

Analysis of the Trend of the Evolution of the Difficulty of Mathematics Questions in the Shaanxi Secondary School Examination

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Abstract: *This paper explores in detail the difficulty evolution trend of Shaanxi's secondary school examination mathematics questions between 2014 and 2023. The study focuses on six test difficulty influencing factors and their several levels such as background, knowledge content, arithmetic ability, thinking ability, reasoning ability, and cognitive ability, and gives the evolution trend of the assessment factors, their levels, and the comprehensive difficulty of Shaanxi's secondary school examination questions, and puts forward several suggestions for guiding teachers' instruction. It also puts forward several suggestions for guiding teachers' teaching and optimizing the preparation strategies for the secondary school examination.*

Keywords: *Mathematics examination questions in the SSCE; Difficulty influencing factors; Evolutionary trend*

1. Introduction

The secondary school examination is a large-scale examination that integrates the selection, streaming, and learning effect of knowledge and ability of junior high school students whether they can go on to higher education, or go on to general or vocational high schools to continue their study. The scientific, fairness, objectivity, comprehensiveness, and effectiveness of the examination questions assessed by millions of students and families concerned are also important to the state. In recent years, the state has introduced a series of requirements on the test proposition. In 2021, the General Office of the Ministry of Education on the strengthening of compulsory education school examination management notice pointed out that: 'shall not exceed the standard proposition and arbitrarily expand and reduce the scope of the examination content, to ensure that according to the standard proposition, teaching and examination convergence. According to the characteristics of different disciplines, it is reasonable to set up the question structure of test papers, reduce the number of memorable questions, increase the number of exploratory, open-ended, and comprehensive questions, and resolutely prevent biased and strange questions, to promote the effective examination of the comprehensive quality of students[1]'. In 2022, the Mathematics Curriculum Standards for Compulsory Education (2022 Edition) stated that 'the preparation of test questions must be strictly based on the curriculum standards, to ensure that the content, difficulty, question types and number of questions in the examination are in line with the requirements of the curriculum standards, and do not exceed or fall below the scope and level set out in the curriculum standards[2]'.

In 2016, Zhang Yi increased the thinking direction factor, formed a comprehensive difficulty model consisting of cognition, background, arithmetic, reasoning, knowledge content, and thinking, systematically analyzed the comprehensive difficulty of the content of the three sets of national papers of the Gao Kao Science Mathematics Examination in 2016, and gave a more scientific analysis at the technical level for the reader's reference[3]. In 2022, Rosa Rossa researched the core literacy oriented Mathematics examination questions and comprehensive difficulty analysis, constructed the Mathematics Core Literacy Evaluation Framework Indicator System and Comprehensive Difficulty Evaluation Framework Indicator System to analyze and study the test questions, which better reflects the importance of Mathematics Core Literacy in the perspective of students' learning and the test questions' proposition, and meets the requirements of the new curriculum standard[4]. This paper analyses the difficulty of the 2014-2023 Shaanxi Mathematics Secondary School Examination questions, discusses the trend of the

difficulty of the questions during this period and the possible regularities, and better understands the characteristics of the Shaanxi Mathematics Secondary School Examination questions in recent years, provides a reference basis for the teaching, research, and development work, and provides scientific guidance for students' preparation for the exams.

2. Difficulty factors of maths questions and their level classification

The study covers the past ten years of secondary school examination questions, which can completely reflect the propositional trend and difficulty changes of Shaanxi Province secondary school examination questions, and then provide valuable references for mathematics teaching and secondary school examination preparation. The article is based on the improvement of Bao Jiansheng's comprehensive difficulty model, including background, knowledge content, arithmetic ability, thinking style, reasoning ability, cognitive ability, and the factors that are classified into levels so that they can be assigned values. i denotes the difficulty factor, j denotes the level under the difficulty factor, L_i denotes the set of levels for the difficulty factor i , l_{11} denotes the 'no background level under the background factor', etc, and d_{ij} denotes the weight of the level of the i -th difficulty factor l_{ij} . Details are shown in Table 1.

