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Modeling and Reliability Analysis of Fuzzy Random Model of End-Bearing Piles

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Abstract: *Based on the fuzziness of random parameters in the reliability analysis model, a fuzzy random reliability analysis model using the first-order second moment method (FOSM) is established. With the decomposition theorem, a series of cut sets are obtained and the fuzzy numbers are converted into a series of interval numbers for calculation, achieving a reasonable fuzzy random reliability calculation method. This method is proved reasonable and practical to analyze the reliability of piles by many actual examples.*

Keywords: *Piles. Fuzzy Random Reliability. Cut Set. Interval Number*

1. Introduction

Pile foundation increasingly appears in high-rise buildings and heavy infrastructure foundation projects, becoming the basis of lots of engineering applications due to its high-bearing capacity and small settlement. It is widely used not only in the soft foundations of coastal areas, but also in inland areas. At present, the study of piles mainly focuses on the calculation of the bearing capacity as well as the calculation of settlement. However, there are few concerns about the reliability analysis. Moreover, existing reliability analysis only involves the random uncertainty factors in the design of pile foundations, while neglecting the fuzzy uncertainties.

Domestic and oversea scholars have made some researches on fuzziness [1-4], most of which are based on the independence of fuzziness and randomness and haven’t taken the fuzziness of random parameters into account. It is well known that the statistical value of random parameters plays a decisive role in the reliability index calculation. Due to the realities that engineering projects tends to be rather complicated, parameters required for the model, especially the soil parameters, would be affected by many factors, such as test conditions, weather conditions, test instruments, etc., and more human factors as well. Therefore, the statistical values of these parameters have the feature of fuzziness to some extent. Fuzziness is a hugely uncertain factor different from random uncertainty. Its objective existence leads to inaccurate results of the conventional analysis model, and even results in inability to handle the conventional model in some cases. Therefore, taking the fuzziness of random parameters into account based on the conventional reliability theory, the establishment of a fuzzy random reliability analysis model has great practical significance for reliability analysis.

2. Fuzzy Reliability Analysis Theory

2.1 Establishment of Fuzzy Reliability Analysis Model

The border between reliability and invalidity described by the traditional reliability theory could be represented by the limit equation Z=R-S=0,as shown in Figure 1. However, it is difficult to distinguish the structure from safety to destruction with clear boundaries. There is an intermediate transitional state, a fuzzy range circle as shown in Figure 2, between reliability and failure. This range indicates that the failure occurs gradually.

The fuzzy function equation is:

$$\tilde{Z} = \tilde{R} - \tilde{S} \tag{1}$$

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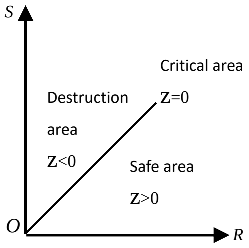

Figure.1: Limit state

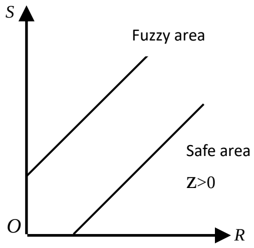

Figure.2: Fuzzy limit state

At present, there are two approaches in the study of fuzzy reliability calculation theory. One is to describe the fuzzy random phenomenon with a fuzzy set. The fuzzy reliability is defined by the probability of the fuzzy event, directly calculated by the integral method. For the other method, the fuzzy variable is used as the basic variable to describe the fuzzy Random phenomena, calculated using the horizontal cut set method. The key to the former method is to determine the membership function of the fuzzy event. The latter method can transform the fuzzy situation into a regular situation thus being able to make full use of various calculation methods in the conventional reliability analysis.

