsilon < 1$, $\theta > 1 - \varepsilon$)

Now, think about the transfer dynamics from the initial position $z(0) > D$. In Figure 1, along the initial value $\chi(0)$, z and χ monotonically decrease in the transfer process. The image of $\dot{\chi}=0$ is determined by $\chi=\psi+(z-D)-(D/\theta)[(z/D)^{-\varepsilon}-1]$. When z value is in the low position, this curve slopes downward, and reaches the minimum at $z=D\cdot[(1-\varepsilon)/\theta]^{\frac{1}{\varepsilon}}$. When $\theta>1-\varepsilon$ and z monotonically decreases, the minimum value is at the left of D; when $\theta<1-\varepsilon$, the opposite happens. Because when $0<\varepsilon<1$ and $\theta\geq 1$, this condition holds. When z approaches infinity, the slope of the image $\dot{\chi}=0$ approaches 1. The curve crosses $z=D$ under $D-(x_2-x_1)-n-\delta$.

Therefore, when the substitution elasticity between capital and labor $\sigma>1$, if effective per capita capital k approaches infinity, marginal product and average product both approach positive constants, then the CES production function generates endogenous growth. Within the framework, the CES model characterizes a long run growth, and convergence in the transfer process, as follows: with the increasing effective per capita capital k, average product of capital z approaches D, while effective per capita capital growth rate $\dot{k}/k$ approaches zero. It is worth noting that for any substitution elasticity σ, with the increase of k, $\dot{k}/k$ approaches zero.

Capital share of model (11) is controlled by the following:

$$k\cdot f'(k)\big/f(k)=\theta A^{\varepsilon}\big/(\theta A+(1-\theta)k^{-\varepsilon})=\theta A^{\varepsilon}\big/(\theta A^{\varepsilon}+(1-\theta)B^{\varepsilon}k_0{}^{-\varepsilon})$$

in which, $k_0=K/L$. For any substitution elasticity σ, if $A=0$, then the above equation equals zero; if $B=0$, In the case of factor substitution elasticity $\sigma>1$ ($0<\varepsilon<1$), if $A>0$ and $B>0$, then with the increasing per capita k_0, capital share approaches 1, labor share reduces to 0. Therefore, in the case of factor substitution elasticity $\sigma>1$, labor-augmenting function generates endogenous growth, and it is important to study the effects of technical bias on factor shares. Next, consider these effects when $\sigma>1$.

3. The effects of technical bias on factor shares with endogenous growth

3.1 Technical progress speed, direction and innovation possibility frontier

Technical progress characterizes its speed and direction through factor shares analysis. Using model (5), according to Diamond[20], technical speed S and direction D are defined as:

$$S=F_t/F=\frac{K\,\partial F_K/\partial t+L\,\partial F_L/\partial t}{KF_K+LF_L},\ \ D=\frac{\partial(F_K/F_L)}{\partial t}\bigg/\frac{F_K}{F_L}=\frac{\dot{F}_K}{F_K}-\frac{\dot{F}_L}{F_L}$$

According to David and Klundert [16] and Sato [17], factor substitution elasticity σ is:

$$\sigma=-\frac{d(K/L)/(K/L)}{d(F_K/F_L)/(F_K/F_L)}=-\frac{F_KF_L}{FF_{KL}} \tag{20}$$

By model (10), define the capital share as a_K, namely

$$a_K=\frac{F_KK}{F} \tag{21}$$

The capital share change rate[1] is

$$\hat{a}_K=\hat{F}_K+\hat{K}-\hat{F} \tag{22}$$

Assume the speed $\hat{A}$ and speed $\hat{B}$ of factor-augmenting technical progress are endogenous variables. According to their definitions and (5)/ (20), we get (for more details, see Appendix 1):

[1] Assume the change rate of any variable x is $\hat{x}=\dot{x}/x$.

