

Numerical Study on the Effect of Tooth Chamfering on the Performance of Linear Conjugate Internal Gear Pumps

Peng Wensheng^{1,a}, Guo Yong^{2,b,*}

¹School of Mechanical and Electrical Engineering, Hunan University of Science and Technology, Xiangtan, 411100, China

²Hunan Provincial Key Laboratory of Mechanical Equipment Health Maintenance, Hunan University of Science and Technology, Xiangtan, 411201, China

^a3256938647@qq.com, ^bhnkjdx_guoy@163.com

Abstract: Straight-conjugate internal gear pumps are widely applied in hydraulic systems due to their compact structure, smooth operation, and high volumetric efficiency. To investigate the influence of tooth-profile chamfering on pump performance, a mathematical model of the chamfered tooth profile was established based on the straight-conjugation principle. A combined approach of numerical simulation and experimental validation was employed to systematically analyze the effects of different chamfer parameters on the internal flow field, pressure distribution, and flow pulsation. The results indicate that an appropriately designed chamfer can significantly improve the pressure gradient and flow stability in the meshing region. Compared with the non-chamfered profile, a 0.5 mm arc chamfer increases the average flow rate by 15%–25% and reduces the flow pulsation rate by approximately 35%, resulting in smoother operation and higher energy efficiency. This study elucidates the mechanism by which chamfering reduces meshing impact and optimizes contact stress, providing a theoretical basis for tooth-profile optimization in high-performance hydraulic gear pumps.

Keywords: linear conjugate internal gear pump; tooth chamfering; flow field characteristics; flow pulsation; performance optimization

1. Introduction

With the continuous advancement of hydraulic transmission technology, gear pumps—one of the most widely used power components in hydraulic systems—play a crucial role in determining overall system efficiency and reliability^[1]. Among various types of gear pumps, linear-conjugate internal gear pumps have been widely applied in construction machinery, aerospace systems, and precision hydraulic equipment owing to their compact structure, low noise, smooth transmission, and high volumetric efficiency^[2]. Compared with conventional involute gear pumps, linear-conjugate gear pumps achieve a constant transmission ratio through linear-conjugate meshing, effectively reducing pressure fluctuation and flow non-uniformity during the meshing process^[3]. However, as hydraulic components evolve toward higher pressure, higher speed, and greater precision, the influence of micro-geometry features on gear meshing behavior and hydrodynamic performance has become increasingly significant. Among these features, tooth-edge chamfering—an essential geometric modification parameter—plays a key role in enhancing flow stability, sealing performance, and noise reduction in gear pumps^[4].

Existing research mainly focuses on tooth-profile modification, meshing error compensation, and flow pulsation control^[5], with most studies emphasizing macro-level profile design and meshing optimization, whereas investigations on tooth-edge chamfering remain relatively limited. Jiang et al. demonstrated that precise tooth-profile modification—such as equivalent tooth-angle correction achieved through forming methods—can significantly reduce surface deviation and improve meshing performance, thus playing a critical role in enhancing gear machining quality^[6]. Wang et al. reported that tooth-profile error compensation combined with machine-tool parameter reverse adjustment can effectively reduce surface deviation and improve meshing accuracy, which is essential for high-precision gear manufacturing and transmission performance optimization^[7]. Ming et al. developed a tooth-surface error identification model and optimized machining parameters using sequential quadratic programming, achieving substantial reductions in face-gear surface deviation and improving processing accuracy and engineering applicability^[8]. Therefore, investigating the influence of chamfer parameters on the meshing

and flow characteristics of linear-conjugate internal gear pumps holds significant theoretical importance^[9].

