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Optimal Control of Agricultural Product Supply Chain Distribution Inventory under E-commerce Model

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Abstract: *Aiming at the dual-channel supply chain system considering manufacturers, suppliers and retailers, the optimal inventory and order control of the second-order dual-channel supply chain system with no time delay and with time delay considering consumer demand is studied by using the finite-time linear quadratic regulator (LQR) based on optimization problem and the disturbance observer based on Kalman filter. Firstly, a controller is designed based on discrete LQR algorithm for the second-order dual-channel supply chain model without time delay. Secondly, for the second-order dual-channel supply chain model with time delay, a disturbance observer is designed based on Kalman filter theory, so that the state of consumer demand can be accurately observed in a limited time. At the same time, the parameters representing inventory and order quantity are constructed with nonlinear constraints, and the second-order discrete supply chain system with time delay is controlled by LQR and loop algorithm to solve the optimal control problem. Finally, the validity of the theorem is verified by a numerical example.*

Keywords: *Dual-channel supply chain model; Kalman filter; LQR control algorithm; Inventory control*

1. Introduction

In order to solve the mismatch between production and demand and the reverse globalization caused by the epidemic, many countries are committed to developing their own agricultural systems. At the same time, with the changes of life and consumption patterns, online shopping and green consumption have gradually become new consumption habits, especially during the epidemic period. This consumption concept has been continued in the post-epidemic period, which shows that consumers' acceptance and dependence on online purchase of agricultural products are increasing. At present, the sales of agricultural products show the development trend of online and offline omni-channel integration, so it is urgent to study the dual-channel agricultural product supply chain considering e-commerce factors in the face of such consumption trends. Only by establishing a stable and lasting dual-channel supply chain can enterprises maintain their advantages in the fierce market competition and win the trust and favor of consumers. Therefore, the optimization and innovation of supply chain has become the key for enterprises to enhance their core competitiveness.

Agricultural products are perishable products with a short life cycle. Scholars have made considerable research results on the deterioration rate of agricultural products supply chain[6,10], but there are still some shortcomings in considering the real situation, such as not considering the stochastic uncertainty of demand. The demand considered in this paper is a constant plus random disturbance, which is more in line with the macro market rules. Online shopping can reduce costs, and e-commerce is a very important environmental factor. Compared with the research object of literature[5], this paper considerers e-commerce, a very important environmental factor. Most of the research on the inventory control of fresh agricultural products is based on a single-objective inventory control model, while there are few multi-objective inventory control models. In this paper, the supplier and retailer are considered.

By establishing the mathematical model of inventory management[3], we can predict the demand and supply of agricultural products, formulate the optimal inventory management strategy, and avoid too much or too little inventory. This study deeply discusses the inventory control of agricultural products supply chain system in the face of unstable demand, aiming at reducing inventory backlog and loss, reducing economic losses, and realizing the optimal allocation of resources through automatic control management, enriching and perfecting the shortcomings of existing research. Measures such as automatic inventory control can effectively improve the circulation efficiency of agricultural products, reduce inventory costs, ensure the quality of agricultural products and enhance the profits of enterprises.

This study takes the dual-channel agricultural product supply chain considering e-commerce as the research object, explores the optimization problem of its inventory decision, and further analyzes the second-order agricultural product supply chain model with time delay and designs a disturbance observer. Compared with the previous research, the main innovations of this paper are to study the decision-making problem of supply chain from a dynamic perspective and analyze the control principle existing in the supply chain of agricultural products from the perspective of automatic control principle. Compared with static research[2], this paper considers the influence of time factor and combines the characteristics of dynamic change of demand disturbance to construct a state space equation to explore the optimal inventory of agricultural products supply chain. In this paper, the supply chain of agricultural products is analyzed systematically, and the linear quadratic regulator (LQR) controller is designed by establishing a mathematical model to make the system achieve stable output results. Compared with the literature[4], the research method of this paper introduces the automatic control principle, which improves the reliability of the control effect. Based on the dynamic perspective and automatic control principle, this study aims at the dual-channel agricultural product supply chain of e-commerce to optimize inventory decision and improve system stability. By constructing the state space equation and designing the controller based on LQR, this paper has achieved innovation in theory and method.

2. Supply chain model and method

2.1. Supply chain model

In this paper, an inventory and profit model of dual-channel supply chain system considering e-commerce factors is established. Producers need to produce fresh agricultural products and supply them to suppliers, while suppliers purchase and supply agricultural products according to certain consumer demand in the consumer market. Suppliers' products can be sold to retailers through offline channels or directly to consumers through e-commerce channels. The quantity of products of retailers and suppliers fluctuates under the influence of market conditions. Consumer demand also fluctuates slightly with the market. At the same time, it is assumed that the sales price of goods in online channels is the same as that in offline channels, which can simplify the coefficient of consumers' preference for different channels.