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$$\hat{F}_K = \hat{A} - \frac{1-a_K}{\sigma}(\hat{A}-\hat{B}) \quad \hat{F}_L = \hat{B} + \frac{a_K}{\sigma}(\hat{A}-\hat{B}) \tag{23}$$

Combined with the direction definition, we have

$$D = \frac{1-\sigma}{\sigma}(\hat{B}-\hat{A}) \tag{24}$$

Therefore, when $\sigma > 1$ (labor and capital are substitutable) and $\hat{A} < \hat{B}$ ($\hat{A} > \hat{B}$), technical progress benefits capital (labor); when $\sigma < 1$ (labor and capital are complementary) and $\hat{A} < \hat{B}$ ($\hat{A} > \hat{B}$), technical progress benefits labor (capital); when $\sigma = 1$, technical progress is Hicks neutral. This is consistent with the direction of technical progress (Antras, 2004; Klump, 2007)[18][19].

By (5) and the speed definition, we obtain

$$S = a_K \hat{A} + (1-a_K)\hat{B} \tag{25}$$

S and D are functions of a_K, that is, $S = S(a_K)$, $D = D(a_K)$.

According to the induced innovation hypothesis proposed by Kennedy[15], we could assume that there is innovation possibility frontier for technical progress, if $\hat{A}$ remains constant, firms will maximize $\hat{B}$. All firms choose instantaneous velocity $(\hat{A},\hat{B})$ to realize the maximization of output $F(AK,BL)$ growth rate. As in Figure 1, innovation possibility frontier is a strict concave function, at its any point, $\hat{A}$ and $\hat{B}$ may reach the maximum, but not beyond the curve. Based on the above assumptions, we observe assumptions, we observe

$$\{(\hat{A},\hat{B}) \in \Re^2 \mid \hat{B} = \psi(\hat{A}), \hat{A} \le \bar{a}, \hat{B} \le \bar{b}\} \tag{26}$$

where, $\psi(0) > 0$, $\psi'(\hat{A}) < 0$, $\psi''(\hat{A}) < 0$; $\bar{a}$ and $\bar{b}$ are upper limits of $\hat{A}$ and $\hat{B}$, both are constants. Restricted by (26), firms will choose the optimal $(\hat{A},\hat{B})$ to maximize technical progress speed S.

$$\underset{\max \hat{A}}{S} = a_K \hat{A} + (1-a_K)\psi(\hat{A}) \tag{27}$$

To obtain the derivation of $\hat{A}$, and set it to zero, we have:

$$\psi'(\hat{A}) = -a_K / (1-a_K) \tag{28}$$

where, $0 < a_K < 1$. As in Figure 2, S curve crosses innovation possibility frontier at $-a_K/(1-a_K)$.

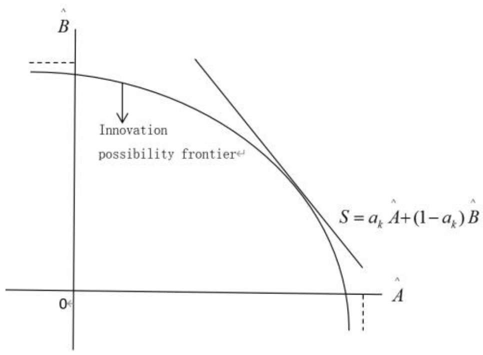

Figure. 2 Innovation possibility frontier

According to the above analysis, $\hat{A}$ is a function of a_K, and $\psi''(\hat{A}) < 0$, using (28), we could attain :

$$\hat{A}'(a_K) = \partial \hat{A} / \partial a_K = -1 / [(1-a_K)^2 \psi''(\hat{A})] > 0 \tag{29}$$

Similarly, $\hat{B}$ is a function of a_K , and $\psi'(\hat{A})<0$, using (26) (28), we could attain :

$$\hat{B}'(a_K)=\partial\hat{B}/\partial a_K=\psi'(\hat{A})\hat{A}'<0 \tag{30}$$

Therefore, when technical progress can speed S reaching the maximum, with the increasing capital share a_K , technical progress is biased towards capital-augmenting technology $\hat{A}$.