Table 1: Level classification, meaning and assignment of the six composite difficulty factors.

i	L_i	connotation	d_{ij}
$i = 1$	l_{11}	Does not rely on any specific application scenarios or real-world problems, but directly examines basic concepts and solutions such as mathematical points, formulas, theorems, etc	1
	l_{12}	Integrate mathematical knowledge with practical problems in daily life as a background, and students apply mathematical knowledge to solve practical problems	2
	l_{13}	Integrate mathematical knowledge with scientific knowledge in the context of problems in scientific areas (e.g. physics, chemistry, biology, etc.) and students apply their mathematical knowledge to solve scientific problems	3
$i = 2$	l_{21}	Only one major point or concept is covered in the question	1
	l_{22}	The questions cover two main knowledge points or concepts	2
	l_{23}	Topics cover three or more major knowledge points or concepts	3
$i = 3$	l_{31}	No maths required	1
	l_{32}	Basic mathematical operations such as addition, subtraction, multiplication and division	2
	l_{33}	Performing algebraic operations involving letters or symbols, evaluating algebraic expressions purely symbolically, combining like terms, solving proportions, and solving inequalities	3
	l_{34}	This refers to more complex algebraic operations involving numerical and symbolic operations	4
$i = 4$	l_{41}	A way of thinking in which the answer to a problem is sought directly from the known conditions, in the natural or logical order of the problem	1
	l_{42}	A way of thinking in which the answer to a problem is sought in reverse, from the opposite side or outcome of the problem	2
$i = 5$	l_{51}	Reasoning processes that require less than three derivation steps	1
	l_{52}	Reasoning processes with three or more required derivation steps	2
$i = 6$	l_{61}	Direct examination of mathematical points, concepts, theorems or formulas	1
	l_{62}	Based on understanding, apply mathematical knowledge, methods, and techniques to specific mathematical problems, and flexibly select and apply what they have learned in the process of solving problems	2
	l_{63}	Classifying, organizing, summarising, and comparing information in a topic, discovering hidden conditions or traps in the topic, and performing logical reasoning and deduction in the solution process	3

According to the factors in the table are divided into levels, and then the natural numbers are assigned to each level, from which Bao Jiansheng proposed a specific formula. l_i is the number of levels of factor i , d_{ij} denotes the weight of the l_{ij} level of the i -th difficulty factor, n_i denotes the total number of

questions at different levels of the i -th factor and d_i is the difficulty coefficient of the i -th factor. The specific formula is as follows:

$$d_i = \frac{\sum_{j=1}^{l_i} n_{il_j} d_{l_j}}{n_i}.$$

When calculating the composite difficulty of each test question, you can take the combined value for the difficulty factors of the six difficulty factors under that test question, using the formula $ID_k = \sum_{i=1}^6 d_i$.

k for the year, d_i for the difficulty factor of the i -th factor, and ID_k for the composite difficulty of the test question of the k -th year.

In this paper, the difficulty factor formula is extended to include specific year units, and D_{ki} denotes the combined weights of the different levels of the i -th difficulty factor in year k . The specific calculation formula is as follows:

$$D_{ki} = \sum_{j=1}^{l_i} n_{kil_j} d_{l_j}, \quad l_{ij} \in L_i.$$

Below n_{ki} denotes the number of questions at different levels for the i -th difficulty factor in year, and d_{ki} represents the difficulty factor for the i -th difficulty factor in year k . The table is shown below:

$$d_{ki} = \frac{D_{ki}}{n_{ki}} = \frac{\sum_{j=1}^{l_i} n_{kil_j} d_{l_j}}{n_{ki}}, \quad l_{ij} \in L_i.$$

From the above equation, the following equation is obtained, with n_k denoting the total number of questions in the test in year k :

$$ID_k = \frac{\sum_{i=1}^6 D_{ki}}{n_k} = \frac{\sum_{i=1}^6 \sum_{j=1}^{l_i} n_{kil_j} d_{l_j}}{n_k}, \quad l_{ij} \in L_i.$$

3. Comparison of Difficulty of Mathematics Test Questions in Shaanxi Secondary School Examination from 2014 to 2023

After analyzing which coefficient of difficulty and which level each of the Shaanxi provincial secondary school mathematics examination questions belonged to from 2014-2023, the number of questions under each level was counted. The percentage of the number of test questions in each year to the total number of test questions was calculated, and the obtained data are shown in Table 2.