There are two trading modes: online e-commerce channels (such as Taobao, Jingdong, Amazon, etc.) and offline channels (such as supermarkets, brand stores, wholesale cities, etc.). Consumers can purchase products directly from suppliers through e-commerce channels. However, agricultural products rotted and damaged in transportation and storage can only be discarded as garbage, so it is assumed that the amount of corruption per unit of agricultural products at each moment is certain. Supplier's profit refers to the profit obtained by selling products to consumers and retailers minus the preservation cost of existing inventory, while retailer's profit refers to the profit obtained by selling products to consumers minus the preservation cost of existing inventory. Based on this principle, the topological structure of the channel supply chain for the production-sales operation of agricultural products enterprises can be established, as shown in Figure 1.

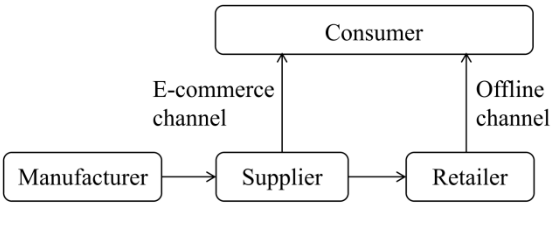

Figure 1: Dual channel supply chain model

For convenience, some variables are used here instead of specific physical meanings. Details are as follows:

- $x_1(k)$ Indicates the actual inventory of the supplier at the time k .
- $x_2(k)$ Indicates the actual inventory of the retailer at the time k ,

- $u_1(k)$ Indicates the quantity of products ordered by the supplier from the manufacturer at the time k,
- $u_2(k)$ Indicates the number of products ordered by the retailer from the supplier at the time k,
- $P_1(k)$ represents the supplier's profit at the time k,
- $P_2(k)$ represents the retailer's profit at the time k,
- d indicates the lag time for retailers to get products after ordering products from suppliers.
- $\varrho(k)$ Represents the total demand of consumers,
- $\nu(0 \le \nu \le 1)$ indicates the customer preference index of e-commerce channel, which is a constant parameter.
- $\xi(0 \le \xi \le 1)$ indicates the deterioration rate of corruption, which is a constant parameter.
- c represents the cost of storing one unit of agricultural products, which is a 1×2 matrix.
- p represents the profit of selling one unit of agricultural products, which is a 1×2 matrix.
- $x(k)$ represents the inventory of the whole supply chain at the time k, which is a 2×1 matrix.
- $P(k)$ represents the profit of the whole supply chain at the first moment, which is a 2×1 matrix.

2.1.1. Supply chain model without time delay

Combined with the production and distribution process of agricultural products, the dual-channel consumer preference and the perishable nature of agricultural products, and considering the small-scale fluctuation of consumer demand, a dynamic model of dual-channel agricultural product supply chain without time delay can be established, which includes an inventory model and a profit model. The equation of state is as follows:

$$\begin{aligned}
&x_1(k+1) = x_1(k) - \xi x_1(k) + u_1(k) - u_2(k) - \nu\varrho(k),\\
&x_2(k+1) = x_2(k) - \xi x_2(k) + u_2(k) - (1-\nu)\varrho(k),\\
&P_1(k+1) = P_1(k) - Cx_1(k) + p\nu\varrho(k),\\
&P_2(k+1) = P_1(k) - Cx_2(k) + p(1-\nu)\varrho(k).
\end{aligned} \tag{1}$$

Equation (1) describes the dynamic process of the respective inventories of suppliers and retailers and the respective profits of suppliers and retailers in the supply chain. Equation (1) can be simplified as the following state space model of inventory and profit of the whole supply chain:

$$\begin{aligned}
&x(k+1) = A_1 x(k) + B_1 u(k) + D\varrho(k),\\
&P(k+1) = P(k) - cx(k) - pD\varrho(k),
\end{aligned} \tag{2}$$

Among them, $A_1 = \begin{bmatrix} 1-\xi & 0 \\ 0 & 1-\xi \end{bmatrix}$, $B_1 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$, $D = \begin{bmatrix} -\nu \\ \nu-1 \end{bmatrix}$.

2.1.2. Supply chain model with time delay

Similarly, the state equation of the dual-channel supply chain model with time delay can be obtained as follows:

$$\begin{aligned}
&x_1(k+1) = x_1(k) - \xi x_1(k) + u_1(k) - u_2(k) - \nu\varrho(k),\\
&x_2(k+1) = x_2(k) - \xi x_2(k) + u_2(k-d) - (1-\nu)\varrho(k),\\
&P_1(k+1) = P_1(k) - Cx_1(k) + p\nu\varrho(k),\\
&P_2(k+1) = P_2(k) - Cx_2(k) + p(1-\nu)\varrho(k).
\end{aligned} \tag{3}$$

Equation (3) describes the dynamic process of the respective inventories of suppliers and retailers and the respective profits of suppliers and retailers in the supply chain. Equation (3) can be simplified to the state space model of inventory and profit of the whole supply chain as shown below:

$$\begin{aligned} x(k+1) &= A_2 x(k) + B_2 u(k) + Cu(k-d) + D\varrho(k), \\ P(k+1) &= P(k) - cx(k) - pD\varrho(k), \end{aligned} \tag{4}$$

Among them, $A_2 = \begin{bmatrix} 1-\xi & 0 \\ 0 & 1-\xi \end{bmatrix}$, $B_2 = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, $D = \begin{bmatrix} -\nu \\ \nu-1 \end{bmatrix}$.