2.2 Fuzzy Mathematics

The fuzzy numbers in projects can be represented by L-R type fuzzy numbers $\tilde{A}$, whose membership functions meet the following equations,

$$\mu_{\tilde{A}}(x)=\begin{cases}1 & a\le x\le b\\ L(x) & x<a\\ R(x) & x>b\end{cases} \tag{2}$$

Where: $L(x)$ is increasing function and right continuous, 0⩽ $L(x)$ <1, $\lim\limits_{x\to+\infty}L(x)=0$; $R(x)$ is decreasing function and left continuous, 0⩽ $R(x)$ <1, $\lim\limits_{x\to+\infty}R(x)=0$.

In equation(2), when $a=b$, L-R type fuzzy numbers can also be represented by triples (m,α,β), where m corresponds to the number of membership degrees 1;α,β are the left and right distributions of fuzzy numbers. L(x),R(x) are called left and right reference functions of fuzzy numbers. The two reference functions commonly used in engineering projects are linear reference functions and normal reference functions. The L(x) and R(x) of the linear reference function are,

$$\begin{cases}L(x)=\max\left\{0,1-\dfrac{m-x}{\alpha}\right\} & \alpha>0, x<m\\ R(x)=\min\left\{0,1-\dfrac{x-m}{\beta}\right\} & \beta>0, x>m\end{cases} \tag{3}$$

As the direct calculation of fuzzy numbers is relatively difficult, it is generally converted into an ordinary set with decomposition theorem for calculation. Suppose $\tilde{A}$ is the fuzzy set on discourse domain U, for any λ∈[0,1],

$$\tilde{A}_\lambda=\{x|x\in U,\mu_\lambda(x)\ge\lambda\} \tag{4}$$

Let $\tilde{A}_\lambda$ be the cut set (or horizonal cut set) of $\tilde{A}$,andλbe cut-set level. The fuzzy set can be expressed as a normal set

$$\tilde{A}=\bigcup_{\lambda\in[0,1]}\lambda\tilde{A}_\lambda \tag{5}$$

Equation (5) is the decomposition theorem [6]. It reveals that any fuzzy set can be represented by a common set. That means any fuzzy mathematical problem can be transformed into an ordinary set problem through a decomposition theorem, namely a non-fuzzy problem. In this paper, the fuzzy number is calculated by converting it into a series of interval numbers.

3. First Order Second Moment Method (FOSM)

FOSM, proposed by A.M. Hasfer and C.Lind, is the simplest and most practical method for approximating the reliability index. It is also referred to as the H-L method [7,8]. It merely uses the mean and standard deviation to describe the statistical characteristics of all the basic random variables. Only the first term of the Taylor series expansion function is considered, and the formula for solving the reliability index is established on the premise that the random variables are relatively independent. The basic principle is to use the average (first-order origin moment) and variance (second order central moment) models of random variables to analyze the structural reliability and extend the limit state function at the center point to make it linear. This method mainly depends on the values of the mean and variance, which however are likely to be fuzzy. In general, they can be described by L-R type fuzzy numbers. The structured fuzzy random function is expressed as,

$$Z = g(\tilde{X}_1, \tilde{X}_2, \cdots \tilde{X}_n) \tag{6}$$

In practical calculations, the limit state equation is taken as

$$Z = g(\tilde{m}_{X_1}, \tilde{m}_{X_2}, \cdots \tilde{m}_{X_n}) + \sum_{i=1}^{n} (X_i - \tilde{m}_{X_i}) \frac{\partial g}{\partial X_i} \tag{7}$$

$$\tilde{\beta} = \frac{\tilde{m}_Z}{\tilde{\sigma}_z} \tag{8}$$

$$\begin{cases} \tilde{m}_Z \approx g(\tilde{m}_{x_1}, \tilde{m}_{x_2}, \cdots \tilde{m}_{x_n}) \\ \tilde{\sigma}_Z^2 \approx \sum_{i=1}^{n} \tilde{\sigma}_{X_i}^2 \left(\frac{\partial g}{\partial x_i} \Big| \tilde{m}_{x_i} \right)^2 \end{cases} \tag{9}$$

Transform the fuzzy numbers in equations (8) and (9) into a series of interval numbers by equation (5), and the expression of the λ horizontal cut set for the fuzzy reliability index can be obtained.

