

Research on the Infiltration of Mathematical Culture Content in Middle School Entrance Examination Mathematics Test Papers: A Case Study of Dongying City from 2019 to 2023

Wang Mengjie*, Zhang Ying

School of Mathematical Sciences, University of Jinan, Jinan, China

**Corresponding author*

Abstract: *With the increasing penetration of mathematical culture in educational evaluation, more typical questions of mathematical culture have emerged in the college entrance examination. Through this highly focused platform of the college entrance examination, mathematical culture has come into the public eye. Under the influence of this major trend, mathematical culture has gradually permeated into various types of questions in the high school entrance examination. By sorting out the questions related to mathematical culture in the Dongying City High School entrance Examination mathematics papers over the past five years, and conducting an analysis from aspects such as the number of questions, question types, knowledge domains, content types, application levels, and core literacy, relevant high school entrance examination questions on mathematical culture are selected for appreciation. The following suggestions are put forward for the future direction of integrating mathematical culture into the high school entrance examination questions: (1) Select diverse materials and evenly distribute the types of mathematical culture; (2) Integrate into current affairs and implement the fundamental task of fostering virtue and nurturing talent; (3) Firmly grasp the essence and steadily enhance the level of core mathematical literacy in the high school entrance examination.*

Keywords: *Mathematical Culture High School Entrance Examination Mathematics Test Questions; Core Literacy in Mathematics*

1. Introduction

The "Compulsory Education Mathematics Curriculum Standards (2022 Edition)" emphasizes that mathematics courses should be oriented towards core literacy, pay attention to mathematical culture, inherit and carry forward the excellent traditional Chinese culture, and at the same time reflect the needs of modern science and technology and social development ^[1]. Professor Zhang Weizhong proposed that the mathematical culture in the curriculum has rich connections with the outside world. The connection between mathematics and the outside can be understood from both vertical and horizontal perspectives. The vertical aspect refers to the connection with the background of students' mathematical learning and the reality of mathematics, while the horizontal aspect refers to the connection with other disciplines ^[2]. He classified the mathematical culture in the college entrance examination into four categories: mathematics and life, history of mathematics, mathematics and technology, and mathematics and art ^[3]. Professor Tang Hengjun, based on the application level of mathematical culture, unified it into additional type and integrated type, among which the integrated type can be further divided into separable type and non-separable type ^[4]. The mathematics test questions of the junior high school academic proficiency examination in Dongying City, Shandong Province, are based on the compulsory education mathematics curriculum standards and the junior high school mathematics textbooks of the People's Education Edition, emphasizing the effective assessment of students' core mathematical literacy. The significance of mathematical culture in mathematics test questions is constantly increasing. Moreover, in recent years, Dongying City has been promoting the high-quality development of junior high school education. Mathematical culture-related test questions have become a major highlight of the mathematics test questions in the junior high school academic proficiency examination in Dongying City, aiming to cultivate students' core mathematical literacy and emphasize the educational value of the mathematics discipline.

This article analyzes the types and distribution characteristics of the mathematical culture questions

in the Dongying City High School entrance Examination mathematics papers from 2019 to 2023 (hereinafter referred to as the "nearly five years"), and selects representative questions for appreciation. It also puts forward some suggestions for the infiltration of mathematical culture in the high school entrance examination mathematics questions in the future.

2. The distribution characteristics of mathematical cultural content in the middle school entrance examination papers of Dongying City in the past five years

In terms of the content types of mathematical culture test questions, this paper, from a macro perspective, refers to the classification results of many scholars and divides mathematical culture test questions into four categories: history of mathematics, mathematics and life, mathematics and technology, and mathematics and humanities. The specific content is shown in Table 1.

Table 1 Classification of Mathematical Cultural Content

Classification of Mathematical Cultural Content	Content
History of Mathematics	Portraits of mathematicians, biographies of mathematicians, the history and background of mathematical concepts, famous historical problems
Mathematics and life	Mathematical content related to personal life, study life, public life, economic production, common sense, etc
Mathematics and Technology	The application of mathematics in science and technology, where science must have its own formulas, theorems, specific knowledge, etc
Mathematics and Humanities	Literature, history, linguistics, and artworks (including paintings, sculptures, music, architecture, etc.)