2.2. Method

2.2.1. LQR algorithm

LQR algorithm[7], called linear quadratic regulator, is the key technology in modern control theory. It mainly deals with linear systems in state space, and finds the best control strategy by optimizing a quadratic objective function involving system state and control input.

The cost function J represents the state vector and the input cumulative value in the process from time 0 to time N. The cost function J is used to describe the control tendency of LQR, and the control characteristics are modified by adjusting the weight matrix. The form of the cost function can be obtained as follows:

$$J = \frac{1}{2} X\left(t_f\right)^T F X\left(t_f\right) + \frac{1}{2}\int_{t_0}^{t_f}\left(X^T Q X + U^T R U\right)dt, \tag{5}$$

Where t_f is the target time, t_0 is the initial time. Matrix F is the weight matrix of terminal cost, which is used to describe the weight of terminal state vector. Matrix Q and matrix R respectively represent the state vector X at that time and the weight of system input U at that time in the process cost. From the form of cost function, we can see that they are all semi-positive definite matrices, that is $J \geq 0$. Therefore, the optimization J will make the state vector converge to zero vector.

Because it is LQR control for discrete systems, it is also necessary to describe the cost function discretely, and the discrete cost function can be obtained as follows:

$$J = \frac{1}{2} X(N)^T F X\left(N\right) + \frac{1}{2}\sum_{k=0}^{N-1}\left(X(k)^T Q X\left(k\right) + U(k)^T R U\left(k\right)\right). \tag{6}$$

2.2.2. Kalman filter

Kalman filter[1] is an efficient recursive autoregressive filter, which is specially used to estimate the state of dynamic system from incomplete measurement data containing noise. This kind of filter uses the measured data at different time points and their joint distribution to estimate unknown variables, and provides more accurate estimation results than single strategy.

Lemma 1. The state of Kalman filter is represented by the following two variables:

- $X_{k|k} = E\left(X_k \mid Y_1, Y_2, \ldots, Y_k\right)$ represents the state estimation at the time k,
- $X_{k|k-1} = E\left(X_{k-1} \mid Y_1, Y_2, \ldots, Y_{k-1}\right)$ represents the state of the known system in the past k time, predicts the state of the $k+1$ time,
- $P_{k|k}$ represents the covariance matrix of posterior estimation error and measures the accuracy of estimation value.

Kalman filtering process includes two main steps: prediction and update.

- Prediction step

In the prediction step, the state of the current moment is predicted according to the state of the previous moment and the control quantity. This predicted value is an estimate, because it has not considered the observed value at the current moment. The error covariance matrix of the predicted value is calculated from the error covariance matrix of the previous moment and the system noise covariance matrix.

$$\begin{aligned} x_{k|k-1} &= F_k x_{k-1} + B_k u_k, \\ P_{k|k-1} &= F_k P_{k-1|k-1} F_k^T + Q_k. \end{aligned} \tag{7}$$

- Update step

In the updating step, the state estimation value of the current moment is calculated according to the observed value and the predicted value of the current moment. This predicted value is a more accurate estimate because it has taken into account the observed value at the current moment. The error covariance matrix of the predicted value is calculated from the error covariance matrix of the previous moment and the system noise covariance matrix.

$$\begin{aligned} K_k &= P_{k|k-1} H_k^T \left(H_k P_{k|k-1} H_k^T + R_k \right), \\ x_{k|k} &= x_{k|k-1} + K_k \left(z_k - H_k x_{k|k-1} \right), \\ P_{k|k} &= \left(I - K_k H_k \right) P_{k|k-1}. \end{aligned} \tag{8}$$

2.2.3. Constraint condition

Because the inventory of suppliers and retailers in this paper cannot be less than 0, and the number of suppliers ordering from manufacturers and retailers ordering from retailers cannot be less than 0. The following constraints can then be obtained:

Definition 1. (Constraints of dual-channel supply chain model). Considering that the number of orders should also have an upper limit in actual situation, the definition h indicates the maximum number of products ordered by the supply chain,

$$0 \le u \le h, 0 \le x \le \infty. \tag{9}$$

Lemma 2. (The upper and lower bounds of control variables and state variables are transformed into standard forms). Considering practical application, the most common constraint is to impose upper and lower bounds on input and state variables. Namely:

$$\begin{aligned} u_{\text{low}} &\le u_k \le u_{\text{high}} \\ x_{\text{low}} &\le x_k \le x_{\text{high}} \end{aligned} \tag{10}$$