Table 2: Number and percentage of questions based on the difficulty factor i and its level L_i in the Shaanxi Secondary School Mathematics Examination, 2014-2023.

i	L_i	2014 ($k=1$)		2015 ($k=2$)		2016 ($k=3$)		2017 ($k=4$)		2018 ($k=5$)	
		quantity	percentage	quantity	percentage	quantity	percentage	quantity	percentage	quantity	percentage
$i=1$	l_{11}	18	72.0%	20	80.0%	21	84.0%	20	80.0%	20	80.0%
	l_{12}	6	24.0%	5	20.0%	4	16.0%	5	20.0%	5	20.0%
	l_{13}	1	4.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%
$i=2$	l_{21}	9	36.0%	7	28.0%	8	32.0%	7	28.0%	4	16.0%

	l_{22}	10	40.0%	8	32.0%	4	16.0%	7	28.0%	9	36.0%
	l_{23}	6	24.0%	10	40.0%	13	52.0%	11	44.0%	12	48.0%
$i = 3$	l_{31}	1	4.0%	2	8.0%	2	8.0%	2	8.0%	1	4.0%
	l_{32}	12	48.0%	10	40.0%	13	52.0%	10	40.0%	18	72.0%
	l_{33}	7	28.0%	8	32.0%	5	20.0%	7	28.0%	4	16.0%
	l_{34}	5	20.0%	5	20.0%	5	20.0%	6	24.0%	2	8.0%
$i = 4$	l_{41}	22	88.0%	23	92.0%	23	92.0%	24	96.0%	24	96.0%
	l_{42}	3	12.0%	2	8.0%	2	8.0%	1	4.0%	1	4.0%
$i = 5$	l_{51}	19	76.0%	20	80.0%	19	76.0%	18	72.0%	18	72.0%
	l_{52}	6	24.0%	5	20.0%	6	24.0%	7	28.0%	7	28.0%
$i = 6$	l_{61}	10	40.0%	7	28.0%	8	32.0%	6	24.0%	8	32.0%
	l_{62}	12	48.0%	16	64.0%	13	52.0%	17	68.0%	15	60.0%
	l_{63}	3	12.0%	2	8.0%	4	16.0%	2	8.0%	2	8.0%
i	L_i	2019 ($k = 6$)		2020 ($k = 7$)		2021 ($k = 8$)		2022 ($k = 9$)		2023 ($k = 10$)	
		quantity	percentage	quantity	percentage	quantity	percentage	quantity	percentage	quantity	percentage
$i = 1$	l_{11}	20	80.0%	18	72.0%	19	73.1%	20	77.0%	18	69.3%
	l_{12}	5	20.0%	6	24.0%	5	19.2%	5	19.2%	7	26.9%
	l_{13}	0	0.0%	1	4.0%	2	7.7%	1	3.8%	1	3.8%
$i = 2$	l_{21}	5	20.0%	9	36.0%	13	50.0%	13	50.0%	9	34.6%
	l_{22}	8	32.0%	7	28.0%	9	34.6%	7	26.9%	11	42.3%
	l_{23}	12	48.0%	9	36.0%	4	15.4%	6	23.1%	6	23.1%
$i = 3$	l_{31}	3	12.0%	1	4.0%	2	7.7%	2	7.7%	3	11.5%
	l_{32}	12	48.0%	13	52.0%	13	50.0%	13	50.0%	13	50.0%
	l_{33}	6	24.0%	8	32.0%	7	26.9%	7	26.9%	6	23.1%
	l_{34}	4	16.0%	3	12.0%	4	15.4%	4	15.4%	4	15.4%
$i = 4$	l_{41}	23	92.0%	23	92.0%	25	96.2%	25	96.2%	24	92.3%
	l_{42}	2	8.0%	2	8.0%	1	3.8%	1	3.8%	2	7.7%
$i = 5$	l_{51}	18	72.0%	19	76.0%	21	80.8%	21	80.8%	17	65.4%
	l_{52}	7	28.0%	6	24.0%	5	19.2%	5	19.2%	9	34.6%
$i = 6$	l_{61}	8	32.0%	9	36.0%	10	38.5%	11	42.3%	9	34.6%
	l_{62}	13	52.0%	13	52.0%	11	42.3%	12	46.2%	13	50.0%
	l_{63}	4	16.0%	3	12.0%	5	19.2%	3	11.5%	4	15.4%

3.1. Comparison of Various Difficulty Factors in Shaanxi Secondary School Mathematics Examination Questions, 2014-2023

Based on the data from the bar chart in Figure 1 the following can be illustrated. All 10 sets of test questions were predominantly examined without context, accounting for more than 60 percent. In the context of life, the 10 sets of test questions accounted for about 20%; in the context of science, the 2015-2019 midterm exams all accounted for 0%, and none of the other years accounted for more than 10%. It can be seen that the test questions focused more on the direct application of knowledge when examining

contextual factors and lacked the integration of mathematical knowledge with life and other subjects.