Consequently, because $\hat{A}$ and $\hat{B}$ are both continuous functions of a_K ($0<a_K<1$), when it could reach the maximum, there are

$$\hat{A}'(a_K)>0,\ \lim_{a_K\to 0}\hat{A}(a_K)=-\infty,\ \lim_{a_K\to 1}\hat{A}(a_K)=\bar{a} \tag{31}$$

$$\hat{B}'(a_K)<0,\ \lim_{a_K\to 0}\hat{B}(a_K)=\bar{b},\ \lim_{a_K\to 1}\hat{B}(a_K)=-\infty \tag{32}$$

Next, analyzing the transfer dynamics of capital share and capital growth speed based on the above consideration.

3.2 Transfer dynamics

Assuming the function is a two-sector model, savings equals investment $S=I$, savings is S, capital is homogeneous, and there is no depreciation. Therefore, the net increase of capital stock at one point equals gross investment: $\dot{K}(t)=I=S=sY$. Thereby, the net increasing is a function of gross output. Assuming K is an exogenous variable and takes the decreasing exponential form:

$$\dot{K}(t)=\gamma e^{-vt}F(t) \text{ or } \hat{K}(t)=\gamma e^{-vt}F(t)/K(t) \tag{33}$$

among which, $0<\gamma<1$, $v\geq 0$, $K(0)>0$.

Using (5), the change rate of gross output is (for more details in Appendix 2):

$$\hat{F}=a_K(\hat{K}+\hat{A})+(1-a_K)(\hat{L}+\hat{B})=a_K\hat{K}+(1-a_K)\hat{L}+S \tag{34}$$

Inserting the above equation, (23) and (25) into (21), (where the growth speed of labor is n), we obtain:

$$\dot{a}_K/a_K=\frac{1-\sigma}{\sigma}(1-a_K)[\hat{B}(a_K)-\hat{A}(a_K)-(\hat{K}-n)]=(1-a_K)[D-\frac{1-\sigma}{\sigma}(\hat{K}-n)] \tag{35}$$

According to (33), the change rate of $\hat{K}$ is:

$$\dot{\hat{K}}/\hat{K}=-v+\hat{F}-\hat{K} \tag{36}$$

Inserting (34) into the above equation, we have

$$\dot{\hat{K}}/\hat{K}=(1-a_K)[\hat{B}(a_K)+\frac{a_K}{1-a_K}\hat{A}(a_K)-\frac{v}{1-a_K}-(\hat{K}-n)]=S-v-(1-a_K)(\hat{K}-n) \tag{37}$$

Then, (35) and (37) forms a set of equations. Restricted by (31), (32), consider whether or not the equilibrium conditions of (35) and (37) are consistent with the steady state. In steady state, $\hat{K}=\hat{K}^*>0$, $a_K=a_K{}^*$ and $0<a_K{}^*<1$.

Meanwhile, there are $D(a_K{}^*)=\frac{1-\sigma}{\sigma}(\hat{K}^*-n)$, $S(a_K{}^*)=v+(1-a_K{}^*)(\hat{K}^*-n)$,equal to the following equations:

$$\hat{K}^*=\hat{B}(a_K{}^*)-\hat{A}(a_K{}^*)+n \tag{38}$$

$$\hat{K}^*=\hat{B}(a_K{}^*)+\frac{a_K{}^*}{1-a_K{}^*}\hat{A}(a_K{}^*)-\frac{v}{1-a_K{}^*}+n \tag{39}$$