$$\tilde{\beta}_\lambda = \left[\beta_\lambda^1, \beta_\lambda^u\right] = \frac{\tilde{m}_{Z\lambda}}{\tilde{\sigma}_{Z\lambda}} = \left[\left(\frac{m_Z}{\sigma_Z}\right)_\lambda^l, \left(\frac{m_z}{\sigma_Z}\right)_\lambda^u\right] \tag{10}$$

$$\begin{cases} \left(\frac{m_Z}{\sigma_Z}\right)_\lambda^l = \min \dfrac{g(\bar{m}_{x_1}, \bar{m}_{X_2} \cdots \bar{m}_{X_n})}{\sum_i^n \bar{\sigma}_{X_i}^2 \left(\frac{\partial g}{\partial x_i} \Big| \bar{m}_{X_i} \right)^2} \\ \left(\frac{m_z}{\sigma_Z}\right)_\lambda^u = \max \dfrac{g(\bar{m}_{x_1}, \bar{m}_{X_2} \cdots \bar{m}_{X_n})}{\sum_i^n \bar{\sigma}_{X_i}^2 \left(\frac{\partial g}{\partial x_i} \Big| \bar{m}_{X_i} \right)^2} \end{cases} \tag{11}$$

Where: $\bar{m}_{X_i}$, $\bar{\sigma}_{X_i}^2$ represent the number of intervals corresponding to the mean value of the fuzzy random number and the fuzzy variance at the λ cut-off level, respectively. Max and min represent the upper and lower bounds of the interval number.

According to the interval reliability calculation [9-11], the structural fuzzy reliability at the horizontal cut set is

$$P_{r\lambda} = \frac{1}{\beta_\lambda^u - \beta_\lambda^l}\left\{\beta_\lambda^u\phi(\beta_\lambda^u) - \beta_\lambda^l\phi(\beta_\lambda^l) + \frac{1}{\sqrt{2\pi}}\left[\exp\left(-\frac{(\beta_\lambda^u)^2}{2}\right) - \exp\left(-\frac{(\beta_\lambda^l)^2}{2}\right)\right]\right\} \quad (12)$$

Therefore, the estimated value of fuzzy reliability is

$$P_r = \int_0^1 P_{r\lambda} d\lambda \quad (13)$$

At the same time, it can also be approximated as

$$P_r \approx \frac{1}{n}\sum_1^n P_{r\lambda_i} \quad (14)$$

4. Analysis of the Fuzzy Random Reliability of Piles

The limit equation of the pile can be [12]

$$z = Q_{su} + Q_{pu} - Q \quad (15)$$

$$\begin{cases} Q_{su} = \sum U_i L_i q_{sui} \\ Q_{pu} = A_p q_{pu} = A_p(\xi_c c N_c + \xi_q \gamma h N_q) \end{cases} \quad (16)$$

Here: Q_{su} is the side friction resistance of piles; Q_{pu} is the end resistance of piles; q_{sui} is the ultimate side resistance of the ith layer of the soil around the pile; q_{pu} is the extreme end resistance of the soil layer at the pile end; Nc,Nq respectively represent the soil cohesion and the bearing capacity coefficient of the foundation depth; ξ_c, ξ_q is the shape factor; c is cohesion of soil; h is the embedded depth of the pile;Ui is the perimeter of the pile section; Li is the thickness of the ith layer of soil; AP is the full area of the cross section of the pile end: Q is the force acting on the top of the pile.