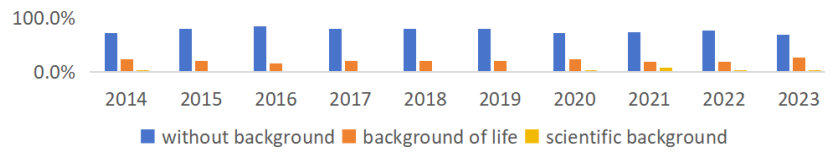


Figure 1: Comparison of levels under the contextual factors of the Shaanxi Secondary School Examination, 2014-2023.

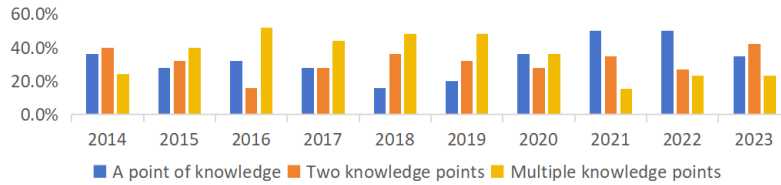


Figure 2: Comparison of each level under the knowledge content factor of Shaanxi secondary school examination questions, 2014-2023.

As can be seen from the Figure 2, one more knowledge point is examined in 2021 and 2022; two more knowledge points are examined in 2014 and 2023; multiple knowledge points are examined in 2015, 2016, 2017, 2018, and 2019, and one knowledge point is not as much examined as multiple knowledge points in 2020. In the past three years, the examination of multiple knowledge points has increased year by year, indicating that education has paid more attention to the cultivation of students' comprehensive abilities, including the ability to analyze problems, solve problems, and transfer knowledge.

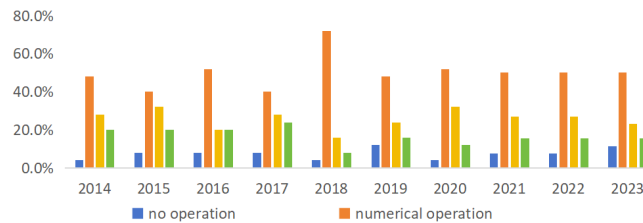


Figure 3: Comparison of each level under the arithmetic ability factor in Shaanxi Secondary School Examination, 2014-2023.

From the data in Figure 3, the following can be analysed. The frequency of numerical operations was the highest in all ten sets of questions, with the highest frequency occurring in 2018, with a percentage of 72.0 percent. For the no-operator level, the percentage was the lowest in all cases, with a gradual increase in the percentage from 2020 onwards. The share of simple symbolic arithmetic is also consistent, with a share of around 25.0 percent. Complex symbolic operations have been relatively stable in recent years. In terms of mathematical arithmetic skills, the difficulty of the test questions showed an upward trend from year to year, and the questions involved complex arithmetic and derivation processes to test students' arithmetic skills and computational abilities.

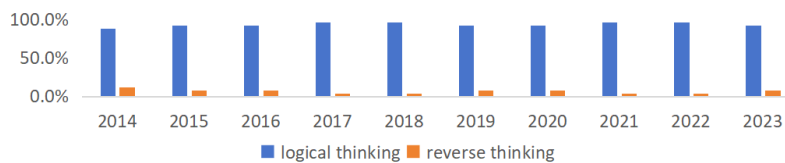


Figure 4: Comparison of each level under the thinking factor of Shaanxi Secondary School Examination, 2014-2023.

As can be seen from the Figure 4, the ten sets of questions are more focused on the examination of positive thinking, and in the case of positive thinking, the smallest percentage, 88.0 percent, was in 2014. For reverse thinking, the test is about 10 percent, and the largest test is in 2014, accounting for 12 percent. This also reflects that the examination angle of the test questions is in line with the direction of students' thinking and that although positive thinking predominates in the examination, there is still a certain degree of examination of reverse thinking.

