The Frontiers of Society, Science and Technology
ISSN 2616-7433 Vol. 1, Issue 8: 125-131, DOI: 10.25236/FSST.2019.010820

Simple Analysis of Split Award Auctions

Zeyu Zhang

Huazhong Agricultural University, China

ABSTRACT. *It is possible, in many purchase settings that a buyer can split a production award between suppliers. In this paper, I tried to introduce a split-award auction model with endogenous split choice. Then I discussed the equilibrium outcome of this model. By analyzing the properties of the equilibria and comparing the equilibria with a sole-source outcome, I arrived at the conclusions showing that the buyer would prefer a split-award auction to a winner-take-all auction because of the efficiency that a split-award auction can offer.*

KEYWORDS*: Split-award auction; Model*

1. Introduction

Government purchase is an interesting phenomenon for social scientists. Actually buyers are always willing to minimize the purchase cost while suppliers are thinking of maximizing their profits. Based on Microeconomics benchmark, the dual problem can be solved easily by optimization process. However, when suppliers are more than one in the market, there would be other ways to bring efficiency, fairness and product quality. Auctions are widely used to solve such problems, for which auctions are used to award contracts for a variety of product and service requirements in the public and private sectors. These auctions can result in a sole-source award, in which a single producer provides all of the required production, or in a split reward, in which production is divided between two or more firms. It is very interesting to question which one is better, not only for the efficiency site, but for the Pareto efficiency as well. In order to get the answer to the question, I focus on the papers about these two kinds of auctions and tried to get an efficient way to modeling them and get a satisfied answer.

Actually Wilson (1979), Bernheim and Whinston (1986) have researches on auctions and they did focus on the winner-take-all auctions with the sole-source outcome. Ely (2001) has talked about the revenue equivalence with endogenous choice model which just matched the assumption of model I would focus on. Anton and Yao (1989) discussed the split-award auctions and give me a lot of essential information on the model, for which the Anton and Yao (1989) is the paper I would talk most about. For extension

ISSN 2616-7433 Vol. 1, Issue 8: 125-131, DOI: 10.25236/FSST.2019.010820

works, I also went through Jehiel and Moldovanu (2001) for the information about revenue maximization and multi-subject auctions. The paper of Anton and Yao (1989) was cited by the paper Klemperer (1999). I found that paper when decided to focus on the auctions, this paper gives me a clear implication of different articles through different time periods.

In this paper, I plan to give four main parts. Part II is the model foundation and basic idea of equilibrium we can get from the model. Part III is derivation of the equilibrium and efficiency of this equilibrium compared with sole-source outcomes. This part also includes a few extension works on the model. Part IV is a brief introduction on the implications of the model.

2. The Model

A buyer, say, the government, must purchase a given quantity of *x* units. There are two potential suppliers: developer *D* and second source *S*; any division of *x* between the suppliers is feasible. The buyer is going to complete cost-minimization while suppliers are going to complete profit-maximization. In the split-award auction, each player submits a sealed bid that specifies prices for varying splits of the total award of *x*. In this paper, to simplify the notation, all variables are in terms of the share of *x* that is awarded to one of the bidders.

Letting $\alpha \in [0,1]$ denote the share of *x* that supplier *D* produces, a bid is a function $P_i: [0,1] \rightarrow \mathcal{R}$. Since then, a split of α implies that for a pair of bids, (P_D, P_S), *D* produces αx units for a payment of $P_D(\alpha)$, while S produces $(1-\alpha)x$ units for $P_s(\alpha)$. In this case, bids are not restricted to a continuous condition, which means not necessarily differentiable. The costs of the suppliers at an arbitrary $\alpha \in [0,1]$ are given by $C_D(\alpha)$ and $C_s(\alpha)$. These cost functions are interpreted as the expected present discounted value of all costs incurred by a bidder, conditional on the split α. Since then, the profits can be written as following:

$$\Pi_i(\alpha) = P_i(\alpha) - C_i(\alpha), \qquad \alpha \in [0,1], \qquad i = D, S.$$