In terms of the application level of mathematical culture test questions, this article classifies the application level of test questions into three levels based on the degree of integration between the context and mathematical knowledge: additional type, separable type, and non-separable type. The specific meanings are shown in Table 2.

Table 2 Level of Application of Mathematical Culture

The application level of mathematical culture	Meaning
Additional type	The "supplementary type" refers to the situation where cultural materials are only attached to the periphery of the test questions in the form of pictures, language, etc. Removing these cultural materials will not affect students' problem-solving
Separable type	"Separable" means that the cultural content of mathematics can be separated from the test questions, and the separation does not affect the solution of mathematical problems
Indivisible type	The term "inseparable" refers to the fact that the content of mathematical culture and the test questions are inseparable, and the solution of problems cannot be separated from the content of mathematical culture

In terms of the knowledge domain of the mathematics culture test questions, this article draws on the classification in the curriculum standards and divides the knowledge domain of the mathematics culture test questions in the high school entrance examination into four parts: "Numbers and Algebra", "Figures and Geometry", "Statistics and Probability", and "Comprehensive and Practical". The specific content is shown in Table 3.

Table 3 Mathematical Culture Knowledge Domains

Numbers and Algebra	Numbers and Expressions	Rational number
		Real number
		Algebraic expression
		Polynomials and fractions
	Equations and inequalities	Equations and Systems of Equations
		Inequalities and systems of inequalities
		Function
	Function	Linear function
		Inverse proportion function
		Quadratic function
Graphics and Geometry	The properties of figures	Point, line, surface, Angle
		Intersecting lines and parallel lines
		Triangle
		Quadrilateral
		Round
		Drawing with a ruler and compass
		Definitions, propositions, theorems
		Axial symmetry of the figure
	The change of the graphic	Rotation of graphics
		Translation of figures
		Similarity of graphics
		Projection of graphics
	Graphics and Coordinates	Coordinates and graphic positions
		Coordinates and graphic motion
Statistics and Probability	Sampling and data analysis	
	The probability of an event	
Synthesis and Practice	Learning activities that take problems as the carrier and mainly involve students' autonomous participation	

In terms of the core literacy of the mathematics culture test questions, this paper analyzes the "degree of mathematization" of the mathematics culture test questions in the high school entrance examination based on the division of core literacy into three progressive levels by Wang Xinyu et al. [5]. The specific content is shown in Table 4.