Equation (10) can be written in the form of the following matrix:

$$\begin{bmatrix} -I_{p\times p} \\ I_{p\times p} \end{bmatrix} u_k \le \begin{bmatrix} -u_{\text{low}} \\ u_{\text{high}} \end{bmatrix}, \begin{bmatrix} -I_{n\times n} \\ I_{n\times n} \end{bmatrix} x_k \le \begin{bmatrix} -x_{\text{low}} \\ x_{\text{high}} \end{bmatrix}, \tag{11}$$

By introducing matrix M、F and M_{N_p}, it can be transformed into standard form through variation (11), and the constraint matrix is a constant matrix, which is irrelevant to k, that is

$$Mx_k + Fu_k \le T, \tag{12}$$

$$M_{N_p} x_{k+N_p} \le T_{N_p}. \tag{13}$$

Among them, $M = \begin{bmatrix} 0_{p\times n} \\ 0_{p\times n} \\ -I_{n\times n} \\ I_{n\times n} \end{bmatrix}$, $F = \begin{bmatrix} -I_{p\times p} \\ I_{p\times p} \\ 0_{n\times p} \\ 0_{n\times p} \end{bmatrix}$, $T = \begin{bmatrix} -u_{\text{low}} \\ u_{\text{high}} \\ -x_{\text{low}} \\ x_{\text{high}} \end{bmatrix}$, $M_{N_p} = \begin{bmatrix} -I_{n\times n} \\ I_{n\times n} \end{bmatrix}$, $T_{N_p} = \begin{bmatrix} -x_{\text{low}} \\ x_{\text{high}} \end{bmatrix}$.

3. Results and analysis

In order to get the optimized inventory and order quantity in the dual-channel supply chain system

(2-1)-(2-4), the following controller design flow is used.

1) For the no demand-disturbed dual-channel supply chain system without delay, the LQR controller is obtained based on LQR algorithm, which keeps the dual-channel supply chain system stable.

2) For the dual-channel supply chain system with demand disturbance without delay, a disturbance observer is designed based on Kalman filter theory, so that the system output can track the state of the observer in a limited time.

3) By analyzing the boundedness of the system, combining the previous disturbance observer and LQR controller, the inventory optimization problem of the supply chain is solved under the actual background. By comparing the profits of the three systems with and without time delay, the form of profit maximization is obtained-the supply chain system with time delay and disturbance observer.

3.1. Result

3.1.1. Time-delay supply chain model with no demand disturbance

When the market demand is stable, that is when customers demand $\varrho(k)=0$, the supply chain model without delay can be simplified as: $X(N)=A_1X(N-1)+B_1U(N-1)$.

Theorem 1. Without considering the demand disturbance and time delay, the LQR controller of the supply chain model is $U=-K(N-k)X$. Among it, $K(N-k)=\left(B_1^TP(k-1)B_1+R\right)^{-1}B_1^TP(k-1)A_1$, $P(k)=([A-BK(N-k)]^T\cdot P(k-1)\cdot\left[A-BK(N-k)\right]+K(N-k)^TRK(N-k)+Q$.

3.1.2. Supply chain model without time delay with demand disturbance

When the market demand is unstable, that is, when the customer demand is unstable, that is $\varrho(k)$ is not zero. When the supply chain system is disturbed by unstable demand $\varrho(k)$, the disturbance observer based on Kalman filter is used to estimate the constants in the demand. Its supply chain model (1-2) shows that the inventory and order quantity of each node in the supply chain will be affected when the demand is disturbed. At the same time, $\varrho(k)$ as a white noise, it has a large amount of interference to the supply chain, so it is necessary to use a disturbance observer to observe the disturbance more accurately.

Theorem 2. When considering the demand disturbance, the disturbance observer designed based on Kalman filter is:

1) Calculate the prior error covariance matrix with the last posterior error covariance matrix $P^-(k+1)=A_1P(k)A_1^T+Q$,

2) Then make a priori estimation $\hat{x}^-(k+1)=A_1\hat{x}(k)+B_1u(k)$,

3) Then calculate Kalman gain $K(k+1)=\dfrac{P-(k+1)H^T}{HP^-(k+1)H^T+R}$,

4) Get a posterior estimate $\hat{x}(k+1)=\hat{x}^-(k+1)+K(k+1)\left(z(k+1)-H\hat{x}^-(k+1)\right)$,

5) Calculate the covariance matrix of posterior error to facilitate the next time $P(k+1)=\left(I-K(k+1)H\right)P^-(k+1)$.

3.1.3. Supply chain model with demand disturbance and time delay

When considering the demand, that is, the consumer market can not fully meet the consumer supply. At the same time, it is considered that the supplier will have a time lag when supplying goods to the retailer, because the retailer will consume time in the process of product delivery after placing an order with the supplier, and the last ordered product will have a time lag when it arrives at the retailer. When considering the above two factors, in the presence of demand disturbance, and considering the time lag phenomenon of agricultural product supply chain, its supply chain model is $x(k+1)=A_2x(k)+B_2u(k)+Cu(k-d)+D\varrho(k)$.