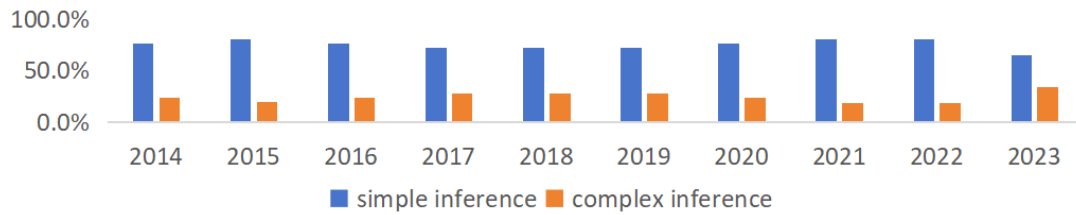


Figure 5: Comparison of each level under the factor of reasoning ability in the Shaanxi Secondary School Examination, 2014-2023.

It can also be seen from the Figure 5 that from 2021 onwards, there was a decrease in the number of questions set on simple reasoning and, on the contrary, an increase in the number of questions set on complex reasoning, which accounted for 34.6 percent in 2023. In terms of the overall number of questions, the lowest number of questions was set in 2023 at 65.4 percent. The highest number of questions was set in 2021 and 2022, at 80.8 percent. It can also be seen from the graph that from 2021 onwards, there is a decrease in the number of questions set on simple reasoning, and on the contrary, there is an increase in the number of questions set on complex reasoning, which accounted for 34.6 percent of the questions in 2023. This requires students to pay more attention to the development of logical reasoning, cause and effect, and other skills.

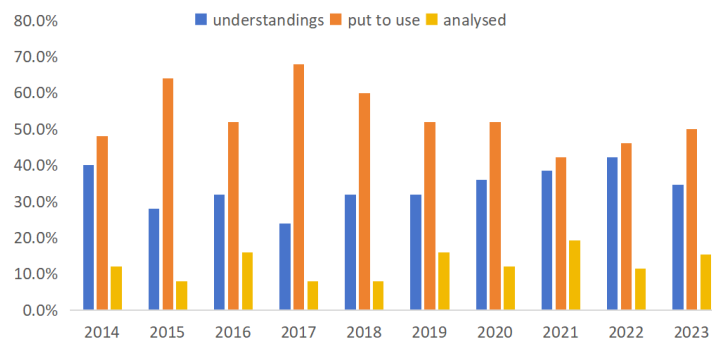


Figure 6: Comparison of each level under the cognitive ability factor in the Shaanxi Secondary School Examination, 2014-2023.

Interpreting the data in Figure 6, we find that ten sets of questions focus more on the assessment of the ability to apply the test, with the percentage fluctuating, reaching a high of 68.0 percent in 2017, from which year there is a decline and then an increase in the level of application. The level of comprehension accounts for 30 percent, reaching a high of 42.3 percent in 2022, but falling to 34.6 percent in 2023. The level of analysis is the least tested in the cognitive skills, and the questions may be more about the knowledge, understanding, and mastery of mathematical points.

3.2. Comprehensive Difficulty Comparison of Mathematics Examination Questions in Shaanxi from 2014 to 2023

From the level of importance of each factor in the test questions, based on the new and improved difficulty coefficients as well as the formula for calculating the combined difficulty coefficients of the test questions, we get Table 3.

From Table 3, we can get the comprehensive difficulty of the ten sets of questions in the following order from high to low: 2017>2019>2015=2016>2014=20>2023>2018>2021>2022. Overall, the difficulty of Shaanxi midterm exams in the past ten years has not fluctuated much, and the difficulty has roughly shown a tendency to rise, then decline, then rise again, which reflects the consistency of midterm exam papers.

In order to analyze the combined difficulty of the 10 sets of maths questions more clearly, the following radar chart of the combined difficulty coefficients was developed based on the difficulty coefficients of the six factors, as shown in Figure 7.

Table 3: Coefficients of the six difficulty factors for each year's test questions and the combined difficulty coefficient for each year's test questions, 2014-2023.