Based on the above analysis, this study focuses on a linear-conjugate internal gear pump and establishes a chamfer mathematical model tailored to the geometry and meshing characteristics of linear-conjugate tooth profiles. The effects of various chamfer radii and geometric forms on the internal flow-field structure, pressure distribution, and flow pulsation are systematically analyzed. Through finite-element numerical simulations, the mechanisms by which chamfering reduces meshing impact, optimizes contact stress, and enhances internal flow stability are thoroughly examined. The main contributions of this work include the development of a parameterized chamfer modeling method for linear-conjugate tooth profiles, a quantitative analysis of the coupled effects between chamfer geometry, flow-field characteristics, and pressure pulsation, and the revelation of key mechanisms by which chamfering improves meshing-zone dynamic performance. The findings provide a solid theoretical foundation for tooth-profile optimization in linear-conjugate gear pumps and offer valuable insights for structural enhancement, noise control, and efficiency improvement in high-performance hydraulic gear pumps.

2. Theoretical Foundations and Mathematical Model

2.1 Meshing Principle of Linear-Conjugate Internal Gears

A fundamental requirement of gear transmission is the maintenance of a constant instantaneous transmission ratio, meaning that the angular velocity ratio between the driving and driven gears must remain invariant throughout the meshing process. To satisfy this condition, the tooth profiles of the mating gears must form a pair of *conjugate curves*. Conjugate curves are defined as profiles whose common normal at every contact point passes through a single fixed point during their relative motion. This geometric characteristic ensures that the law of gearing is fulfilled, enabling smooth power transmission without slipping.

According to the law of gear meshing, at any instant, the common normal $\mathbf{n}-\mathbf{n}$ at the contact point P between the two tooth profiles must intersect a fixed point O , which lies on the pitch circles of both gears and is known as the pitch point. If the angular velocities of the driving and driven gears are denoted as ω_1 and ω_2 , respectively, the equality of normal components of velocity at the contact point leads to the following relationship:

$$v_{1n} = v_{2n} \quad (1)$$

$$\omega_1 O_1 P \sin \alpha = \omega_2 O_2 P \sin \alpha \quad (2)$$

Here, O_1 and O_2 denote the centers of the two gears, and α represents the angle between the common normal at the contact point and the radii drawn from O_1 and O_2 to the contact point. This condition ensures that the instantaneous angular velocity ratio of the two gears remains constant, yielding:

$$\frac{\omega_1}{\omega_2} = \frac{O_2 P}{O_1 P} = i \quad (3)$$

Therefore, as long as the tooth profiles satisfy the geometric condition that the common normal always passes through a fixed point during meshing, conjugate engagement can be achieved.

2.1.1 Tooth Profile Generation Principle

In gear design, if the tooth profile of one gear is known, the conjugate profile of the mating gear can be obtained using the envelope method. Specifically, when the known gear performs pure rolling along its actual motion trajectory, the envelope of its tooth profile in the coordinate system of the mating gear forms the conjugate curve. Linear-conjugate gears represent a special class of conjugate gears in which the tooth profile is a straight line, and the direction of the common normal remains constant and always passes through the fixed pitch point. This ensures a constant transmission ratio and smooth operation. Compared with conventional involute gears, linear-conjugate gears exhibit more uniform contact stress, reduced noise, and smaller meshing impact, making them suitable for high-precision and high-stability internal gear pumps.

2.1.2 Derivation of Tooth Profile Equations

According to the geometric modeling method for arc-chamfered tooth profiles reported in the literature, the tooth profile equation after chamfering can be expressed accordingly. Unlike involute gears, the tooth profile of the pinion in a linear-conjugate internal gear pump is a straight line. Based on the geometric relationship between the tooth profile half-angle and the tooth thickness, the tooth profile equation for the left-side flank of the pinion can be written as:

$$y_1 = x_1 \operatorname{ctg} \beta + r_1 \cos \theta + r_1 \sin \theta \operatorname{ctg} \beta \quad (4)$$