By augmenting the disturbance into the state through the augmented system, we can obtain

$$\begin{bmatrix} x(k+1) \\ \varrho(k+1) \end{bmatrix} = \begin{bmatrix} A_2,0 \\ I,0 \end{bmatrix} \begin{bmatrix} x(k) \\ \varrho(k) \end{bmatrix} + \begin{bmatrix} B_2,0 \\ 0,0 \end{bmatrix} \begin{bmatrix} u(k) \\ \varrho(k) \end{bmatrix} + \begin{bmatrix} C,0 \\ 0,0 \end{bmatrix} \begin{bmatrix} u(k-d) \\ \varrho(k-d) \end{bmatrix}, \tag{14}$$

Then the supply chain model can be equivalently written as a two-delay discrete linear system with two inputs and constraints:

$$x_3(k+1)=A_0x_3(k)+B_0u_3(k)+C_0u_4(k-d), \tag{15}$$

Among it, $u_3(k)=u_4(k)$. Accordingly, the performance index function of the supply chain model can be equivalently written as:

$$\begin{aligned} J_N = {} & x(N+1)^T P_{N+1} x(N+1) + \sum_{k=0}^{N} x(k)^T Q_k x(k) \\ & + \sum_{t=0}^{N} u_3^T(k) R_3 u_3(k) + \sum_{t=0}^{N-d} u_4^T(k) R_4 u_4(k), \end{aligned} \tag{16}$$

Among it, $R_t = \begin{cases} R_3+R_4, 0\leqslant k\leqslant N-d \\ R_3, k>N-d \end{cases}$ .For convenience of expression[8,9], the following symbols are introduced:

$$\begin{aligned} U(t) &\triangleq \begin{cases} u_3(k), \\ \begin{bmatrix} u_3(k), 0\leqslant k<d \\ u_4(k-d), k\geqslant d \end{bmatrix}, \end{cases} \\ \bar{u}(k) &\triangleq \begin{cases} C_0u_4(k-d), 0\leqslant k<d, \\ 0, k\geqslant d, \end{cases} \\ \bar{R}_k &\triangleq \begin{cases} R_3, 0\leqslant k<d, \\ \operatorname{diag}\{R_3,R_4\}, k\geqslant d, \end{cases} \\ B_k &\triangleq \begin{cases} B_0, 0\leqslant k<d, \\ [B_0,C_0], k\geqslant d. \end{cases} \end{aligned} \tag{17}$$

Using these symbols, the system can be written as

$$x(k+1)=\begin{cases} A_0x(k)+B_kU(k)+\bar{u}(k), 0\leqslant k<d_2, \\ A_0x(k)+B_kU(k), k\geqslant d_2. \end{cases} \tag{18}$$

The performance index function (16) can be written as

$$J_N = x^T(N+1)P_{N+1}x(N+1)+\sum_{k=0}^{N} x^T(k)Q_kx(k)+\sum_{k=0}^{N} U^T(k)\bar{R}_kU(k). \tag{19}$$

Corresponding dual stochastic state space model;

$$\begin{aligned} x(k) &= A^T x(k+1)+u(k), \\ y(k) &= B^T x(k+1)+v(k), \end{aligned} \tag{20}$$

Among it $k=0,1,\cdots,N-1$, the sum of $u(t)$ and $v(t)$ is zero mean white noise and satisfy

$$\left\langle \begin{bmatrix} x(N+1) \\ u(i) \\ v(i) \end{bmatrix}, \begin{bmatrix} x(N+1) \\ u(j) \\ v(j) \end{bmatrix} \right\rangle = \begin{bmatrix} P_{N+1} & & \\ & Q_i\delta_{i,j} & \\ & & R_i\delta_{i,j} \end{bmatrix}. \tag{21}$$

Then under the supply chain model, the performance index function can be equivalently written as the following quadratic form:

$$\begin{aligned} J_N &= \begin{bmatrix} \xi \\ U \end{bmatrix} \begin{bmatrix} R_{x_0} & R_{x_0 y} \\ R_{y x_0} & R_y \end{bmatrix} \begin{bmatrix} \xi \\ U \end{bmatrix}, \\ &= \xi' R_{x_0} \xi + 2\left(R_{y x_0} \xi\right)' U + U' R_y U. \end{aligned} \tag{22}$$

Among it

$$\begin{aligned} \xi &= \operatorname{col}\left\{x(0), \bar{u}(0), \bar{u}(1), \cdots, \bar{u}(d-1)\right\}, \\ U &= \operatorname{col}\left\{U(0), U(1), \cdots, U(N)\right\}, \\ x_0 &= \operatorname{col}\left\{x(0), x(1), \cdots, x(d)\right\}, \\ y &= \operatorname{col}\left\{y(0), y(1), \cdots, y(N)\right\}, \\ R_{x_0} &= \left\langle x_0, x_0 \right\rangle, \\ R_{x_0 y} &= \left\langle x_0, y \right\rangle, \\ R_{y x_0} &= \left\langle y, x_0 \right\rangle, \\ R_y &= \left\langle y, y \right\rangle. \end{aligned} \tag{23}$$

From the above discussion, the goal of this paper can be equivalently stated as: to solve an optimal control input $\left\{u^*(k)\right\}_{k=0}^{N}$ sequence that minimizes the objective function of the system under constraints.