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According to (33), (38) and (39) meet the conditions:

$$\hat{A}(a_K{}^*) = v \tag{40}$$

Taking the above equation and $\hat{B} = \psi(\hat{A})$ into (38), we have

$$\hat{K}^* = \psi(v) - v + n \tag{41}$$

Therefore, the equilibrium conditions of (35)and (37) are consistent with the steady state. When $\dot{a}_K = 0$ and $\dot{\hat{K}} = 0$, we seek its transfer dynamics through establishing the phase diagram of space $(a_K, \hat{K})$, as in Figure 3. When $\dot{a}_K = 0$, we get

$$\hat{K} = \hat{B}(a_K) - \hat{A}(a_K) + n = z_1(a_K) \tag{42}$$

Taking the derivation of time, we obtain

$$z_1{}'(a_K) = \hat{B}'(a_K) - \hat{A}'(a_K) \tag{43}$$

According to (36) and (37), when $0 < a_K < 1$, there are $z_1{}'(a_K) < 0$, $\lim_{a_K \to 0} z_1(a_K) = +\infty$, $\lim_{a_K \to 1} z_1(a_K) = -\infty$. So, when $a_K \to 0$, the corresponding trace of $\dot{a}_K = 0$ approaches $+\infty$; when $a_K \to 1$, the corresponding trace of $\dot{a}_K = 0$ approaches $-\infty$. In phase space $(a_K, \hat{K})$, equations $\dot{a}_K = 0$ and $\dot{\hat{K}} = 0$ divide the first quadrant into four areas, marked as I, II, III and IV. If $\sigma > 1$, in areas I and II, capital share a_K is more than $a_K{}^*$. $a_K = a_K{}^*$ corresponds to the condition $\dot{a}_K = 0$, therefore, when $a_K < a_K{}^*$, $\dot{a}_K < 0$. Thereby, in areasIand IV, capital share decreases.

Similarly, when $\dot{\hat{K}} = 0$, there is

$$\hat{K} = \hat{B}(a_K) + \frac{a_K}{1-a_K}\hat{A}(a_K) - \frac{v}{1-a_K} + n = z_2(a_K) \tag{44}$$

Taking the derivation of time, we have

$$z_2{}'(a_K) = \hat{B}'(a_K) + \frac{a_K}{1-a_K}\hat{A}'(a_K) + \frac{\hat{A}(a_K) - v}{(1-a_K)^2} = \frac{\hat{A}(a_K) - v}{(1-a_K)^2} \tag{45}$$

In the steady state, if $v = 0$, then $\hat{A}(a_K{}^*) = 0$, $\hat{K}^* = \hat{B}(a_K{}^*) + n$. From Figure 3, it can be observed that the trace of $\dot{\hat{K}} = 0$ is in the first quadrant, first decreasing then increasing. In areas of II and III, the corresponding capital growth speed $\hat{A}(a_K)$ is larger than that at $\dot{\hat{K}} = 0$. Therefore, capital growth speed will slow down; in contrast, in areas of I and IV, capital growth speed will increase. By (44), there is

$$z_2(a_K{}^*) = \hat{B}(a_K{}^*) + n > 0 \tag{46}$$

Subtracting (44) from (42), we have

$$z_1(a_K) - z_2(a_K) = -\frac{1}{1-a_K}\hat{A}(a_K) \tag{47}$$

The above equation demonstrates that, when $a_K < a_K{}^*$, the trace of $\dot{\hat{K}} = 0$ is lower than that of $\dot{a}_K = 0$; when $a_K > a_K{}^*$, the trace of $\dot{\hat{K}} = 0$ is higher than that of $\dot{a}_K = 0$. Correspondingly, combining (36) and (37), we know that, when $a_K = 0$, the trace of $\dot{\hat{K}} = 0$ begins with $\bar{b} + n$.