β -the half-angle of the tooth profile

θ -the corresponding tooth-thickness angle

Since the tooth profiles of the pinion and the internal gear form a conjugate pair, the conjugate profile of the internal gear can be derived using the tooth-profile reversal method. When the pinion rotates by an angle φ_1 , the internal gear rotates by an angle φ_2 according to the meshing principle. Consequently, the tooth profile equation of the mating internal gear can be obtained as:

$$\begin{cases} x_2 = x_1 \cos(\varphi_1 - \varphi_2) - y_1 \sin(\varphi_1 - \varphi_2) + e \sin \varphi_2 \\ y_2 = x_1 \sin(\varphi_1 - \varphi_2) + y_1 \cos(\varphi_1 - \varphi_2) + e \cos \varphi_2 \end{cases} \quad (5)$$

e -the center distance

2.1.3 Meshing Characteristics Analysis

From the above equations, it can be concluded that the direction of the common normal at the contact point remains constant during the meshing process of linear-conjugate gears, resulting in a stable transmission ratio and a continuous, smooth contact region. This effectively avoids the velocity fluctuations and impact phenomena that typically occur at the beginning and end of meshing in conventional involute gears. Therefore, the linear-conjugate meshing approach significantly enhances the transmission stability of the gear pump, reduces noise, and prolongs service life.

In summary, the linear-conjugate principle provides a solid theoretical foundation for the tooth-profile design of internal gear pumps, and the corresponding pump housing structure is shown in Figure 1. By satisfying the law of gearing and the envelope condition, it not only ensures a constant transmission ratio and smooth operation but also establishes the basis for subsequent optimization of chamfer geometry and performance improvement.

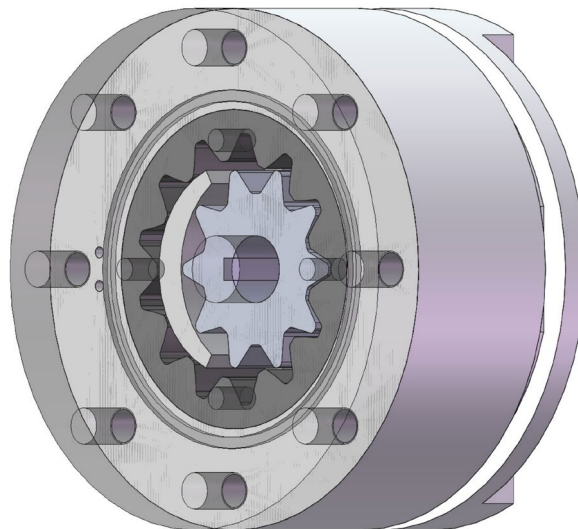


Figure 1. Pump housing structure

In conclusion, the principle of conjugate-curve generation provides the theoretical foundation for gear meshing design. By satisfying the law of gearing and the envelope condition, a conjugate tooth profile capable of delivering a constant transmission ratio and smooth operation can be achieved. This establishes the theoretical basis for subsequent studies on tooth-edge chamfer optimization and

performance enhancement.

2.2 Geometric Characteristics of Tooth Chamfering

To reduce stress concentration and impact during meshing and to improve lubrication, chamfers are typically applied to the tooth edges. According to the type of geometric transition, tooth chamfers can be classified into linear chamfers and rounded (arc) chamfers. The former features a simple structure and easy manufacturability, making it suitable for general operating conditions. However, due to the discontinuity of the normal direction, slight impact may occur at the beginning of meshing. In contrast, rounded chamfers provide smooth geometric continuity through arc transitions, effectively reducing peak contact stress and meshing impact, thereby significantly improving transmission smoothness and lubrication performance. This makes them especially advantageous for high-speed, high-precision, and high-reliability gear pumps.

The selection of chamfer type should comprehensively consider operating conditions, meshing characteristics, manufacturing processes, and performance requirements. Linear chamfers are preferable for low-speed, low-pressure, or cost-sensitive applications, whereas rounded chamfers are more suitable for medium- to high-speed, high-pressure conditions where noise and vibration control are critical. Overall, linear chamfers are appropriate for moderate load conditions, while rounded chamfers are more suitable for high-performance gear pumps. In practical design, optimal chamfer geometry and parameters should be determined by combining numerical simulation results with manufacturing feasibility to achieve a balanced compromise between meshing performance and manufacturability.