Table 4 Core Literacy in Mathematics

Core literacy	Horizontal division	Weight
Mathematical abstraction	A can recognize and recall mathematical concepts and rules in simple situations, abstract simple mathematical problems and solve them	1
	B can distinguish mathematical concepts and rules in more complex situations, abstract mathematical problems and solve them with conventional methods. Be able to describe the characteristics of mathematical objects in complex situations in mathematical language; Be able to perform operations and reasoning with the aid of symbols and solve simple rule problems	2
	C can abstract new mathematical concepts and rules in comprehensive situations and understand the generality of mathematical conclusions. Be able to draw parallels with existing experiences and select and apply new conclusions to solve problems in new situations. Be able to perform operations and reasoning with the aid of symbols and solve complex pattern problems	3
Logical reasoning	A can infer certain simple conclusions from existing facts in simple situations through methods such as induction and analogy, and can conduct reasoning within three steps using a single knowledge point	1
	B can explore the train of thought for argumentation through the analysis of conditions and results in more complex situations, and can conduct more than three steps of reasoning with accurate mathematical language. During the process of argumentation, one can understand the logical relationships among relevant concepts, propositions and theorems, and initially establish a networked knowledge structure	2
	C can put forward hypothetical premises through reasonable reasoning in comprehensive situations, use appropriate thinking methods (such as auxiliary lines, classification discussions) to analogize existing models, and conduct more than three steps of reasoning and proof using multiple knowledge points to establish a networked knowledge structure	3
Mathematical modeling	A can list equations based on the quantitative relationships in specific problems in simple situations	1
	B can identify and raise questions in practical situations and establish mathematical models for the problems. Be capable of analyzing the functional relationships in simple problems in combination with images and solving mathematical problems in models	2
	C can apply mathematical knowledge to solve models in comprehensive situations, and attempt to verify and improve the models based on real contexts, ultimately solving complex practical problems with mathematics	3
Intuitive imagination	A can understand the positional relationship, morphological changes and movement laws of things through space. Be able to draw geometric figures based on the description of written language	1
	B can accurately understand mathematical concepts, describe and analyze mathematical problems by means of geometric intuition. Be able to conduct intuitive geometric exploration and find problem-solving approaches through complex physical hands-on operations or graphic movement operations	2
	C can establish the connection between shapes and numbers, and through the intuitive exploration of geometric figures, describe and analyze problems in other mathematical fields beyond geometry	3
Mathematical operation	A can perform simple operations on numbers and expressions based on rules and formulas, and solve simple equations (systems) and inequalities (systems).	1
	B can choose a reasonable method for calculation and explain the calculation principle	2
	C can understand and perform operations based on the new definition	3
Data analysis	A can understand the information contained in the data and make judgments. Be able to understand the concepts of various terms in statistical probability	1
	B can select appropriate analytical methods based on the background of the problem and draw conclusions through simple data processing. Be able to calculate probabilities in simple situations	2
	C can obtain rich information from the situation and make reasonable assumptions or suggestions. Be able to calculate probabilities in complex situations	3

For the questions involving mathematical culture in the Dongying City senior high school entrance

examination from 2019 to 2023, the mathematical core competencies assessed and their three levels were coded and assigned values. The weight of each level of the mathematical core competencies in the total score for each set of test papers was then organized and calculated. Taking the calculation of Level 1 of the Mathematical Operation competency in 2019 as an example, the weight was determined by dividing the score for Level 1 of the Mathematical Operation core competency in all mathematical culture-related questions from the 2019 senior high school entrance examination mathematics test by the total score, and converting it into a percentage (retaining one decimal place). The total score for the Dongying City senior high school entrance examination mathematics test is consistently 120 points. This process resulted in a summary of the scores and weights for the 18 levels across the six core competencies.

If a question examines multiple core literacy levels, the total score of the question will be divided based on the primary and secondary qualities examined. In order to reflect the primary and secondary relationship of the literacy tested in a question, the levels of mathematical core literacy that are simply and sparsely tested in some questions are not assigned values. Other key literacy that are tested are marked first. Generally, the core literacy assigned values in a question is no more than three. In addition, when assigning values to the core competencies examined in the questions in this paper, the competencies that are emphasized in the test questions will be assigned higher scores, and the higher the degree of emphasis, the higher the proportion of the assigned scores. By using this method, a detailed analysis was conducted on the questions related to mathematical culture in the middle school entrance examination of Dongying City from 2019 to 2023.

This article takes the mathematical culture questions in the mathematics test papers of the Dongying High School Entrance Examination in the past five years as the research object, and conducts a specific analysis from aspects such as question types, background, content types, knowledge fields, application levels, and the examination of core literacy. Among them, the knowledge domain is designed to study the examination content of mathematical culture questions. The content type is designed to study the cultural background of mathematical culture test questions. The application level is to study the degree of correlation between the content of mathematical culture and test questions. The core literacy is to study the types of core literacy examined in the mathematical culture test questions and their examination levels. The following chart is listed.

As can be seen from Table 5, there have been a total of 36 questions related to mathematical culture in the middle school entrance examination papers of Dongying City in the past five years. The distribution of the content has the following characteristics:

In terms of the number of questions, there are a total of 25 questions in the middle school entrance examination papers each year. Among them, in 2019, there were 6 questions related to mathematical culture, accounting for 24%. From 2020 to 2022, there were 7 cases each year, accounting for 28%. In 2023, there were 9 channels, accounting for 36%. Overall, in the past five years, the number of questions related to mathematical culture in the high school entrance examination mathematics has been increasing, which is in line with the requirements of the new curriculum standards.