Therefore, the pile limit state is expressed as

$$z = g(U_i, L_i, q_{sui}, A_p, \xi_c, c, N_c, N_q, \xi_q, \gamma, h, Q) \quad (17)$$

Where:Nc,Nq can be expressed as a function of the friction angle φ;qsui is related to coefficient of earth pressure at rest in the clay soil. According to the degree of uncertainty of each parameter and the impact on the reliability index value, the basic variables in the model can be divided into three types: fuzzy random variable, random variable and deterministic variable. Since the reliability index is sensitive to changes in the mean and of c,φ,K,Q and changes in variance. Therefore, c,φ,K and their associated correction coefficients and Q could be considered as fuzzy random variables, and other parameters are regarded as random quantities or certainty quantity. Describe the pile's fuzzy limit state equation with basic variables as,

$$Z = g(\tilde{c}, \tilde{\varphi}, \tilde{K}, \tilde{Q}, a_i, b_i) \quad (18)$$

In the formula: ai is basic parameter of which regarded as random quantity; bi is the basic parameter of which regarded as deterministic quantity. Using equations (7)-(14), the fuzzy random reliability of the pile can be obtained.

5. Examples

There is a concrete friction pile embedded into a sand layer as shown in Figure 3, which is set to have a concrete strength that meets requirements, and its supporting ability is determined by the supporting ability of the soil. Assuming that the end face of the pile is circular, the corresponding function to the lack of soil supporting capacity is,

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$$g = 2\pi RL(\gamma L / 2 + q)K \tan \varphi - P \tag{19}$$

Where: R is the radius of the pile; L is the embedded depth of the pile; q is the surface additional load; K is the lateral pressure coefficient of the soil; φ is the average internal friction angle of the soil; P is the force acting on the pile; γ is the density of the soil.

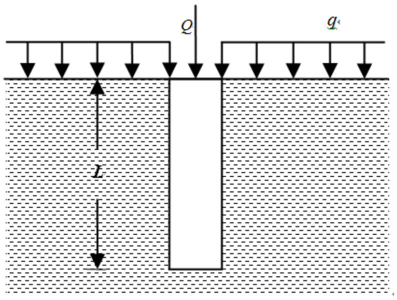

Figure.3: Friction pile

Let R,L,q and γ be constants,which are, R=0.153m,L=12.2m,q=14.63kN/m,γ=17.59kN/m, $\tilde{\varphi}, \tilde{K}, \tilde{P}$ are fuzzy random quantities.

$$\tilde{K} = ((1.5, 0.15, 0.225)_{LR}, (0.1, 0.01, 0.015)_{LR}),$$

$$\tilde{\varphi} = ((30°, 3°, 4.5°)_{LR}, (0.1, 0.01, 0.015)_{LR}),$$

$$\tilde{P} = ((907, 90.7, 136.05)_{LR} KN, (0.1, 0.01, 0.015)_{LR}),$$

Table 1: Comparison between calculated results and conventional results

Classification	Reliability index	Reliability
Conventional reliability model	1.649 2	0.950 4
Random reliability model	1.531 0	0.932 4

From the calculation results in Table 1, it can be seen that ignoring the fuzziness will lead to a large deviation in the results, while the fuzziness is an objective existence. Therefore, taking fuzziness into consideration is more reasonable. At the same time, it can be seen that the fuzzy reliability index and fuzzy reliability obtained with the FOSM method are all smaller than those without considering the fuzziness. Therefore, this method is safer to evaluate the pile reliability.

6. Conclusion

Pile foundation projects play a decisive role in geotechnical engineering, thus important to study the reliability of its practical application. Due to the inherent complexity of engineering practice, randomness and fuzziness often exist simultaneously, making reliability calculations more challenging.

(1) In this paper, the uncertainties are described by fuzzy random variables. Based on FOSM method, a fuzzy random reliability analysis model is established. The model fully considers the two widely-existing uncertainties of fuzziness and randomness, making it more according with engineering practice.

(2) Based on the decomposing theorem of fuzzy mathematics, the calculation of the fuzzy set is converted into a regular set, achieving a method for solving the fuzzy random reliability. This method not only reduces the computational complexity, but also makes it accessible to make full use of the various methods in conventional reliability calculation model.

(3) The proposed model is specifically applied to the reliability analysis of friction piles. And fuzziness is turned out to be not negligible by calculation results.

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