3. Simulation Model Construction and Validation

3.1 Model Construction

To investigate the influence of tooth chamfer parameters on the meshing performance of a linear-conjugate internal gear pump, a fluid-domain model and computational mesh of the pump were developed using the specialized pump simulation software PumpLinx, based on the preceding theoretical analysis. This provides the geometric foundation for the subsequent numerical simulations.

3.1.1 Fluid-Domain Model Development

In this study, the working fluid region within the pump chamber was selected as the simulation domain. According to the structural characteristics of the pump housing and the meshing behavior of the gears, the internal fluid-domain geometry was extracted using three-dimensional modeling software. The model includes key regions such as the inlet zone, meshing zone, and outlet zone. The geometric deformation of the fluid domain induced by gear rotation and meshing was fully considered to ensure realistic and continuous fluid motion during the simulation.

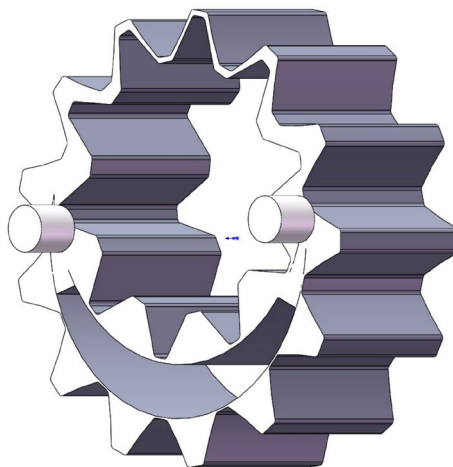


Figure 2. Fluid domain of the gear pump

Figure 2 presents the three-dimensional fluid-domain model of the gear pump. As shown in the figure, the geometry of the fluid domain conforms to the internal cavity of the pump housing, and the gear rotation region is clearly represented, enabling an accurate depiction of the fluid behavior during the

suction, compression, and discharge phases.

3.1.2 Mesh Generation

To ensure both simulation accuracy and computational efficiency, the fluid domain was discretized using the finite-volume method. According to the geometric complexity and flow characteristics, local mesh refinement was applied in the meshing zone and regions with high pressure or velocity gradients near the outlet. An unstructured tetrahedral mesh was employed to balance geometric adaptability and numerical convergence. Mesh quality evaluation showed that the minimum element quality remained above 0.6, indicating that the overall mesh meets the requirements for numerical stability.

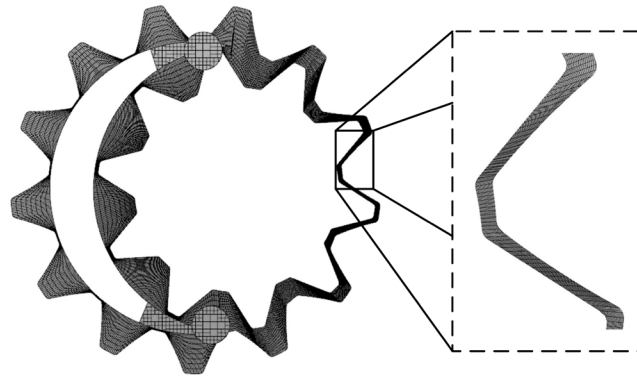


Figure 3. Overall mesh distribution and locally refined regions

Figure 3 shows the mesh distribution of the fluid domain. It can be observed that the overall mesh is uniformly generated, and local refinement has been applied near the meshing zone, which helps capture the localized variations in pressure and velocity within the flow field.

3.1.3 Boundary Conditions and Physical Properties

In the numerical simulations, the inlet of the gear pump was specified as an atmospheric pressure boundary condition, while a constant pressure load was applied at the outlet, with several pressure-difference conditions considered. The surfaces of the gears and pump housing were defined as no-slip boundaries. The oil flow in the inter-tooth region was governed by the continuity equation and momentum conservation equation. The working fluid used in the simulation was No. 46 hydraulic oil, with a density of $\rho = 870 \text{ kg/m}^3$ and a dynamic viscosity of $\mu = 0.041 \text{ Pa}\cdot\text{s}$ under ambient temperature. To ensure numerical stability, a combined steady–transient solution strategy was adopted, and iterative convergence analysis was conducted for all operating conditions.

