

Higher education evaluation system based on grey clustering evaluation model

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Abstract: In order to solve the problem of evaluating the quality of a country's higher education, this thesis establishes a higher education evaluation system. By selecting eight indicators such as higher education capital investment, higher education gross enrollment rate and higher education fairness and etc. And then using the grey clustering evaluation model and analytic hierarchy process, whereby this thesis designs a higher education evaluation system, which can comprehensively and objectively evaluate the quality of a country's higher education. Next, the thesis collects data and then uses the established evaluation system to evaluate the higher education level of the United States, Germany, China and India, and comes to the conclusion that the higher education quality of the United States, Germany is excellent and China is middle and India is poor, which is basically consistent with the public understanding. For that reason, system proves that it has achieved good results.

Keywords: Mathematical modeling, Higher Education Quality Evaluation System, Grey Clustering Evaluation Model, Analytic Hierarchy Process, Central Point Mixing Possibility Degree Function

1. Introduction

For a country's education system, higher education is a very important part. Therefore, how to quantify the quality of higher education has become a new problem. At present, the quality evaluation of higher education in China is still in the stage of qualitative evaluation, and there is no very practical quantitative evaluation system. This paper attempts to establish a quantitative quality evaluation system for the quantitative evaluation of the quality of higher education, in order to objectively evaluate the quality of higher education in a country, and the method is simple and practical.

In mathematics, evaluation problems are often transformed into classification problems, that is, problems in which a certain object belongs to a certain class. Based on this idea, this paper studies the establishment of higher education quality evaluation system using grey clustering evaluation model[1-2].

2. Introduction of Grey Clustering Evaluation Model

Gray clusters can be divided into two types: one is gray associated clusters, which are used for the merger of similar factors; The other is the gray white weight cluster, which is used to detect what kind of observation object belongs to. The grey cluster evaluation model belongs to the grey whitening weight cluster. The modeling steps are as follows:

1) Set the number of evaluation gray classes s

2) Determine the center point of gray class 1 and gray class $k(k = 2, 3, \dots, s)$ and gray class s . The center point is represented by λ_j^k the point most likely to belong to the gray class k , which can be the midpoint or not, based on the greatest probability of belonging to the gray class.

3) Construct the probability function

For gray cluster evaluation models, the center point-based blend probability function is often used. The construction process is as follows:

(1) For the gray class 1 and the gray class s , the corresponding lower limit measure probability function and the upper limit measure probability function are constructed respectively as $f_j^1[-, -, \lambda_j^1, \lambda_j^2], f_j^s[\lambda_j^{s-1}, \lambda_j^s, -, -]$.

(2) For gray classes $k(k \in \{2,3, \dots, s - 1\})$, construct a trigonometric probability function.

The probability function calculation formula is as follows:

$$f_j^1(x) = \begin{cases} 0 & x \notin [a_j, \lambda_j^2] \\ 1 & x \in [a_j, \lambda_j^1] \\ \frac{\lambda_j^2 - x}{\lambda_j^2 - \lambda_j^1} & x \in [\lambda_j^1, \lambda_j^2] \end{cases}$$

$$f_j^s(x) = \begin{cases} 0 & x \notin [\lambda_j^{s-1}, b_j] \\ \frac{x - \lambda_j^{s-1}}{\lambda_j^s - \lambda_j^{s-1}} & x \in [\lambda_j^{s-1}, \lambda_j^s] \\ 1 & x \in [\lambda_j^s, b_j] \end{cases}$$

$$f_j^k(x) = \begin{cases} 0 & x \notin [\lambda_j^{k-1}, \lambda_j^{k+1}] \\ \frac{x - \lambda_j^{k-1}}{\lambda_j^k - \lambda_j^{k-1}} & x \in [\lambda_j^{k-1}, \lambda_j^k] \\ \frac{\lambda_j^{k+1} - x}{\lambda_j^{k+1} - \lambda_j^k} & x \in [\lambda_j^k, \lambda_j^{k+1}] \end{cases}$$

$k = 2, 3, \dots, s - 1$

The image of the blended probability function based on the center point is shown in the figure 1:

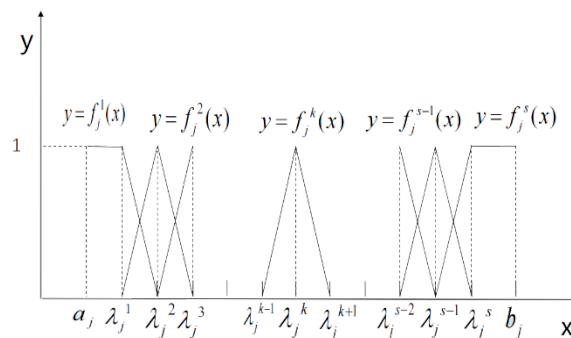


Figure 1: Mixed Probability Function Image Based on Center Point

- 4) Determine the weight of each indicator
- 5) Calculate the composite clustering coefficients

Use the following formula to calculate the composite clustering coefficient of an object for a gray class:

$$\sigma_i^k = \sum_{j=1}^m f_j^k(x_{ij}) \eta_j$$

- 6) Determine the category to which the object belongs

The $\max_{1 \leq k \leq s} \{\sigma_i^k\} = \sigma_i^{k^*}$ is judged object i to belong to the gray class k^* .

When multiple objects belong to the same gray class k^* , you can further determine the advantages and disadvantages and bits of each object based on the size of the composite cluster coefficient[3].

