The Application of Fusion Algorithm based on Matrix Analysis in Evidence Theory

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ABSTRACT. This paper directly adopts evidence reasoning formula to calculate sensor information fusion result. The amount of calculation and calculation time delay increase with the increasing number of target found, uses two recursive calculation ways of evidence combination to calculate results, and proposes a fusion algorithm based on matrix analysis, using matlab software and C language programming to realize the method and calculate by an example. The results prove that the fusion result calculated by the method gets the same result as that of evidence reasoning synthesis formula, but the time needed for calculation will be reduced.

KEYWORDS: Evidence reasoning; Matrix analysis; Fusion algorithm. Sensor information

1. Introduction

With the rapid development of sensor technology, multi-sensor data fusion has become a key technology in many areas development. Data fusion is a process to put a number of data of different types and sources for comprehensive treatment, in order to obtain more effective information. In multiple sensor information fusion system, sensor information contains a lot of uncertainty, and information fusion center must reason based on uncertainty reasoning information to improve the reliability of information and increase the complementary information, so as to achieve the purpose of target identification [1]. Uncertainty reasoning methods mainly include Bayes reasoning, evidence theory, and so on. Its characteristic is easy to implement, especially evidence theory. Its main advantage is: satisfy Bayes probability theory’s weaker condition and has a direct expression "uncertainty" and "unknown". Because of these characteristics, evidence reasoning is the most ideal method dealing with uncertainty. If evidence reasoning combination formula is directly adopted for fusion result, the amount of calculation and calculation time delay will increase with target number increase. So this paper proposes a kind of fusion algorithm based on matrix analysis in order to solve this problem.
2. Evidence Theory

Evidence theory is put forward by Dempster in 1967 first, and his students Shafer further developed a theory of inexact reasoning in 1976, which is also known as Dempster/Shafer theory evidence (D-S evidence theory), belonging to the category of artificial intelligence. It was first used in expert systems, and it has the ability of dealing with uncertain information.

In DS evidence theory, the incompatible basic proposition (assumption) makes up complete set as recognition framework, which tells all the possible answers to a question, but only one answer is correct. The framework’s subset is known as proposition. The trust proposition assigned to each proposition is known as basic probability assignment (BPA, called m function); \( m(A) \) is basic credible number, reflecting the reliability of \( A \) size. Trust function Belgium \( (A) \) reflects trust proposition of \( A \); likelihood function \( Pl(A) \) reflects trust proposition of \( A \), which establishes the uncertainty of measurement of \( A \). In fact, \([\text{Bel}(A), \text{Pl}(A)]\) denotes \( A \)’s uncertain interval; \([0, \text{Bel}(A)]\) means proposition \( A \)’s supporting evidence interval; \([0, \text{Pl}(A)]\) means proposition \( A \)’s quasi letter interval; \([\text{Pl}(A), 1]\) means proposition \( A \)’s refused evidence range. \( M_1 \) and \( m_2 \) are basic probability distribution functions made by two independent evidence sources (sensors). Dempster joint rules can calculate new basic probability distribution function reflecting fusion information produced by the combination of evidences.

3. Fusion Algorithm Based on Matrix Analysis

3.1 Bayesian approximation method

Voorbraak found that the synthesis of mass functions will produce a Bayes trust function (a probability measure on recognition framework), then mass functions are replaced by Bayes theorem approximation, which will not affect the result of Dempster synthesis rule. Voorbraak provided mass functions’ Bayes theorem approximate calculation formula, namely

\[(1)\] Voorbraak proved the conclusions as follows:

Mass functions’ Bayes theorem approximate synthesis = mass functions synthesis Bayes approximation

The meaning of Voorbraak "Bayes approximation method":

For those who only care about the final conclusion of "elements" (that is, single hypothesis) in recognition framework rather than its "subsets" (that is, multiple hypothesis subsets) is very useful, and greatly simplifies the amount of calculation.

Bayesian approximation has the following properties:
(1) The focal element is element, so the number of focal elements after approximation is less than or equal to $|\Theta|$.

(2) $m_1 \oplus m_2$'s Bayesian approximation $\frac{m_1 \oplus m_2}{\Theta}$ is the same as $m_1 \oplus m_2$, namely the combination rule of Dempster has nothing to do with evidence combination order.

(3) If the original trust function is a Bayesian function, then Bayesian combination result is accurate.

(4) If there are $N$ focal elements in mass functions, Bayesian approximate calculation time is $O(N/|\Theta|)$.

### 3.2 Fusion algorithm based on matrix analysis

Sensor network’s $n$ sensors identify a goal. Assume that recognition results have $m$ possible cases, then trust distribution can adopt a matrix of $n \times m$ to identify:

$$\begin{bmatrix}
M_1 \\
M_2 \\
\vdots \\
M_n
\end{bmatrix} =
\begin{bmatrix}
m_{11} & m_{12} & \cdots & m_{1m} \\
m_{21} & m_{22} & \cdots & m_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
m_{n1} & m_{n2} & \cdots & m_{nm}
\end{bmatrix}(1)$$

In it, any element $m_{ij}$ in matrix $M$ represents the goal of sensor is the first $j$ possible trust. Due to the sum of trust of the same sensor assigned to $m$ possible recognition results should be 1, the sum of each row elements of the matrix should satisfy normalization conditions. That is:

$$m_{i1} + m_{i2} + \cdots + m_{im} = 1 \quad (i=1, 2, 3, \cdots, n) \quad (2)$$

With a turn of a row in matrix is multiplied by another row:

$$M_i^T \times M_j =
\begin{bmatrix}
m_{i1} & m_{i2} & \cdots & m_{im}
\end{bmatrix} \bullet
\begin{bmatrix}
m_{j1} & m_{j2} & \cdots & m_{jm}
\end{bmatrix}(3)$$