3.2 Simulation Condition Setup

To analyze the influence of different chamfer parameters on the internal flow characteristics of the linear-conjugate internal gear pump, numerical simulations were conducted based on the established fluid-domain model. The objective was to examine how chamfer geometry affects fluid motion, pressure distribution, and flow stability, and to clarify the role of chamfering in improving meshing-zone flow performance.

By comparing the flow characteristics under various chamfer configurations, this study reveals the effects of chamfer design on flow stability, local pressure distribution, vortex formation, and the evolution of the fluid flow path inside the pump, thereby providing theoretical support for chamfer parameter optimization.

To systematically investigate the influence of tooth chamfering on the internal flow behavior of the gear pump, four representative simulation cases were established, as summarized in the table.

Case 1: No chamfer, used as the baseline to analyze the natural flow field and flow characteristics of the pump.

Case 2: Small circular chamfer with a radius of 0.2 mm and a chamfer angle of 15° , used to examine the effect of a slight chamfer on flow behavior.

Case 3: Medium circular chamfer with a 0.5 mm radius and 15° angle, used to analyze improvements in flow stability and pressure distribution.

Case 4: Large circular chamfer with a 1.0 mm radius and 15° angle, used to evaluate the influence of a larger chamfer on pressure and velocity fields in the meshing zone.

4. Results and Discussion

4.1 Comparison of Flow-Field Characteristics under Different Chamfer Parameters

To investigate the influence of tooth-edge chamfering on the internal flow characteristics of the gear pump, numerical simulations were conducted for four cases: no chamfer, small chamfer (0.2 mm), medium chamfer (0.5 mm), and large chamfer (1.0 mm). The simulation results show that as the chamfer radius increases, the pressure distribution in the meshing zone of the gear pump improves significantly.

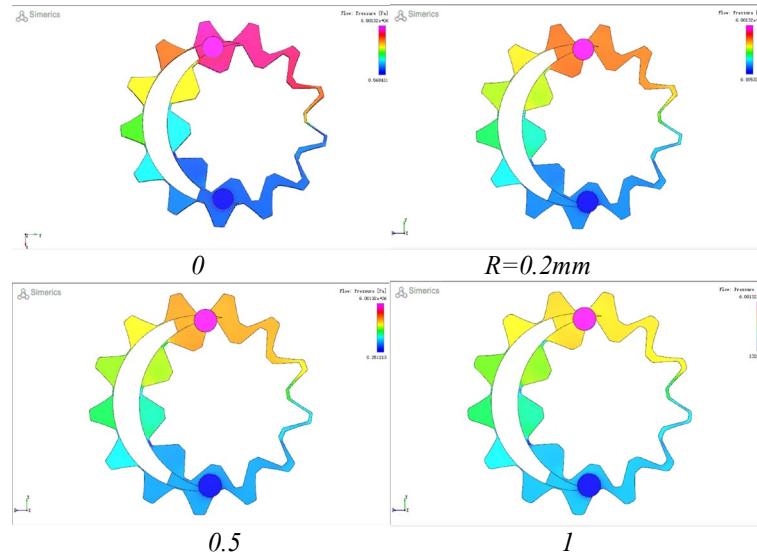


Figure 4. Pressure contour distribution

As shown in Figure 4, the pressure field inside the gear pump undergoes significant changes as the chamfer radius increases. Chamfering improves the uniformity of the flow field in the meshing zone and indirectly enhances volumetric efficiency. In the case without chamfering, the high-pressure region is highly concentrated and unevenly distributed, leading to large pressure gradients near the tooth tip and root, which can cause meshing impact and leakage. When the chamfer radius increases to 0.2 mm, the distribution of the high-pressure region becomes more uniform, the meshing transition becomes smoother, and local pressure spikes are mitigated. With a 0.5 mm chamfer, the pressure field uniformity reaches its optimum, characterized by reduced peak pressure values, a smaller low-pressure region, and overall smoother flow. Further increasing the chamfer radius to 1.0 mm leads to a more flattened pressure distribution; however, the deterioration of sealing performance results in pressure decay and leakage near the outlet.