(1) In terms of question types, in the past five years' high school entrance examination questions, multiple-choice questions, fill-in-the-blank questions, and problem-solving questions all involve mathematical culture. From 2019 to 2022, the number of the three types of questions that permeate mathematical culture was relatively balanced, each consisting of 2 to 3 questions. In 2023, the number of fill-in-the-blank questions related to mathematical culture increased significantly, while the number of multiple-choice and problem-solving questions remained largely unchanged.

(2) In terms of content types, the past five years' high school entrance examination questions in Dongying City have covered the history of mathematics, mathematics and life, mathematics and science and technology, and mathematics and humanities. Among them, there are two questions about the history of mathematics, accounting for 5.6%. There are 27 subjects in mathematics and life, accounting for 75%. There are two subjects in mathematics and Technology, accounting for 5.6%. There are five subjects in mathematics and humanities, accounting for 13.9%. It can be seen from this that the cultural questions in the high school entrance examination for mathematics focus on the assessment of mathematics and life.

(3) In terms of application level, there are 6 additional types, accounting for 16.7%. There are 12 separable types, accounting for 33.3%. There are a total of 18 non-separable types, accounting for 50%, which is the highest proportion. It can be seen that, on the whole, the integration of mathematical culture and mathematical knowledge is not close enough and is still in a relatively fragmented state. However, this situation is gradually improving.

Table 5 Distribution Characteristics of Mathematical Cultural Content in the Middle School Entrance Examination Papers of Dongying City in the Past Three Years

Year	Question Number	Question Type	Background	Content type	Application Level	Knowledge Domain
2021	4	Multiple-choice questions	Shopping issues	Mathematics and life	Separable type	Real number
	6	Multiple-choice questions	Traffic light problem	Mathematics and life	Separable type	Probability
	11	Fill-in-the-blank questions	Census	Mathematics and Humanities	Additional type	Scientific notation
	13	Fill-in-the-blank questions	The age structure of interest groups	Mathematics and life	Separable type	Statistics
	16	Fill-in-the-blank questions	Construction issues	Mathematics and life	Indivisible type	Equation
	20	Problem-solving questions	Hand-written newspaper competition	Mathematics and life	Indivisible type	Statistics and Probability
	22	Problem-solving questions	Rice yield	Mathematics and life	Indivisible type	Equation
2022	4	Multiple-choice questions	Tree planting	Mathematics and life	Separable type	Real number
	6	Multiple-choice questions	Square painting	Mathematics and life	Indivisible type	Axial symmetry of the figure
	11	Fill-in-the-blank questions	The number of viewers of the Winter Olympics	Mathematics and life	Additional type	Scientific notation
	13	Fill-in-the-blank questions	Survey on Homework Duration	Mathematics and life	Separable type	Statistics
	20	Problem-solving questions	Communist Youth League activities	Mathematics and life	Indivisible type	Statistics and Probability
	22	Problem-solving questions	The structure of the Yellow River Bridge	Mathematics and Humanities	Separable type	Trigonometric function
	23	Problem-solving questions	Fruit sales	Mathematics and life	Indivisible type	Equations and inequalities
2023	4	Multiple-choice questions	Paper-cutting	Mathematics and Humanities	Separable type	Axial symmetry of the figure
	5	Multiple-choice questions	Buy flour	Mathematics and life	Separable type	Equation
	11	Fill-in-the-blank questions	An approximation of π	Mathematics and Humanities	Additional type	Scientific notation
	13	Fill-in-the-blank questions	Light reflection	Mathematics and life	Indivisible type	Coordinates and graphic positions
	14	Fill-in-the-blank questions	Shooting test	Mathematics and life	Indivisible type	Statistics
	15	Fill-in-the-blank questions	Navigation issues (not involving speed)	Mathematics and life	Separable type	Trigonometric function
	16	Fill-in-the-blank questions	Round timber embedded in the wall	History of Mathematics	Additional type	Round
	20	Problem-solving questions	Study tour activity	Mathematics and life	Indivisible type	Statistics and Probability
	23	Problem-solving questions	Fence problem	Mathematics and life	Indivisible type	Equations and inequalities