Subsequently, the similar LQR control and disturbance observer design are carried out through the iterative algorithm in MATLAB, and finally, the corresponding results can be obtained by simulation with the constraint of realistic conditions, that is, the inventory and order quantity should be greater than 0.

3.2. Analysis

In this simulation, three different control strategies are concerned for inventory management, including: the strategy considering disturbance observer and time delay (type I), the strategy not considering disturbance observer (type II) and the strategy not considering time delay (type III). By analyzing the performance of inventory, purchase volume, demand estimation and accumulated profit of these three strategies in a limited time, we can better understand the advantages and disadvantages of each strategy and the applicable scenarios. In this simulation, we assume the following parameters:

3.2.1. Simulation parameter setting

First, we assume the initial parameters:

- Initial inventory: the initial inventory of each commodity is 50 pieces, that is $x_0 = [50;50]$,
- Optimization step size: each step is optimized in 10 steps, that is $N_{op} = 10$,
- Demand model: demand is modeled as the average of 10 plus random noise, that is $wk = 10 + \operatorname{randn}(1, N)$,
- Profit and cost: the profit of selling a commodity is 10, that is $p = [10,10]$; The cost of storing a commodity is 0.1, that is $c = [-0.1, -0.1]$,
- Lag time: In the model with time delay, the supplier needs to lag three days to meet the retailer's order demand, that is $d = 3$,

- E-commerce channel customer preference index: the percentage of consumers who choose e-commerce channels to total consumers, that is $\nu = 0.2$,

- Deterioration rate: the ratio of deteriorated agricultural products to total agricultural products per unit time, that is $\xi = 0.2$,

- Inventory maintenance: the inventory index of both suppliers and retailers is 20 pieces, that is $x_r = [20; 20]$.

Then, suppose the optimization function and the relevant parameters in Kalman filter:

- Suppose the state cost matrix is $\begin{bmatrix} 1,0 \\ 0,1 \end{bmatrix}$. Used to calculate the cost of the state deviating from the target value, and used to punish the inventory state deviating from the target value in the optimization process.

- Assume that the control cost matrix is $\begin{bmatrix} 1,0 \\ 0,1 \end{bmatrix}$. It is used to control the cost of input, punish the change of purchase quantity and control the smoothness of the system in the optimization process.

- Assuming the measurement covariance matrix, that is, measuring the intensity of noise, setting the measurement covariance matrix to a small value shows that the system can improve the high-precision state measurement, that is $R_k = \begin{bmatrix} 0.00001,0 \\ 0,0.00001 \end{bmatrix}$.

- Assuming that the initial value of the error covariance matrix is identity matrix, P will be updated dynamically with the simulation.

3.2.2. Simulation analysis

There are three types of controllers for dual-channel supply chain, namely type I (considering disturbance observer and time delay), type II (not considering disturbance observer) and type III (not considering time delay).

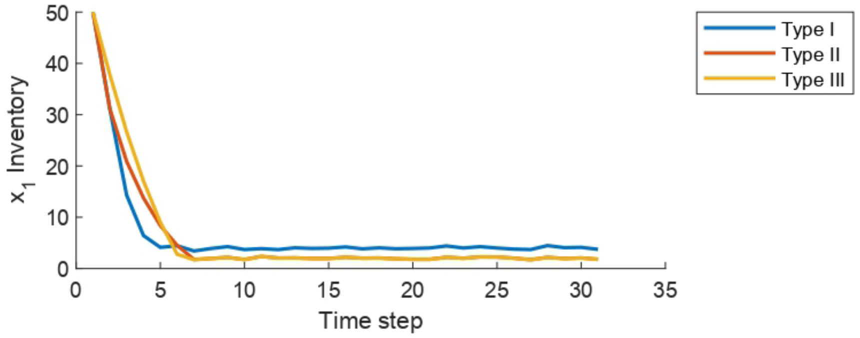

Figure 2: Changes of supplier inventory under different control strategies

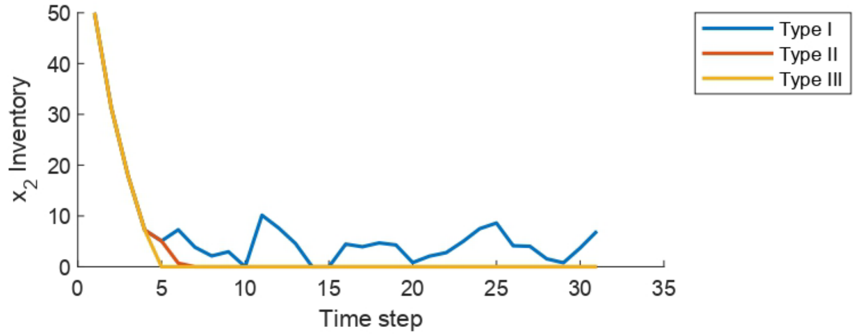

Figure 3: Changes of retailer inventory under different control strategies

These two charts show the inventory changes under different control strategies. As shown in Figure