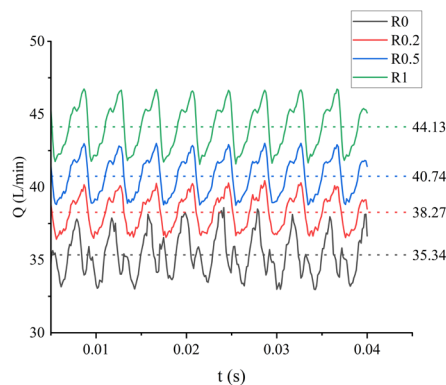


Figure 5. Outlet flow rate curves

According to Figure 5, the average outlet flow rate of the gear pump increases significantly as the

chamfer radius becomes larger, while the amplitude of flow fluctuations gradually decreases. In the absence of chamfering, severe pulsations occur, which are likely to induce meshing impacts. When the chamfer radius reaches 0.5 mm, the outlet flow becomes the most stable, exhibiting the smallest fluctuation. Further increasing the chamfer radius to 1.0 mm continues to enhance the average flow rate; however, localized leakage causes a slight increase in pulsation. Overall, the 0.5 mm circular chamfer provides the optimal balance between flow enhancement and pulsation suppression.

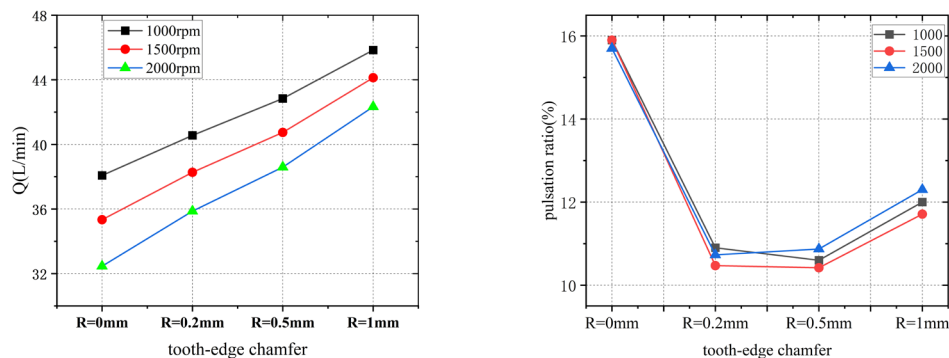
Table 1. Comparison of outlet flow rate and pulsation ratio for various chamfer radii

	R0mm	R0.2mm	R0.5mm	R1mm
$Q_{sh} / L \cdot \min^{-1}$	35.34	38.27	40.74	44.13
$\delta_Q \%$	15.9	10.47	10.42	11.71

In summary, chamfer design has a significant impact on the internal pressure distribution of the gear pump. The medium chamfer (0.5 mm) yields the most uniform pressure gradient in the meshing zone and the smallest pressure fluctuation (as indicated in Table 1), making it the optimal chamfer parameter for achieving smooth meshing and reducing energy losses.

4.2 Analysis of Pressure and Flow Stability at Different Rotational Speeds

Under an outlet pressure of 6 MPa and an atmospheric inlet boundary condition, simulations were performed at rotational speeds of 1000 r/min, 1500 r/min, and 2000 r/min. The results indicate that the average flow rate of the gear pump increases linearly with increasing speed, while the pulsation ratio is strongly influenced by the chamfer parameters.