(5) In terms of the knowledge points examined, the knowledge points and question types tested in each year's test questions are basically the same. The examination content mainly focuses on scientific notation, statistics and probability, equations and inequalities, trigonometric functions, etc. It also involves knowledge points of figures and geometry such as circles and axial symmetry of figures. Some question types are relatively fixed, and students can be guided to adopt targeted review strategies. And the coverage of knowledge points is showing an expanding trend.

(6) From the perspective of core mathematical literacy, as can be seen from Table 6, the core literacy tested in the annual questions related to mathematical culture mainly focuses on mathematical modeling, data analysis, and mathematical operations. Occasionally, intuitive imagination and mathematical abstraction literacy are examined, and logical reasoning literacy is basically not tested. For mathematical abstraction, mathematical operation, and intuitive imagination literacy, the examination focuses on levels 1 and 2, and the difficulty of the questions is relatively low. As for mathematical modeling and data analysis, all levels 1, 2 and 3 are comprehensively examined. There are both medium and low difficulty

questions as well as high difficulty questions. The examination of level 3 mainly focuses on statistics and probability in the solution questions and application questions related to selling goods.

Table 6 Summary Table of the Weights of Core Mathematical Literacy and Proficiency in Dongying City's High School Entrance Examination Mathematics Papers from 2019 to 2023

Core literacy in mathematics	Quality level	2019	2020	2021	2022	2023	Average weight
Mathematical abstraction	1	0	0	2.5%	2.5%	0	2.5%
	2	0	2.5%	0	0	3.3%	2.9%
	3	0	0	0	0	0	0
Logical reasoning	1	0	0	0	0	0	0
	2	0	0	0	0	0	0
	3	0	0	0	0	0	0
Mathematical modeling	1	2.5%	0	0	0	2.5%	2.5%
	2	2.5%	0	3.3%	0	0	2.9%
	3	6.7%	8.3%	8.3%	6.7%	6.7%	7.34%
Intuitive imagination	1	0	0	0	2.5%	2.5%	2.5%
	2	0	0	0	0	2.5%	2.5%
	3	0	0	0	0	0	0
Mathematical operation	1	2.5%	2.5%	2.5%	2.5%	2.5%	2.5%
	2	0	6.7%	0	6.7%	3.3%	5.57%
	3	0	0	0	0	0	0
Data analysis	1	2.5%	2.5%	2.5%	2.5%	2.5%	2.5%
	2	0	2.5%	2.5%	0	0	2.5%
	3	6.7%	6.7%	6.7%	6.7%	6.7%	6.7%

3. Appreciation of the High School Entrance Examination Questions in the Past Five Years

Example 1: As shown in the figure 1, if two of the switches are randomly closed K_1 , K_2 , K_3 , the probability that the two bulbs can light up simultaneously is ()

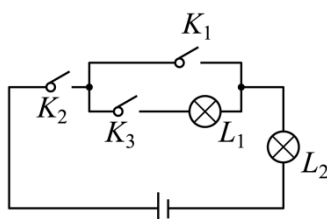


Figure 1 Circuit diagram

- A. $\frac{1}{6}$ B. $\frac{1}{2}$ C. $\frac{1}{3}$ D. $\frac{2}{3}$

Analysis: This question belongs to the category of situational application questions. The interdisciplinary content in situational application questions not only provides the background of the problem but also expects students to abstract the mathematical research object from the context and solve it by integrating interdisciplinary knowledge with mathematical knowledge, thinking and methods. This problem creates a physical context of a circuit. It requires students to accurately calculate the number of combinations of two switches selected from three and understand the impact of the switch's closure on the circuit, thereby judging the light emission of the light bulb. This process requires students to have clear logical thinking and strong analytical skills. Although the question tests the knowledge of probability theory and circuits, it uses light bulbs and switches in daily life as carriers, making the question more practical and enhancing students' interest and motivation in learning. It reflects the connection between mathematics and physics, allowing students to feel that the development of each discipline is not independent or closed, but rather promotes and interacts with each other.

