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Application of the Planning Model in Mathematical Modeling

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Abstract: *In recent years, planning models have been deeply loved by participants in mathematical modeling competitions. The basic idea is to determine the decision variable and the target function and solve it, and then use the traditional numerical solution method or the current relatively emerging intelligent optimization algorithm to obtain the optimal solution. Planning models are widely used in many fields such as physics, chemistry, dynamics and optics. This paper discusses the practical application of the model in the last three years, and discusses the application of the model in daily life. It provides reference opinions for the subsequent participants to systematically learn this model.*

Keywords: *planning model; mathematical modeling competition; optimal solution*

1. Introduction

Planning model has high application value in many fields such as economic prediction, engineering computing and scientific research and development. As an important mathematical model, the planning model has frequently appeared in various mathematical modeling competitions in previous years. For example, question B in 2021 (Optimization of Thermal Transmitter Material Parameters in 2021), question A of American College Students (Optimization of Cycling Track) in 2022, question B of Mathematical Modeling in 2021 (Optimization of C4 Olefin Catalyst), etc.

In this paper, we analyze and solve the programming-related problems of planning and some classical exercises in recent mathematical modeling competitions, and establish the linear programming, non-linear programming model and multi-objective programming model for multi-degree of freedom independent variables.It mainly discusses the application of planning model in mathematical modeling. First, we summarize the concept of planning model, an important mathematical modeling model, and systematically elaborate the basic principles of linear planning model, nonlinear planning model and multi-objective planning model.On the basis of familiarity with the concept, the deep analysis of mathematical modeling classic examples.The problems chosen in this paper are those for the major mathematical modeling competitions of the last three years. It has a strong timeliness and authority. The induction and solution of the real problem of this problem have certain reference significance for the teachers and students who are interested in the mathematical modeling competition.

2. Overview of planning model

2.1 Linear programming model

To establish a mathematical model of the optimization problem, first determine the decision variable of the problem, with the n-dimensional vector, and then construct the target functions of the model $f(x)$ and x the allowed value range Ω, Ω is called the feasible domain, often used a set of inequalities (or equations) $g_i(x) \le 0(i=1,2,...,m)$, called the constraint condition.In general, such models can be formulated in the following form:

$$\begin{aligned} &\min z = f(x) \\ &s.t. g_i(x) \le 0(i=1,2,\cdots,m) \end{aligned} \tag{1}$$

The model composed of (1) and (2) belongs to constrained optimization, if only equation (1) is

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unconstrained optimization. The $f(x)$ is called an objective function and the $g_i(x) \le 0$ is called a constraint.

In an optimization model, if the objective function $f(x)$ and both the $g_i(x)$ in the constraints are linear functions, then the model is called linear programming. Decision variable, objective function and constraint condition constitutes the three basic elements of linear planning.

2.2 Nonlinear programming model

The standard shape of the nonlinear programming is

$$\min F(X)$$

$$s.t.\begin{cases} AX \le b \\ Aeq \cdot X = beq \\ G(X) \le 0 \\ Ceq(X) = 0 \\ VLB \le X \le VUB \end{cases} \tag{2}$$

Where C is the vector of n dimension variable, $G(X)$ and $Ceq(X)$ are both vectors composed of nonlinear functions

2.3 Multi-objective planning model

The establishment of a multi-goal planning model is generally divided into three big steps. The first step is to determine the decision variable of the problem to be solved $x = (x_1, x_2, \dots, x_3)^T$.The second step is to select the appropriate multi-degree of freedom variables according to the problem requirements to construct the objective function $f(x)$.Finally, a set of inequality (or equations) is commonly used $g_i(x) \le 0 (i = 1,2,\dots,m)$to represent the constraints. In general, the model can be expressed as follows:

$$\min z = f(x) \tag{3}$$

$$s.t. g_i(x) \le 0 (i = 1, 2, \cdots m) \tag{4}$$

The model composed of (1) and (2) belongs to constrained optimization, and if only formula (1), it is unconstrained optimization.$f(x)$ is called objective function and $g_i(x) \le 0$ is called constraint

$$Z = F(X) = \begin{bmatrix} \max(\min) f_1(x) \\ \max(\min) f2(x) \\ \vdots \\ \max(\min) f_k(x) \end{bmatrix} \tag{5}$$

$$s.t. \phi(X) = \begin{bmatrix} \varphi_1(x) \\ \varphi_2(x) \\ \vdots \\ \varphi_m(x) \end{bmatrix} \le G = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_m \end{bmatrix} \tag{6}$$

Where $X = \left[x_1, x_2 \cdots x_n\right]^T$ is the decision variable.