a. Flow rate at different rotational speeds

b. Flow pulsation ratio at different rotational speeds

Figure 6. Flow rate and pulsation ratio at different rotational speeds

As shown in Figure 6, under the condition of an outlet pressure of 6 MPa and atmospheric inlet pressure, clear trends can be observed in the variation of average flow rate and pulsation ratio with different chamfer radii at various rotational speeds. As the speed increases, the overall displacement rises, while flow pulsation is strongly affected by the chamfer geometry. Without chamfering, the pulsation ratio is approximately 15.9%, indicating large fluctuations. When the chamfer radius increases to 0.2 mm, the pulsation ratio decreases to about 10.5%, resulting in a more stable flow. At 0.5 mm, the pulsation reaches its minimum value of around 10.4%, providing the smoothest operation. Further increasing the chamfer radius to 1.0 mm enhances the average flow rate, but localized leakage causes a slight increase in pulsation (about 11.7%). These results demonstrate that an appropriate chamfer can effectively suppress pressure fluctuations while maintaining sealing performance, with the 0.5 mm circular chamfer offering the best overall balance.

4.3 Flow Rate and Pulsation Ratio under Different Pressure Differences

To investigate the effects of various inlet–outlet pressure differences on the internal flow and pressure distribution of the gear pump, numerical simulations were conducted under rotational speeds of 1000 r/min, 1500 r/min, and 2000 r/min with pressure differences of 1 MPa, 6 MPa, and 10 MPa. For each pressure-difference condition, four tooth-profile configurations were analyzed with chamfer radii of 0

mm, 0.2 mm, 0.5 mm, and 1.0 mm, enabling a comparative evaluation of flow characteristics and loading behavior under both low- and high-pressure operating conditions.

Compared with the unchamfered tooth profile, applying a 0.5 mm circular chamfer increases the average flow rate by approximately 15%, with the improvement reaching up to 25% under high-pressure conditions. For a constant pressure difference, the average flow rate increases nearly linearly with rotational speed, indicating a proportional relationship between displacement and speed. As the pressure difference increases from 1 MPa to 10 MPa, the enhanced filling of the meshing zone by high-pressure oil results in an overall increase in displacement. In the unchamfered case, severe flow separation at tooth sharp edges leads to significant energy loss and the lowest flow output. A small chamfer (0.2 mm) improves flow passage smoothness, while a medium chamfer (0.5 mm) produces the smoothest flow field and lowest flow resistance, yielding the highest flow rate. However, an excessively large chamfer (1.0 mm) reduces the effective sealing area, increasing leakage and causing a slight decrease in flow output.

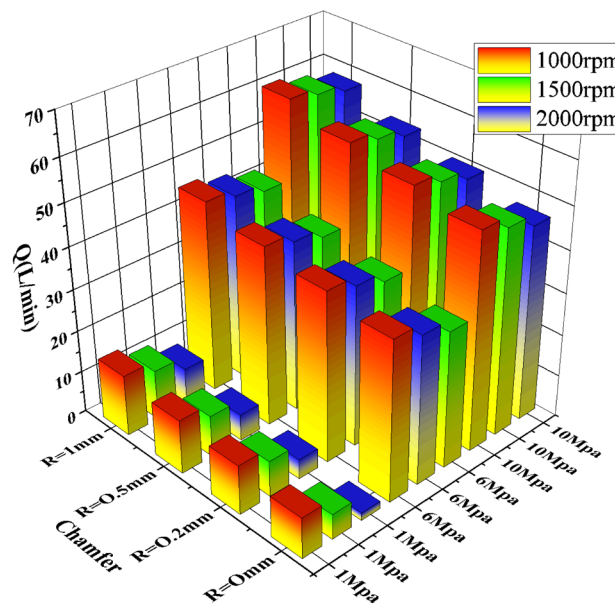


Figure 7. Flow rate under different pressure differences and rotational speeds

As illustrated in Figure 7, the comparison under different pressure-difference conditions shows that the influence of chamfering is relatively weak at low pressure, whereas its regulating effect becomes much more pronounced under medium to high pressures (6–10 MPa). In particular, under the high-pressure condition of 10 MPa, the 0.5 mm chamfer maintains the highest flow-rate output, demonstrating excellent flow stability and sealing performance.