We assume that $C_D(0) = 0$ and $C_S(1) = 0$. Since none of the suppliers can get a positive payment from zero production, we can set $P_D(0) = 0$ and $P_s(1) = 0$. What is more, there should be a restriction that $P_i \geq C_i$ for each bidder. This restriction rules out equilibrium payoffs that are supported by threats involving negative profits. Also suppliers are having the information of all costs when they bid. This assumption allows us to abstract. from complications associated with uncertainty about relative costs. The assumption does not appear to be essential to the qualitative results of the model, I will extend this problem later in this paper. The joint cost function can be written by the following:

$$B(\alpha) = C_D(\alpha) + C_S(\alpha), \qquad \alpha \in [0,1]$$

ISSN 2616-7433 Vol. 1, Issue 8: 125-131, DOI: 10.25236/FSST.2019.010820

Given the cost functions, the joint cost function as defined by B provides a simple cost-efficiency criterion for comparing various splits, which means that, when $B(\alpha) < B(\hat{\alpha})$, there must be a reallocation such that all parties would benefit from a change from the split $\hat{\alpha}$ to the split α. Given a pair of bids, (P_D, P_S), the buyer determine the outcome of the split-award auction. Say, the total payment by the buyer at an arbitrary split of α is given by the following:

$$G(\alpha) = P_D(\alpha) + P_S(\alpha), \qquad \alpha \in [0,1]$$

To minimize the cost of purchase, buyer selects a split result α by choosing the optimal level of α:

$$\alpha \in \mathrm{argmin}_{\alpha\in[0,1]}\{G(\alpha)\} \tag{1}$$

In (1), without any information about suppliers' costs, the buyer is making comparison between the submitted prices before and after. When there're ties, which can lead to an award choice from (1) that is not unique, in order to break the tie, we have regulate that in the case where there're ties, α has to be chosen randomly from among those splits in the set $\mathrm{argmin}_{\alpha\in[0,1]}\{G(\alpha)\}$ with the smallest $\mathrm{B}(\alpha)$ value. Since a lower-cost bidder will avoid a tie by reducing his bid by a small amount, then the procedure above mimics the outcome that occurs when bid prices are discrete instead of continuous. In order to make both suppliers produce a positive amount, I added a restrict that $\alpha \in (0,1)$ and not equals to 0 or 1.

In this model, Nash equilibrium is a pair of bids, (P_D^*, P_S^*), that are mutual best response for the suppliers. If the equilibrium α satisfies (1) and the regulation of tie-breaking, then it is an equilibrium outcome. The best-response property provides that none of the bidders could increase her profits above their realized values of profits as following:

$$\Pi_i^*(\alpha) = P_i^*(\alpha) - C_i^*(\alpha), \qquad i = D, S$$

Supplier can take her opponent's bid as given by altering P_i^*. Actually, by raising her price at α^* and by lowering her price at some $\hat{\alpha}$, a bidder can try to induce the buyer to select $\hat{\alpha}$, indeed, a bidder can change the prices at many splits simultaneously by altering a bid. Let the following denote the equilibrium purchasing price:

$$g^* = \mathrm{G}(\alpha^*) = P_D^*(\alpha^*) + P_S^*(\alpha^*)$$

A number of economic situations are consistent with the above model. These situations are specified by describing the factors that determine costs for each bidder. Actually, by the study of costs, when the determination of C_D and C_s is too complicated, the equilibrium structure of a split-award auction is completely determined by the joint production cost of the suppliers.

ISSN 2616-7433 Vol. 1, Issue 8: 125-131, DOI: 10.25236/FSST.2019.010820

3. Equilibrium Bids

3.1 Calculation of equilibrium bids

A necessary condition for a particular split to be an equilibrium outcome is that the price of purchase for these split equals each other of the sole-source prices.

Lemma 1 (Price equivalence). If choice bundle (P_D^*, P_S^*) is a Nash equilibrium for this model, and let g^* be the associated total price to the buyer. Then, the equilibrium bids satisfy the following:

$$g^* = P_D^*(1) = P_S^*(0)$$

If "*Lemma 1*" fails to hold, then one bidder's sole-source price is strictly greater than g^*. The other bidder can increase her profit by raising all of her bid prices slightly without change the original choice of buyer at α^*. As a result, the buyer is always indifferent with respect to at least two outcomes in equilibrium; and at least three if $\alpha^* \in (0,1)$, a split-award outcome.