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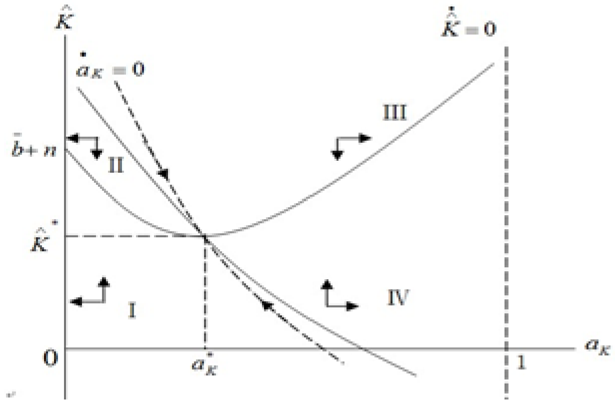

Figure. 3 The saddle paths of two traces when $v=0$, $\sigma>1$

As in Figure 3, when $v=0$ (or s is a constant) and factor substitution elasticity $\sigma>1$, if technical progress is Harold neutral, then there is the optimal path between capital share and capital growth speed, and the path is a saddle one, with equilibrium point $(a_K^*, \hat{K}^*)$ the saddle point. Simultaneously, by the definition of technical progress direction, when $\sigma>1$, $v=0$, then technical progress is biased towards capital.

Next, consider the case of $v>0$ (or s is not a constant), then, the trace of $\dot{\hat{K}}=0$ moves downward, the initial value is $\bar{b}+n-v$ (as in Figure 4). According to (44), if $\hat{A}(a_K)<v$, there is

$$z_1(a_K)-z_2(a_K)=\frac{v-\hat{A}(a_K)}{1-a_K} \tag{48}$$

If $\hat{A}(a_K)>v$, there is one a_K^* to make $\hat{A}(a_K^*)=v$, and the traces of $\dot{a}_K=0$ and $\dot{\hat{K}}=0$ cross at a_K^*. Meanwhile, by the definition of technical progress direction, when $\sigma>1$, $\hat{A}(a_K^*)=v$, then technical progress is biased towards capital. By (41), if two traces cross at the first quadrant, then it meets the following: $\psi(v)-v+n>0$ 。

As in Figure 4, under the condition of $v>\psi(v)+n$, two traces cross at the fourth quadrant, this point doesn't have any economic meaning. In that case, there may be a quasi-equilibrium point $(a_K^0, \hat{K}^0)$, determined by the crossing point of the trace of $\dot{a}_K=0$ and horizontal axis a_K. Similar to Figure 3, when factor substitution $\sigma>1$, equilibrium point $(a_K^0, \hat{K}^0)$ is the saddle point, and meets: $\hat{K}^0=0$, $\hat{B}(a_K^0)-\hat{A}(a_K^0)+n=0$. By the definition of technical progress direction, when $\sigma>1$, $v>\psi(v)+n$, then technical progress is biased towards labor.