Then get a new composite matrix $A$: 

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The main diagonal elements are the two sensor target recognition confidence accumulation. The sum of diagonal elements constitutes the uncertain factors of the evidence, namely:

\[ k = \sum_{p \neq q} m_{ip} \times m_{jq} (p, q = 1, 2, 3, \cdots, m) \]  

(5)

The fusion results are:

\[ m_i = \frac{m_{ii}}{1 - k} (i = 1, 2, 3, \cdots, m) \]  

(6)

4. Matlab Programming Based on Matrix Analysis Fusion Algorithm

The fusion algorithm of matrix analysis gets the same result as that of evidence reasoning synthesis formula, but it will reduce the time needed for calculation. In this paper, matlab mathematical software is used, and based on matrix analysis of fusion algorithm, program and realize multiple sensors identify a goal at the same time. The fusion program (two evidences) process is shown in Figure 1:

Figure 1. fusion program (two evidences) process

Calculation example:

There are five sensors identifying a diesel engine, and the possible outcomes of the target are the following six circumstances:

1. normal; 2. fuel injector needle valve failure state; 3. injection pressure failure condition; 4. fuel delivery advance angle fault state; 5. delivery valve seal cone fault state; 6. unable to identify

The basic probability allocation value obtained from various sensors is in the following Table 1:

<table>
<thead>
<tr>
<th>Situation</th>
<th>Sensor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>0.15</td>
</tr>
<tr>
<td>4</td>
<td>0.2</td>
</tr>
<tr>
<td>5</td>
<td>0.1</td>
</tr>
<tr>
<td>6</td>
<td>0.15</td>
</tr>
</tbody>
</table>
According to the flow chart, write MATLAB program. First fuse sensor 1 and 2, and so on to get the final fusion results as in Table 2:

Table 2 Fusion results

<table>
<thead>
<tr>
<th>Situation</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fusion</td>
<td>0.94736</td>
<td>0.010634</td>
<td>0.010956</td>
<td>0.008055</td>
<td>0.003665</td>
<td>0.019334</td>
</tr>
</tbody>
</table>

By fusion results, it can be seen that normal diesel engine reliability is 0.94736. According to the credibility, it is trust that diesel engine is in normal condition.

Two sensors are programmed by C language, which are also algorithm realizations of fusion algorithm based on matrix analysis. The algorithm description and process are as follows:

Algorithm description:
Input: trust distribution matrix a[2][M], uncertainty factor k.
Output: Combination of output data (ab), identify possible trust distribution fusion results of target

Calculation example:
Two sensors basic probability allocation value is in Table 3:

Table 3 Two sensors basic probability allocation value

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Type1</th>
<th>Type2</th>
<th>Type3</th>
<th>Type4</th>
<th>Type5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor 1</td>
<td>0.4</td>
<td>0.2</td>
<td>0.15</td>
<td>0.1</td>
<td>0.15</td>
</tr>
<tr>
<td>Sensor 2</td>
<td>0.5</td>
<td>0.14</td>
<td>0.16</td>
<td>0.05</td>
<td>0.15</td>
</tr>
</tbody>
</table>

According to the algorithm process, write C language program, and get the combination data as in the following Table 4:
### Table 4 Combining data

<table>
<thead>
<tr>
<th></th>
<th>0.2000</th>
<th>0.0560</th>
<th>0.0640</th>
<th>0.0200</th>
<th>0.0600</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1000</td>
<td>0.0280</td>
<td>0.0320</td>
<td>0.0100</td>
<td>0.0300</td>
<td></td>
</tr>
<tr>
<td>0.0750</td>
<td>0.0210</td>
<td>0.0240</td>
<td>0.0075</td>
<td>0.0225</td>
<td></td>
</tr>
<tr>
<td>0.0500</td>
<td>0.0140</td>
<td>0.0160</td>
<td>0.0050</td>
<td>0.0150</td>
<td></td>
</tr>
<tr>
<td>0.0750</td>
<td>0.0210</td>
<td>0.0240</td>
<td>0.0075</td>
<td>0.0225</td>
<td></td>
</tr>
</tbody>
</table>

The fusion results are in the following Table 5:

### Table 5 Fusion results

<table>
<thead>
<tr>
<th>Type1</th>
<th>Type2</th>
<th>Type3</th>
<th>Type4</th>
<th>Type5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fusion results</td>
<td>0.71556</td>
<td>0.10018</td>
<td>0.085868</td>
<td>0.017889</td>
</tr>
</tbody>
</table>

By Matlab program testing, the fusion result is in following Table 6:

### Table 6 Matlab program testing fusion results

<table>
<thead>
<tr>
<th>Type1</th>
<th>Type2</th>
<th>Type3</th>
<th>Type4</th>
<th>Type5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fusion results</td>
<td>0.71556</td>
<td>0.10018</td>
<td>0.085868</td>
<td>0.017889</td>
</tr>
</tbody>
</table>

Comparing the two results, it can be seen that the fusion results of the two procedures are the same.

### 5. Conclusion

Two evidence combination methods recursive calculation way are used for fusion results calculation. A fusion algorithm based on matrix analysis is proposed, and matlab software and C language programming are used to achieve so as to overcome the faults of the amount increase of calculation and calculation time delay with target increases found when fusion result is obtained by using of evidence reasoning combination formula directly. At last, through instance substitution program verification, results show that method calculation fusion results are the same as that of evidence reasoning synthetic formula, but it will reduce the time needed for calculation, namely the fusion algorithm based on matrix analysis applying to evidence reasoning is effective and feasible.

### References