3. Establishment of a higher education evaluation system

The first step in the establishment of an evaluation system is to select appropriate evaluation indicators. Evaluation indicators must not only be able to comprehensively and objectively reflect the quality of higher education, but also evaluate indicators and have easy access to corresponding data. After research,

it is believed that it is more appropriate to select the following indicators, as shown in Table 1:

Table 1: Selected metrics

Indicator 1	Indicator 2	Indicator 3	Indicator 4	Indicator 5	Indicator 6	Indicator 7	Indicator 8
Higher education funding	The cost of attending higher education	Gross enrolment ratio of tertiary education	Educational equity	Course quality	Scientific research level	International level	International reach

The next step is to model the gray cluster evaluation model according to the modeling steps.

3.1 Determine the number of gray classes evaluated

After research, it is believed that it is more appropriate to divide into four gray categories as excellent, good, medium and poor.

3.2 Construct the probability function

For different indicators, the center point should be set reasonably and respectively.

3.3 Determining the weight of the indicator

There are several ways to determine the weights. This paper uses the analytic hierarchy method to determine the weights of the indicators.

3.3.1 Introduction to Analytic Hierarchy Process

Analytic Hierarchy Process(AHP) is a hierarchical weighted decision-making analysis method proposed by American operations research scientist Satty in the early 1970s by applying network system theory and multi-objective comprehensive evaluation method. Combining quantitative analysis and qualitative analysis, the method uses the experience of decision makers to judge the relative importance between the criteria for measuring whether the goals can be achieved, and reasonably gives the weight of each criterion of each decision-making plan, and then uses the weight to find the order of advantages and disadvantages of each plan. This method can be applied to problems that are difficult to solve quantitatively[4-5].

The steps of the analytic hierarchy method are:

- 1) Establish a hierarchical model
- 2) Construct a judgment matrix

The most commonly used method of constructing the judgment matrix in the analytic hierarchy method is the consistent matrix method, that is, not to compare all the factors together, but to compare two and two with each other; Relative scales are used at this point. This method can minimize the difficulty of comparing factors of different natures with each other in order to improve accuracy. The consistent array has the following characteristics:

$$(1).a_j^i = \frac{1}{a_i^j}, a_{ii} = 1, i, j = 1, 2, \dots, n$$

(2) A^T is also a consistent matrix

(3) The rows of the matrix A are proportional.

(4) The maximum feature root (value) of A is $\lambda = n$, and the remaining feature roots are equal to 0.

(5) Any column (row) of matrix A is a feature vector that corresponds to the root of the feature n , has $AW = nW$

The following is an example of a judgment matrix:

$$A = \begin{bmatrix} 1 & 3 & 6 \\ \frac{1}{3} & 1 & 4 \\ \frac{1}{6} & \frac{1}{4} & 1 \end{bmatrix}$$

3) Consistency check

In practice, due to the complexity of objective things and the ambiguity of people's judgments and comparisons of things, it is difficult to construct a completely consistent judgment matrix. Therefore, when Saaty constructed the analytic hierarchy method, he proposed the consistency test, which refers to the range of judgment matrices that are allowed to have certain inconsistencies. The *A* more inconsistencies in the judgment matrix are. The greater the degree of inconsistency of the eigenvector corresponding to the maximum eigenvalue value as the weight vector of the degree of influence of the compared factor on a factor in the upper layer, the greater the judgment error caused. Therefore, consistency indicators are introduced $CI = \frac{\lambda - n}{n - 1}$ to measure the degree of *A* inconsistency[6].

To measure the size of *CI*, the stochastic consistency indicator *RI* is introduced, as shown in Table 2.

Table 2: Section Stochastic Consistency Indicator *RI* values

n	1	2	3	4	5	6	7
RI	0	0	0.58	0.90	1.12	1.24	1.32

Here the consistency ratio is introduced, $CR = \frac{CI}{RI}$ generally, when $CR < 0.1$, when the *A* degree of inconsistency is within the allowable range, there is satisfactory consistency, through the consistency test, its normalization feature vector can be used as a weight vector, otherwise it is necessary to adjust and construct a new judgment matrix. a_j^i

After experimentation, a reasonable judgment matrix was finally constructed ($CR = 0.0294 < 0.1$), and the weights of each indicator at this time were as follows Table 3:

Table 3: Weight values for each metric

Index	Higher education funding	The cost of attending higher education	Gross enrolment ratio of tertiary education	Educational equity
weight	0.2005	0.2320	0.0541	0.1217
Index	Course quality	Scientific research level	International level	International reach
weight	0.0809	0.0360	0.2482	0.0266

4. Application

Table 4: Composite clustering coefficients for the four countries

Country	United States	Germany	China	India
Gray Group				
excellent	0.6256	0.2759	0.2165	0.0541
good	0.1599	0.0377	0.1331	0.0000
middle	0.1983	0.0851	0.2805	0.0000
poor	0.0000	0.2364	0.0859	0.8780

In order to test the effectiveness of the higher education evaluation system established in this paper, this paper selects the higher education systems of the United States, Germany, China and India for evaluation, as shown in Table 4.

This paper finds the required data from the websites of the Ministry of Education and the World Bank of each country, scores each country's indicators based on the data found (see Appendix B for specific data), and finally uses the evaluation system we have established to classify them. The final calculation of the comprehensive cluster coefficient of each country and the gray class to which it belongs is shown in the following table 5:

Table 5: Evaluation results for the four countries

Country	United States	Germany	China	India
Evaluation results	excellent	excellent	middle	poor

It can be seen from the results that the results obtained by the quantitative research using the higher education evaluation system are basically consistent with the public's understanding, which proves that the evaluation system established is accurate.

5. Conclusion

The higher education evaluation system established in this paper can better evaluate the quality of higher education in a country, and it is simple and practical. Therefore, this evaluation system is a relatively good evaluation system.

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