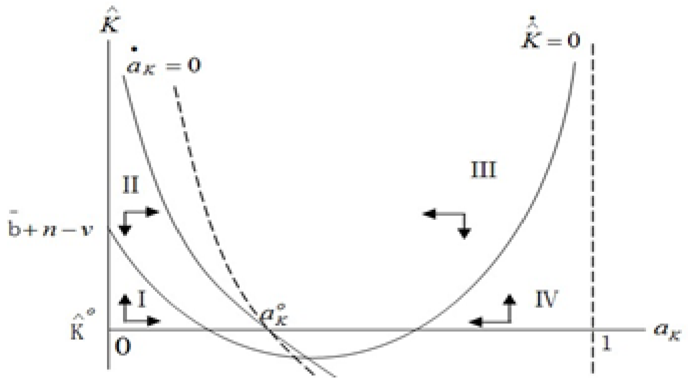

Figure. 4 The saddle paths of two traces when $v=0$, $\sigma>1$

When $\hat{K}^0=0$, $\hat{B}(a_K^0)-\hat{A}(a_K^0)+n=0$, and within the innovation possibility frontier, when $0\le v<\psi(v)+n$, if $\hat{F}_K=\hat{A}(a_K^*)=v$, $\hat{F}_L=\hat{B}(a_K^*)=\psi(v)$, then interest rate and wage rate increase with the increasing v. Therefore, when $v > 0$, interest rate is not a constant, thereby the whole economic system is enabled to move along the corresponding path. If innovation possibility frontier is symmetric, then when $\hat{K}^* - n = 0$ ($\hat{A}(a_K{}^*) = \hat{B}(a_K{}^*)$), there is $a_K{}^* = 1/2$; similarly, when $\hat{K}^* - n > 0$, then $a_K{}^* < 1/2$. Therefore, when v is relatively small, $\hat{K}^* - n = \psi(v) - v > 0$, within the innovation possibility frontier, capital share will be less than $1/2$, namely, long run factor shares distribution is consistent with the theory.

Therefore, under the innovation possibility frontier, when the speed of factor-augmenting technical progress reaches the maximum, technical progress is more biased towards capital, with the increasing capital share; when the substitution elasticity of labor and capital is larger than 1, and savings rate is a constant, if technical progress is Harold neutral, then there is the optimal path between capital share and capital growth speed, and it takes the saddle path, with capital share remaining constant, and technical progress biased towards capital; when the substitution elasticity of labor and capital is larger than 1, and savings rate is not a constant, if technical progress is not Harold neutral, then there still is the optimal path between capital share and capital growth speed, and it takes the saddle path, but with equilibrium point being the half equilibrium point, and zero capital growth speed, and technical progress biased towards labor; furthermore, if innovation possibility frontier is symmetric, capital share will be less than1/2, and long run factor shares distribution is consistent with the theory.

4. Conclusion

This paper first studies the conditions for the CES production function to generate endogenous growth: when the substitution elasticity between capital and labor $\sigma > 1$, if effective per capita capital k approaches infinity, marginal product and average product both approach positive constants, then the CES production function generates endogenous growth. Under this framework, the CES model characterizes the long run growth, and convergence during the transfer process, more specifically, with the increasing effective per capita capital k, average product of capital z approaches a constant D, while effective per capita capital growth rate $\dot{k}/k$ approaches zero. Note that for any substitution elasticity σ, with the increasing k, $\dot{k}/k$ approaches zero. Meanwhile, when $\sigma > 1$, if $A > 0$ and $B > 0$, then with the infinite increasing per capita capital k_0, capital share approaches 1, labor share 0. Therefore, in the case of $\sigma > 1$, the factor-augmenting model generates endogenous growth, and it embodies significance to study the effects of technical progress on factor shares.

After meeting the endogenous growth conditions, we consider the transfer dynamics of factor shares caused by technical progress bias. Results show that, first, within the innovation possibility frontier, firms pursue the maximization of technical progress speed, with the increasing capital share a_K, technical progress is more biased towards capital-augmenting technical progress; secondly, when the substitution elasticity between capital and labor is larger than 1, and savings rate is a constant, if technical progress is Harold neutral, then there is the optimal path between capital share and capital growth speed, it takes the saddle path, and technical progress is more biased towards capital; thirdly, when the substitution elasticity between capital and labor is larger than 1, and savings rate is not a constant, if technical progress is not Harold neutral, then there is still the optimal path between capital share and capital growth speed, and it takes the saddle path, but with equilibrium point being the half equilibrium point, and zero capital growth speed, and technical progress biased towards labor; furthermore, if innovation possibility frontier is symmetric, capital share will be less than $1/2$, and long run factor shares distribution is consistent with the theory.

From the tests of the model, it can be observed that in the factor-augmenting CES model, if capital-augmenting technical progress is larger than 0 and labor-augmenting technical progress is larger than 0, then with the infinite increasing per capita capital, capital share approaches 1, labor 0. If capital is narrowly defined as buildings and equipment, then the meaning of the model will not match the data; but if human capital is incorporated into the definition of capital, then the meaning of the model will be more reasonable, that is, with the development of economy, the initial labor share of gross output will reduce to 0.