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We can write the $s.t.\phi(X)=\begin{bmatrix}\varphi_1(x)\\ \varphi_2(x)\\ \vdots\\ \varphi_m(x)\end{bmatrix}\le G=\begin{bmatrix}g_1\\ g_2\\ \vdots\\ g_m\end{bmatrix}$ as follows: $s.t. x\in\Omega$. Where the space Ω where the $X=\left[x_1,x_2\cdots x_n\right]$ is located is called the decision space.The space in which $F(x)$ is located is called a reduced form of the target space: $\begin{matrix}Max(\min)Z=F(X)\cdots(1)\\ s.t.\phi(X)\le G\end{matrix}$ If there are n decision variables, k objective functions, and m constraint equations, then

$$\begin{aligned}&Z=F(x)\cdots k \text{ dimensional function variables}\\ &\phi(\mathrm{X})\cdots \text{The m-dimensional vector variable}\\ &\mathrm{G}\cdots \text{The m-dimensional constant variable}\end{aligned} \tag{7}$$

3. Practical application of the planning model

3.1 Instances of linear programming

Example 1: Making cooking utensils requires two kinds of resources: labor and raw materials. A company production plan, production of three different products, production management department provides data is as follows: A produce·1 8 pieces per hour, 5kg raw materials, each profit 6 yuan per piece, B produce 3 pieces per hour, 4kg raw materials, profit 5 yuan per piece, C produce 6 pieces per hour, 4kg raw materials, profit 6 yuan per piece.

300kg of raw materials were supplied every day, and the available labor force was 180h. Establish a linear planning model to maximize the total revenue, and find the daily output of various products.

Step 1: Determine the decision variable. The required unknown variable is the daily output of the three products with algebraic symbols, that is, the daily output of the three products A, B and C by x_A, x_B, x_C.

Step 2: Determine the constraints, where the constraints can be constrained by labor and raw materials:

$$\text{raw material: } 5x_A+4x_B+4x_C\le 300 \tag{8}$$

$$\text{labour force: } 8x_A+3x_B+6x_C\le 180 \tag{9}$$

Step 3: Determine the objective function. The goal of this problem is to maximize the total return, which is:

$$\max X=6x_A+5x_B+6x_C \tag{10}$$

According to the above three steps, the linear planning of this production combination problem is:

$$\max X=6x_A+5x_B+6x_C \tag{11}$$

$$s.t.\begin{cases}5x_A+4x_B+4x_C\le 300\\ 8x_A+3x_B+6x_C\le 180\\ x_A,x_B,x_C\ge 0\end{cases} \tag{12}$$

3.2 Non-linear programming instance

Example 2: $C4$ olefin is widely used in the production of chemical products and medicine, and ethanol is the raw material for the production and preparation of $C4$ olefin. In the preparation process,

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the combination of catalyst combination (That is, the combination of Co load, $Co\ /\ SiO2$ and HAP loading ratio, and ethanol concentration) and temperature will affect the selectivity of $C4$ olefin and $C4$ olefin yield.The appropriate catalyst combination and temperature can maximize the yield of $C4$ olefins under the same experimental conditions.Under the condition of the temperature below 350 degrees, how to choose the catalyst combination and temperature, so that the value of $C4$ olefin yield may be large.

Analyse: We first found the ethanol conversion rate and $C4$ olefin selectivity with temperature, $Co\ /\ SiO2$ content, and ethanol Linear equation between five indicators of concentration, ethanol surplus in unit, and HAP quality. The nonlinear relationship between the $C4$ olefin yield and the five indices was obtained by multiplying the two together. The simulated annealing algorithm was used to solve the maximum value of the $C4$ olefin yield and the values of the five indexes.

Model building: In the above problems, we found the linear relationship between ethanol conversion rate, $C4$ olefin selectivity and temperature, ethanol unit residual quantity, $Co\ /\ SiO2$ quality, ethanol concentration, and HAP quality.

$$y_1 = -80.712 + 0.335x_1 + 0.449x_2 - 0.04x_3 \text{-} 8.781x_4 + 0.139x_5 \tag{13}$$

$$y_2 = -51.069 + 0.181x_1 - 2.039x_2 + 0.035x_3 + 1.761x_4 + 0.081x_5 \tag{14}$$

The yield of $C4$ olefin is the product of the ethanol conversion rate and the selectivity of $C4$ olefin, where we construct the objective function,

$$Y = \mathrm{y}_1 \times y_2 \tag{15}$$

According to the data given in the title, we can know that the value range of each variable is:

$$\begin{cases} 250 \le x_1 \le 450 \\ 0.1 \le x_2 \le 9.52 \\ 9.9 \le x_3 \le 199 \\ 0.3 \le x_4 \le 2.1 \\ 0 \le x_5 \le 200 \end{cases} \tag{16}$$

Model solution:

We used the simulated annealing algorithm toolbox of MATLAB, and tried to derive multiple sets of local optimal solutions many times, and selected the optimal group, as shown in the **Table 1** below:

Table 1: Parameter optimal value table table questions

target	Co content /(mg)	temperature $/^\circ C$ (mg)	U quality/(mg)	Ethanol concentration(ml/min)	HAP quality(mg)
optimal value	450	0.1	183.7	0.3	200

We obtained an optimal value of 4633.13 with 10,522 iterations, as follows as **Figure 1**:

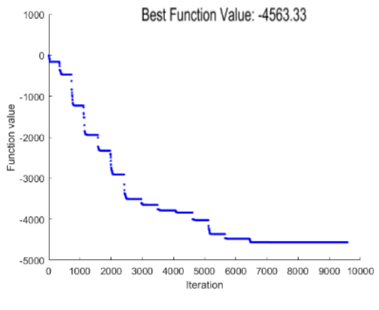

Figure 1: Iterative map problem

As described above, we use the simulated annealing algorithm to solve the nonlinear planning model. The results have high accuracy, and the implementation process is relatively simple in the solution process. Later, we can try other intelligent algorithms for nonlinear planning.

3.3 Multi-objective planning instance

In addition to the above linear planning model and nonlinear planning model, the application of multi-objective planning model is also very extensive. The following are the common solutions to the multi-objective planning model.

(1)Target planning law

The basic idea is: when facing the multi-goal problem, we need to build a goal planning model. Such models need to give each objective function an expected value, but also need to determine the priority of each target and the corresponding weight coefficient of each target in the whole model[1].

(2) Linear weighting

The basic idea is: according to the priority degree of multi-objective$f_i(x)(i=1,2,\ldots,m)$, multiply by a group of weight coefficient $\lambda_j(j=1,2,\ldots,m)$ and then sum as an objective function to constitute a single objective planning problem [2].then:

$$\min \mathrm{f} = \sum_{j=1}^{m} \lambda_j f_j(x), \text{ in } \lambda_j \geq 0 \sum_{j=1}^{m}, \lambda_j = 1 \tag{17}$$

(3) Minimax method

The basic idea is that for minimized multi-objective planning, just write the objective function with a value range as a constraint with bidirectional constraints. At this point, for each $x \in R$, we first find the maximum value of the objective function value$f_i(x)$, and then find the minimum value of the maximum value[3]. That is, construct single target planning:

$$\min \mathrm{f} = \max_{1 \leq j \leq m} \{\mathrm{f_j}(\mathrm{x})\} \tag{18}$$

(4)Target reach method (step method)

For multi-objective planning:$\min[\mathrm{f_1(x), f_2(x), \ldots, f_m(x)}]$ $\mathrm{s.t.}$ $\mathrm{g_i(x)} \leq 0$ $\quad \mathrm{j} = 1,2,\ldots,\mathrm{n}$.Design a set of target value idealized vector $(f_1^*, f_2^*, \ldots, f_m^*)$ corresponding to the target function first,, Then let γ be a relaxation factor scalar.Let the $\mathrm{W} = (\mathrm{w_1, w_2, \ldots, w_m})$ be the vector of the weight coefficient[4].So the multi-goal planning problem is reduced to:

$$\min_{x,\gamma} \gamma \tag{19}$$

$$\mathrm{f_j}(x) - w_j\gamma \leq f_j^* \qquad j = 1,2,\ldots,\mathrm{m} \tag{20}$$

$$g_j(x) \leq 0 \qquad j = 1,2,\ldots,\mathrm{k} \tag{21}$$

(5) Dictionary order method

Sort the importance of the target, solve each single target plan in turn (the optimal solution of the previous target is not unique, the result is used as the next target constraint), and end when there is a unique solution.

(6) Intelligent algorithm (simulated annealing algorithm, particle swarm algorithm, genetic algorithm, etc.)

In addition to the traditional numerical solution method, the application of intelligent algorithms in multi-objective optimization models is also relatively common. It includes simulated annealing algorithms, particle swarm algorithms and genetic algorithms [5].

4. The results of the outlook

This paper first summarizes several common classification models of planning models, and helps readers to get familiar with the operation process of various planning models. On the basis of being familiar with the principles, I have carefully selected the heavyweight mathematical modeling subject competitions and some classic exercises in recent years. The linear planning model, nonlinear planning model and multi-objective planning model are analyzed and solved respectively. Through the problem

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analysis, it can be clearly seen that the multi-planning model is widely used in mathematical modeling. Several examples listed in this paper include many different fields, such as physics, chemistry and mechanics. The idea of using goal planning can solve many practical problems and has a very high practical application value. In this paper, the exercises of different models are established, but in the mathematical modeling competition, the solution of the model is also very important. The next step will continue to explore the different solutions of the planning model. In the current research, the solution of the planning model mainly includes the traditional numerical solution method and some relatively popular intelligent solution and optimization algorithms. For example, simulated annealing algorithm, particle swarm algorithm, genetic algorithm, and so on. The discussion of multi-objective optimization model based on these algorithms is worth studying.

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