2(x1 inventory), at the beginning of the time step, the inventory under the three strategies decreased rapidly. After time step 5, the inventory level of type I quickly stabilized at a low level, showing strong inventory control ability. In contrast, type II and type III show slight fluctuations after the initial inventory decline, but the overall situation tends to be stable, indicating that although these strategies are not as good as type I in response speed and control accuracy, they can still maintain a relatively stable inventory level. However, the inventory of type I is higher than that of type II and type III, which can meet certain requirements in the case of market fluctuation, that is, it is more anti-jamming. As shown in Figure 3(x2 inventory), the inventory change of x2 is similar to that of x1, which further verifies the consistency of inventory change of supply chain and retailer under different control strategies. During this period, the inventory fluctuation of type III strategy is larger, which reflects its advantages in market dynamic adaptability. Similarly, although type I is not as stable at 0 as type II and type III, a certain inventory can achieve better anti-disturbance ability.

Although type I strategy is the best in maintaining inventory stability, it may involve higher management cost and complexity. On the contrary, although type II and type III strategies are not as good as type I in control accuracy and reaction speed, they may have advantages in cost and operation simplicity. Therefore, choosing the most suitable inventory control strategy needs to be decided according to the specific business needs and cost-benefit trade-off.

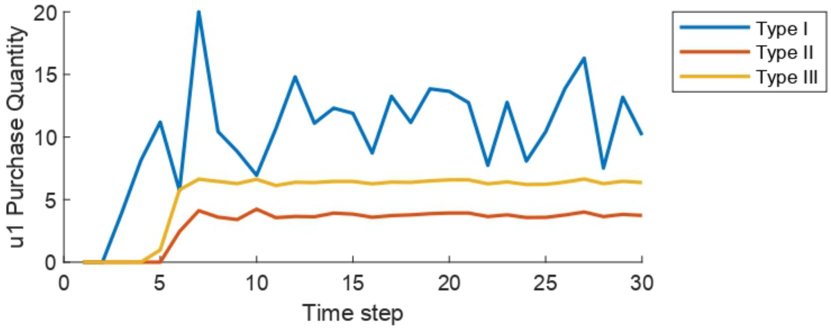

Figure 4: Changes of supplier purchase quantity under different control strategies

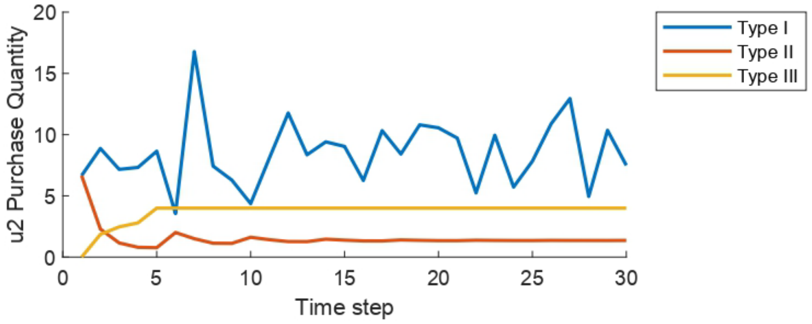

Figure 5: Changes of retailer purchase quantity under different control strategies

These two charts show the changes of inventory order quantity under different control strategies. As shown in Figure 4(u1 purchase volume), the fluctuation of purchase volume under the strategies of type II and type III is small, indicating that these strategies are more stable in purchasing decision. Under the strategy of type I, the purchase quantity fluctuates greatly, which further indicates that in the random market environment, the purchase quantity considering the disturbance observer definitely fluctuates more than the other two, and the purchase quantity of type I is higher than the other two, which implies that type I can maintain stability at a higher purchase quantity. At the same time, we find that after considering the disturbance observer in two systems with time delay, the purchase volume has been greatly improved compared with before, which implies the improvement of profit. As shown in Figure 5(u2 purchase volume), the change trend of u2' s purchase volume is similar to that of u1, which further shows that the purchase decisions of supply chain and retailers are consistent under different control strategies. Similarly, type I has higher purchasing ability and better anti-interference ability.

The inventory and purchase quantity under different control strategies are combined, and the system considering disturbance observer and time delay has the ability to maintain inventory stability at a higher purchase level. The system considering the disturbance observer has a much higher purchase volume than the system not considering the disturbance observer. Although the inventory at the retailer's end is not stable enough, it can fluctuate within a certain range, and higher purchase volume means higher revenue.