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Appendix 1

Because of $Y = F(AK, BL)$, there are

$$F_K = AF_1,\ a_K = \frac{F_K K}{F} = \frac{AF_1 K}{F},\ 1 - a_K = \frac{BF_2 L}{F} \tag{1a}$$

$$\sigma = -\frac{F_K F_L}{FF_{KL}} = -\frac{AF_1 BF_2}{FAF_{12}B} = -\frac{F_1 F_2}{FF_{12}} \tag{2a}$$

According to the definition of technical progress speed:

$$\hat{F}_K = \hat{A} + \frac{\partial F_1}{\partial t}\frac{1}{F_1} = \hat{A} + (F_{11}K\dot{A}_t + F_{12}L\dot{B}_t)/F_1 = \hat{A} + [(F_{11}AK)\hat{A} + (F_{12}BL)\hat{B}_t]/F_1 \tag{3a}$$

Because there is no difference between AK and BL for F_1, $F_{11}AK = -F_{12}BL$, and we get

$$\hat{F}_K = \hat{A} + \frac{F_{12}BL}{F_1}(\hat{A} - \hat{B}_t) = \hat{A} + \frac{F_2 BL \cdot FF_{12}}{FF_1F_2}(\hat{A} - \hat{B}_t) = \hat{A} - \frac{1 - a_K}{\sigma}(\hat{A} - \hat{B}) \tag{4a}$$

Similarly, we get $\hat{F}_L = \hat{B} + \frac{a_K}{\sigma}(\hat{A} - \hat{B})$.

ISSN 2616-5902 Vol. 3, Issue 2: 74-85, DOI: 10.25236/AJBM.2021.030214

Appendix 2

According to $Y = F(AK, BL)$, change rate is created as follows:

$$\frac{\dot{F}}{F} = \frac{\partial F}{\partial (AK)} \cdot \frac{\partial (AK)}{\partial K} \cdot \frac{K}{Y} \cdot \frac{\dot{K}}{K} + \frac{\partial F}{\partial (AK)} \cdot \frac{\partial (AK)}{\partial K} \cdot \frac{A}{Y} \cdot \frac{\dot{A}}{A}$$

$$+ \frac{\partial F}{\partial (BL)} \cdot \frac{\partial (BL)}{\partial L} \cdot \frac{L}{Y} \cdot \frac{\dot{L}}{L} + + \frac{\partial F}{\partial (BL)} \cdot \frac{\partial (BL)}{\partial L} \cdot \frac{B}{Y} \cdot \frac{\dot{B}}{B} \quad (1b)$$

To get the derivations of AK and BL for the both sides of the equation respectively, we have

$$\frac{\partial F}{\partial K} = K \cdot \frac{\partial F}{\partial (AK)}, \; \frac{\partial F}{\partial A} = A \cdot \frac{\partial F}{\partial (AK)}, \; \frac{\partial F}{\partial L} = L \cdot \frac{\partial F}{\partial (BL)}, \; \frac{\partial F}{\partial B} = B \cdot \frac{\partial F}{\partial (BL)} \quad (2b)$$

By the above equations, there are

$$\frac{\partial F}{\partial K} \cdot \frac{K}{Y} = \frac{\partial F}{\partial A} \cdot \frac{A}{Y} = \frac{\partial F}{\partial (AK)} \cdot \frac{AK}{Y}, \; \frac{\partial F}{\partial L} \cdot \frac{L}{Y} = \frac{\partial F}{\partial B} \cdot \frac{B}{Y} = \frac{\partial F}{\partial (BL)} \cdot \frac{BL}{Y} \quad (3b)$$

Because capital share $a_K = \frac{F_K K}{F}$, labor share $(1 - a_K)$, inserting (3b) into (1b), we obtain

$$\frac{\dot{F}}{F} = a_K \left(\frac{\dot{K}}{K} + \frac{\dot{A}}{A}\right) + (1 - a_K)\left(\frac{\dot{L}}{L} + \frac{\dot{B}}{B}\right) \quad (4b)$$