Example 2: On May 11, 2021, the data from the seventh national census showed that the national population had increased by 72.06 million compared with that from the sixth national census. Why is 72.06 million expressed in scientific notation?

Analysis: This question demonstrates the connection between mathematics and the humanities. The national census is a comprehensive survey and registration of the current population across the country, organized by the state in accordance with the law, conducted on a universal basis, household by household and individual by individual. The key point of the census is to grasp the changes in the current population in various regions, the gender ratio, and the urban-rural population, etc., so as to enable the country to formulate the next development policies. It is one of the key tasks in our country. This question is set against the backdrop of the population census and mainly examines scientific notation. It is hoped that students can understand some of the country's measures and concepts during the problem-solving process, and appreciate the application value and humanistic value of mathematics.

4. Summary and Suggestions

In response to the above issues, the following suggestions are put forward for the future compilation of the high school entrance examination questions in Dongying City:

(1) Select diverse materials and evenly distribute the types of mathematical culture

To change the current situation where mathematical culture test questions overly emphasize those related to mathematics and daily life, and to balance the types of mathematical culture in the test questions, the compilation of test questions should recognize the rich diversity of mathematical culture materials and further expand the selection range. For instance, the golden section in sculpture and the graphic transformation in weaving are all closely related to geometry. Ancient mathematical historical materials such as calculation sticks and Qin Jiushao's algorithms are also highly suitable for the examination of numbers and formulas. The sound waves in music and the variable relationships in scientific experiments are all inseparable from function problems. Incorporating the above materials into the test question compilation can significantly increase the proportion of the three types of mathematical culture questions, namely, the history of mathematics, mathematics and humanities, and mathematics and science and technology, making the situational characteristics of the test questions more balanced.

(2) Integrate into current affairs and implement the fundamental task of fostering virtue and nurturing talent

The compilation of test questions should correctly recognize and address the issue that current mathematics and culture-related test questions do not pay sufficient attention to current social affairs. Integrating current hot topics into mathematics education can, on the one hand, bring mathematics closer to society and better showcase the form of mathematics education. On the other hand, it is also one of the important methods to implement the fundamental task of fostering virtue and nurturing talent in the discipline of mathematics. Therefore, the formulation of test questions should keep pace with The Times, break away from old scenarios such as rolling dice, planting flowers, and buying tickets, and incorporate more social hotspots like the core socialist values, national spirit, ecological civilization, and national development to create more novel questions.

For instance, Question 20 of the 2017 Lishui City High School Entrance Examination Mathematics Paper took the ecological protection measure of "completely eliminating water of grade V or below" as the material. It used data statistics methods to record the implementation process of sewage treatment. While adopting the traditional dual-graph complementary presentation method, it also set up an open evaluation question, which not only tested students' data analysis abilities at different levels. It also prompts students to think deeply about social issues such as environmental protection, making it a very good exemplary proposition.

(3) Firmly grasp the essence and steadily enhance the level of core mathematical literacy in the high school entrance examination

Steadily enhancing the level of core mathematical literacy in the high school entrance examination does not mean denying basic questions, but rather further improving the current situation where the overall mathematical cultural questions tend to focus on the basics. To develop more profound mathematical cultural test questions, we should reject the practice of "culture for the sake of culture and innovation for the sake of innovation", and instead emphasize "both virtue and talent". For instance, in statistics and probability questions, open-ended questions based on data judgment can be set to examine students' data analysis literacy in obtaining information from the context and making reasonable assumptions, or inquiry questions from specific to general based on mathematical models can be set to cultivate students' logical reasoning literacy in solving new problems by analogy with existing experience.

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