Vintages factors	2014 ($k = 1$)	2015 ($k = 2$)	2016 ($k = 3$)	2017 ($k = 4$)	2018 ($k = 5$)	2019 ($k = 6$)	2020 ($k = 7$)	2021 ($k = 8$)	2022 ($k = 9$)	2023 ($k = 10$)
d_{k1}	1.32	1.20	1.16	1.20	1.20	1.20	1.32	1.35	1.27	1.35
d_{k2}	1.88	2.12	2.20	2.16	2.32	2.28	2.00	1.65	1.73	1.88
d_{k3}	2.64	2.64	2.52	2.92	2.28	2.44	2.52	2.50	2.50	2.42
d_{k4}	1.12	1.08	1.08	1.04	1.04	1.08	1.08	1.04	1.04	1.08
d_{k5}	1.24	1.20	1.24	1.28	1.28	1.28	1.24	1.19	1.19	1.35
d_{k6}	1.72	1.80	1.84	1.84	1.76	1.84	1.76	1.81	1.69	1.81
ID_k	9.92	10.04	10.04	10.44	9.88	10.12	9.92	9.54	9.42	9.89

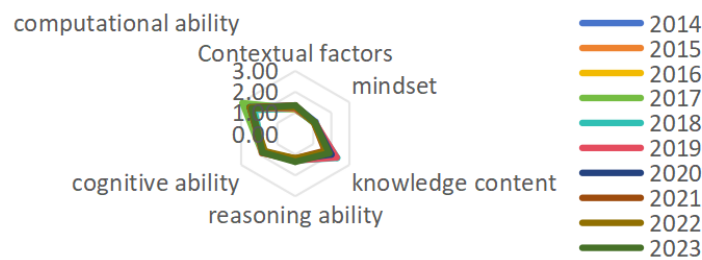


Figure 7: Radar chart of six coefficients of difficulty of Shaanxi secondary school examination questions, 2014-2023.

Based on the shape of the radargram in Figure 7, the following pattern can be summarised. The 2014-2023 Shaanxi provincial secondary school mathematics examination questions do not have a large gap in thinking styles and cognitive abilities. The next largest gap was in contextual factors and reasoning ability, and the last was in arithmetic ability and knowledge content. It shows that the paper is unstable in the examination of mathematical numbers and symbols, and the examination of arithmetic ability and knowledge content is the most difficult. However, the overall presentation of a similar hexagonal shape suggests that each year's question was largely stable in its examination of the factors, with no major ups and downs.

4. Conclusions of the study

Based on Figures 1 to 6, the following conclusions and recommendations can be drawn. The difficulty coefficients of the analyses of the ten sets of test questions have both similarities and differences. (1) The difficulty coefficients for each of the factors of knowledge content and arithmetic ability are larger for all ten sets of test questions, with the difficulty coefficient for arithmetic ability being above 2.00. (2) Cognitive ability shows a rising trend year by year, while the level of use of cognitive ability factors in each set of test questions accounted for the highest percentage, the level of analysis accounted for the lowest percentage. (3) Reasoning ability, background factors, thinking accordingly are simple reasoning, no background, the level of smooth thinking accounted for the highest percentage. It shows that the test questions are not very difficult, all of which are designed to examine students' basic knowledge.

4.1. Focus on integrative and systematic knowledge

In teaching, teachers should focus on the comprehensiveness and systematicity of knowledge, and organically integrate different knowledge points and skills to enhance students' comprehensive application ability. Students are guided to create individual or group knowledge trees or mind maps to systematize and visualize what they have learned, making it easier to understand and remember. In different grades or stages of learning, the same topic or knowledge point is studied in different depths and breadths, and students' knowledge system is gradually built up through repetition and deepening.

4.2. Strengthening basic arithmetic and improving the development of complex arithmetic skills

In daily teaching, teachers should focus on strengthening students' training in basic arithmetic skills such as addition, subtraction, multiplication, and division. Ensure that students are proficient, and gradually introduce complex algebraic and symbolic operations. Given the higher difficulty of arithmetic skills, teachers should design more practice problems involving complex symbolic operations. Students practice and improve their arithmetic skills and calculation ability in solving practical problems. Regular tests and exercises are organized to help students identify and correct common errors in arithmetic, and provide timely feedback and guidance.

4.3. Enhancement of cognitive skills and diversification of teaching methods

As the requirements of cognitive ability increase year by year, teachers can gradually increase the cognitive difficulty of topics in teaching, encourage students to understand and analyze problems in depth, and improve their logical thinking and problem-solving abilities. Diversified teaching methods, such as inquiry-based learning and project-based learning, are used to promote active thinking and exploration, thus improving students' cognitive abilities. Given the relatively low level of students' analysis, teachers should strengthen the cultivation of students' analytical skills.

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