Chamfer design significantly improves the flow stability of the gear pump. The pulsation ratio with a 0.5 mm circular chamfer is reduced by approximately 35% compared with the unchamfered case, resulting in noticeably smoother operation. Under low pressure difference (1 MPa), the unchamfered profile exhibits the highest pulsation ratio (approximately 15%–17%). When the chamfer radius is increased to 0.2 mm and 0.5 mm, the pulsation ratio decreases to about 10%–11%, indicating that chamfering enhances flow continuity in the meshing zone and mitigates impact. However, when the chamfer radius becomes excessively large (1.0 mm), the reduction in sealing area and the increase in localized leakage cause the pulsation ratio to rise slightly to about 11%–12%.

Under medium and high pressures (6–10 MPa), the internal pressure gradient within the pump increases, and the stabilizing effect of chamfering becomes even more significant. Without chamfering, the pulsation ratio is around 16%, whereas the 0.5 mm chamfer consistently yields the lowest pulsation ratio (approximately 10.3%–10.5%) across all rotational speeds. This demonstrates its ability to effectively suppress flow fluctuations and stabilize output flow under high-pressure conditions.

Table 2. Pulsation ratios under different rotational speeds and pressure differences

rotation rate / r/min	1000	1500	2000	1000	1500	2000	1000	1500	2000
pressure difference /Mpa	1			6			10		
0mm	15.10	15.90	17.07	27.49	15.90	17.96	82.00	15.70	16.74
0.2mm	10.18	10.90	11.22	11.63	10.47	10.55	16.19	10.73	10.34
0.5mm	10.02	10.60	11.03	11.11	10.42	10.44	13.06	10.87	10.28
1mm	12.40	12.00	11.89	12.04	11.71	11.61	11.72	12.30	11.67

Table 3. Summary of parameter effects and trend characteristics

Operating Parameter	Trend Characteristics	Optimal Chamfer Radius	Major Effects
Pressure difference from 1–10 MPa	Average flow rate increases significantly; pulsation increases slightly	0.5 mm	Enhances pressure-bearing capacity and flow stability
Increasing chamfer radius	Pulsation ratio decreases first and then increases	0.5 mm	Smoothens flow and reduces meshing impact
Rotational speed from 1000–2000 rpm	Flow rate increases linearly; pulsation shows slight fluctuation	0.5 mm	Maintains stable flow output

The results indicate that an appropriately designed tooth-edge chamfer can significantly enhance the hydrodynamic performance of a linear-conjugate internal gear pump. Detailed pulsation ratios under different operating conditions are listed in Table 2. Compared with the unchamfered tooth profile, adopting a 0.5 mm circular chamfer increases the average flow rate by approximately 15%–25% and reduces the flow-rate pulsation ratio by about 35%. This effectively mitigates meshing impact and pressure fluctuations, thereby substantially improving operational smoothness and volumetric efficiency. Finally, Table 3 summarizes the parameter effects and trend characteristics, confirming the 0.5 mm chamfer as the optimal choice. These findings provide valuable guidance for the tooth-profile optimization of high-performance hydraulic gear pumps.

5. Conclusions

This study investigates the design of tooth-edge chamfers for linear-conjugate internal gear pumps and systematically reveals the influence of chamfer geometry on pump hydrodynamic performance through theoretical modeling, numerical simulation, and parameter analysis. The results demonstrate that chamfering effectively mitigates local high-pressure regions and flow separation in the meshing zone, thereby improving flow continuity and sealing performance. Among the tested configurations, the 0.5 mm circular chamfer exhibits the best overall performance, increasing the average flow rate by approximately 20% and reducing the pulsation ratio by about 35%.

This research confirms the critical role of chamfer geometry in enhancing the overall performance of internal gear pumps and provides valuable guidance for micro-geometry optimization and high-efficiency design of hydraulic gear pumps. The findings also establish a solid foundation for the engineering application of linear-conjugate internal gear pumps.

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