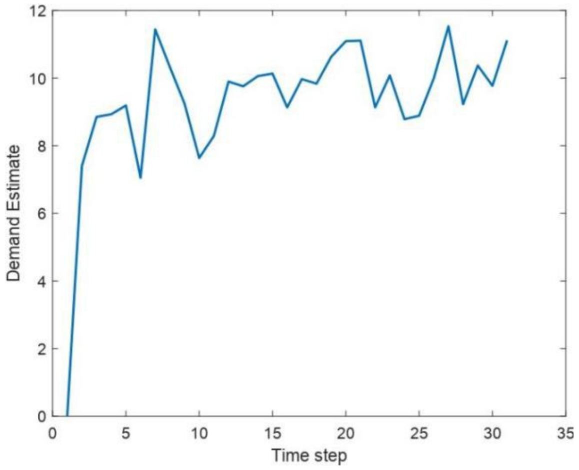

Figure 6: Demand estimation

It can be observed from Figure 6 that it remained at a high level on the whole, showing the instability of demand estimation. This fluctuation may reflect the uncertainty of actual market demand. This disturbance mode is closer to the real market demand, because there are many competing products in today's society and the competition pressure is great, so the demand is greatly disturbed. Studying the demand with this random disturbance has important reference value for making inventory strategy and optimizing supply chain management. By choosing this trend of demand estimation, managers can better predict the market dynamics and make more effective decisions.

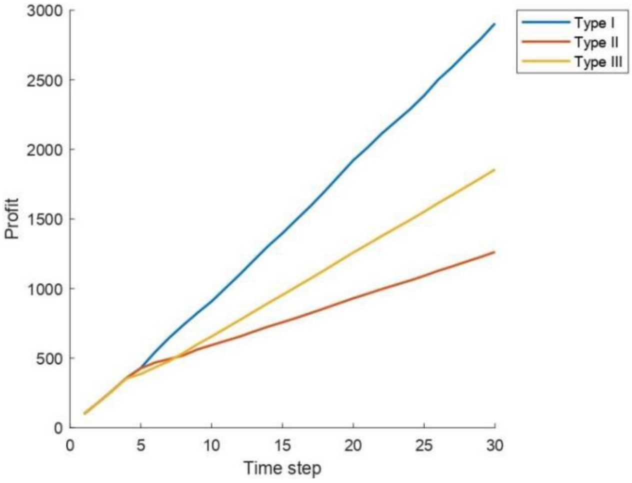

Figure 7: Accumulated profit

This chart(Figure 7) shows the changes of accumulated profits under different control strategies. Among the accumulated profits, type I strategy has the highest accumulated profit, which shows that the strategy considering disturbance observer and time delay can bring higher profits, which shows that this strategy performs best in inventory management and purchasing decision, and can better balance inventory and demand. The cumulative profit of type III strategy is the second. Although it is an idealized model without considering time lag, the profit of supply chain model without considering disturbance and time lag is high, but it can still maintain a good profit level. The cumulative profit of type II strategy is the lowest, and the supply chain model with time delay, which ignores the observation disruptor, has the greatest impact on the profit, resulting in poor system performance and evidently reduced profit.

From the simulation analysis, we can draw the following conclusions:

1) type I strategy with disturbance observer and time delay is the best. This strategy can well balance the inventory and purchase quantity, thus obtaining the highest accumulated profit. Its demand estimation is accurate, and the inventory and purchase volume change smoothly, which shows that the strategy has good adaptability and stability in the face of demand fluctuation.

2) type III strategy without considering time delay is the second. Although it is slightly inferior to type I in demand estimation and inventory management, it can still maintain a good profit level. Ignoring the delay will significantly reduce the system performance and realistic reliability.

3) The performance of type II strategy without considering disturbance observer is the worst. This strategy has not performed well in inventory and purchase quantity management, resulting in a evident reduction in profits. It shows that it is very important to consider the disturbance observer in inventory management with time delay, which shows that the disturbance observer can improve the system performance to some extent.

To sum up, the influence of disturbance observer and time delay should be given priority in inventory management system to improve the stability and profitability of the system.

4. Discuss

On the basis of predecessors, this paper further solves the problem of dual-channel supply chain model with disturbance and time delay. The supply chain topology considered here is a second-order discrete system. Through the LQR controller design for the dual-channel supply chain system without interference and time delay and the observation disturber design for the dual-channel supply chain with disturbance based on Kalman filter, we can further deduce the relevant controller and disturbance observer and apply them to the simplified dual-channel supply chain model with time delay and disturbance. Finally, the simulation results prove that the dual-channel supply chain system with time delay and disturbance observer has the highest profit.

Combined with the research work of this paper and the time lag of the actual situation, it is found that the dual-channel supply chain system with disturbance observer is the best. In fact, the deterioration rate of agricultural products is also a variable that changes with time. In this paper, it is regarded as a constant, so it is a problem to be solved to design a supply chain control algorithm that considers the time deterioration rate of products. At the same time, for the profit distribution of supply chain and retailer, this paper regards it as a whole, and the interest contract design in supply chain is also a point studied by many scholars. In the future, we can study how to allocate the overall profit by using the automatic control principle to maximize the profit, taking into account the fresh-keeping efforts of suppliers and retailers.

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