3.2 Efficiency and sole-source outcomes

If the outcome turns to be a sole-source award, then one supplier produces all x units, and the joint cost turns to be the cost of the only supplier with a lower cost as following

$$\begin{cases} B(0) = C_S(0), & \alpha = 0 \\ B(1) = C_D(1), & \alpha = 1 \end{cases}$$

Generally saying, bidder *D* has a lower cost with the sole-source outcome, then we have the proposition as following:

Proposition 1. Suppose $B(1) < B(0)$. Then $\alpha^* = 1$ is an equilibrium outcome, and *D* is the sole-source supplier. If $B(1) < B(\alpha)$ for $\forall\alpha \in [0,1)$, then $\alpha^* = 1$ is the unique equilibrium outcome. Equilibrium bids satisfy the following rules:

$$g^* = B(0) = P_S^*(0) = P_D^*(1)$$

$$\Pi_D^* = B(0) - B(1)$$

$$\Pi_S^* = 0$$

It's easy to understand that the sole-source supplier who wins the total award has a lower cost while higher-cost supplier loses with zero profit. At the same time, buyer pays the total price equals to the production cost of the higher-cost supplier. The buyer daces the same price at the outcomes of 0 and 1, and since $B(1) < B(0)$, then the regulation of

ISSN 2616-7433 Vol. 1, Issue 8: 125-131, DOI: 10.25236/FSST.2019.010820

tie-breaking selects $\alpha = 1$. Particularly, as assumed in *Proposition 1* that $B(1) < B(0)$, bidder *D* can always offer a sole-source price that is arbitrarily close to $B(0)$. Also when no cost advantage exists, such that $B(1) = B(0)$, then a coin toss decides between $\alpha^* \in \{0,1\}$, and the buyer purchases x at the production cost.

We can observe that from Proposition 1, a sole-source outcome is a unique equilibrium no matter if sole-source production is efficient. Proposition 1 also shows that a sole-source equilibrium exists regardless of the structure of the joint production costs at split-award outcomes $\alpha \in (0,1)$. As a result, each bidder can unilaterally deny any interior split by submitting a higher price relative to sole-source price for that split. So sole-source outcome might be efficient yet the fairness has not been checked.

Through the above analysis, I could conclude that the split-award outcomes are always more efficient than the sole-source outcome.

3.3 Implicit price collusion and split-award outcomes

Begin with this analysis based on the condition that a split-award is an equilibrium outcome. Also a split-award is efficient relative to a sole-source outcome when $B(\alpha) \leq B(1)$, which is also the sufficient and necessary for α to be an equilibrium outcome.

Proposition 2. Let $N = \{\alpha | B(\alpha) \leq B(1), 0 < \alpha < 1\}$ be the set of outcomes for which joint production costs are less than sole-source production costs. Then, *N* is the set of split-award equilibrium outcomes.

Proposition 3. Let $\alpha^* \in N = \{\alpha | B(\alpha) \leq B(1), 0 < \alpha < 1\}$. If (P_D^*, P_S^*) is a Nash equilibrium with α^* as the equilibrium outcome, then we have the following conditions. First, as the total price to the buyer at α^*, g^* satisfies the following:

$$g^* \in [B(0), B(0) + B(1) - B(\alpha^*)] \tag{2}$$

Second, as the profit of *S* at α^*, Π_S^* and $\Pi_S(0)$ satisfy the following:

$$\Pi_S^* \in [g^* - B(0), B(1) - B(\alpha^*)] \tag{3}$$

$$\Pi_S(0) = g^* - B(0) \tag{4}$$

Third, as the profit of *D* at α^*, Π_D^* and $\Pi_D(1)$ satisfy the following:

$$\Pi_D^* = g^* - \Pi_S^* - B(\alpha^*) \tag{5}$$

$$\Pi_D(1) = g^* - B(1) \tag{6}$$

Furthermore, there is an equilibrium for any g^*, Π_D^* and Π_S^* that satisfy conditions (10)-(14).

From the above, we can see that *Proposition 3* establishes the range of payoffs that is

ISSN 2616-7433 Vol. 1, Issue 8: 125-131, DOI: 10.25236/FSST.2019.010820

associated with a given split-sward equilibrium outcome. As the price to buyer, g^*determines the joint profits $\Pi_D^* + \Pi_S^*$; and in this case, g^* accompany with division of the joint profits between the bidders are two degrees of freedom.

Say, in this case, the buyer price z is greater than g_m. From *Lemma 1*, we know that the sole-source bid prices are equal to the equilibrium split price. As a result, every bidder must receive a profit of at least $z + profit$ at g_m, in order to prevent the sole-source deviation to a price just below $g_m + z$. Also in order to prevent the sole-source deviation to a just below $g_m + z$ price, joint profits must be at least $2z$ higher than the joint profits at g_m. Since the new split price has only increased by z, joint profits cannot be $2z$ higher than it at g_m, which would lead to a result that there is no split-award equilibrium at a price above g_m. Based on the above, we can get *Proposition 4* as following.

Proposition 4. Let $\alpha^* \in N$. Then, over the range of equilibrium payoffs at α^*, the highest purchase price occurs at the bids that generate the highest individual profits for each bidder and the highest joint profits.

As a result, there is a unique Pareto-efficient pair of payoffs at the equilibrium α^*, for which the payoffs involve charging the buyer the highest possible equilibrium prices.

4. Implications

4.1 Information considerations

Earlier in this paper, I clarify that suppliers are fully informed about each other's costs. Actually the assumption of full informed suppliers doesn't need to be taken literally. When suppliers are fully informed, extensive bidding coordination is feasible. For instance, in the highest-price equilibrium, each supplier takes her opponent's cost into consideration and then gets her opponent's incentives. However, the existence of collusive split-award equilibria does not depend on the bidders' information base. Consider a two-bidder model in which government purchase is limited to three awards $\{0, \frac{1}{2}, 1\}$, and each supplier has a private cost parameter, θ_i, drawn from a common knowledge uniform distribution on [0.9,1]. If sole-source costs are equal to θ and the split cost equals to $4\theta/9$ for each supplier, so the following exist:

$$B(\frac{1}{2}) < B(1)$$

Then it is straightforward to verify the following:

$$P(0) = P(1) = 1$$

ISSN 2616-7433 Vol. 1, Issue 8: 125-131, DOI: 10.25236/FSST.2019.010820

$$P\left(\frac{1}{2}\right) = \frac{1}{2}$$

And the above is a split-award equilibrium in which both suppliers make positive profits and the buyer price exceeds the price that would occur in a winner-take-all auction.

As a conclusion, realistically, split-award auction is an efficient way to solve production quality problem. However, as an auction design, it cannot achieve the purpose for the buyer, and might be a worst condition in some cases. On the suppliers' site, split-award auction might lead to an equilibrium that really maximized their benefits, no matter by efficiency or Pareto efficiency.

References

[1] Anton J James, Yao DA (1987). Second Sourcing and the Experience Curve: Price Competition in Defense Procurement, The RAND Journal of Economics. The RAND Corporation, vol.18, no.1, pp.57-76.

[2] Anton J James, Yao D A (1989). Split-Awards, Procurement, and Innovation. The RAND Journal of Economics, vol.20, no.4, pp.538-552.

[3] Anton J James, Yao D A (1989). Coordination in Split Award Auctions. The Quarterly Journal of Economics, vol.107, no.2, pp.681-707.

[4] Bernheim B D, Whinston M D (1986).Menu Auctions, Resource Allocation, and Economic Influence. Quarterly Journal of Economics, vol.101, pp.1-31.

[5] Carbajal Juan Carlos, Ely Jeffrey C (2010). Mechanism Design Without Revenue Equivalence.

[6] Ely Jeffrey. C (2001). Revenue Equivalence Without Differentiability Assumptions.

[7] Jehiel Philippe, Moldovanu Benny (2001). Revenue Maximization and Efficiency in Multi-object Auctions.

[8] Klemperer Paul (1999). Auction Theory: A Guide to the Literature. Journal of Economic Surveys, vol.13, no.3, pp.227-286.

[9] Mathewson G.F, Winter R A (1987). The Competitive Effects of Vertical Agreements: Comment. American Economics Review, vol. 77, pp. 1057-1062.

[10] Rob R (1986).The Design of Procurement Contracts. American Economic Review, vol. 76, pp. 378-389.