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Discrete-Time Behavioural Portfolio Choice in Frictionless Markets

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Abstract: *In the exploration of discrete-time behavioural portfolio choice under Cumulative Prospect Theory, this study, conducted within a single-period economy, establishes a portfolio optimization model with the objective of maximizing distorted expected utility. Analytic solutions for optimal allocation are derived under two key assumptions: prohibiting short-selling while allowing borrowing, and prohibiting both short-selling and borrowing. Under the assumption of forbidding short-selling and allowing borrowing, a unique optimal portfolio emerges when the risk aversion on gains dominates the risk propensity on losses. In contrast, in cases where the latter prevails, there may be multiple optimal portfolios or none. Transitioning to the scenario where both short-selling and borrowing are forbidden, dominance of risk aversion on gains results in two optimal portfolios. When risk aversion equals risk propensity on losses, multiple portfolios may arise; otherwise, a unique optimal portfolio emerges or none at all. If risk propensity on losses dominates, two optimal portfolios exist; otherwise, uniqueness or absence of an optimal portfolio may occur. Concluding the study, sensitivity analysis is presented, highlighting the impact of the CPT-ratio and the level of loss aversion on the optimal portfolio, provided it exists and is unique.*

Keywords: *Behavioural Finance; CPT Model; Portfolio Choice; Utility Function; Weighting Distortions*

1. Introduction

Within the Expected Utility Theory (EUT) framework, the conventional portfolio optimization model assumes rationality among investors. EUT posits three key perspectives: uniform risk aversion reflected in a concave utility function, evaluation of wealth based on final asset positions, and reliance on objective probabilities.

However, empirical experiments consistently reveal deviations from these classical EUT principles when investors confront risk uncertainty. Notably, phenomena such as the Allais Paradox [1] and Ellsberg Paradox [2] indicate non-uniform risk preferences and challenges in objectively evaluating probabilities.

In response to these observations, Tversky and Kahneman (1992) [4] established Cumulative Prospect Theory (CPT) which departs from EUT by proposing that individuals evaluate assets relative to certain criterion, instead of focusing solely on final capital. Moreover, investors exhibit distinct attitudes toward gains and losses, showcasing risk-averse when they are gaining and risk propensity when they are losing. Additionally, CPT acknowledges the tendency of individuals to overrate unlikely events and under rate likely events, represented by the inverse S-shaped probability distortion function. Concurrently, investors display heightened sensitivity to losses compared to gains—a phenomenon termed loss aversion. This is reflected in the fact that the slope of the loss value function is larger than the slope of the return value function.

Numerous researchers have delved into the intricacies of the choice of investment combination under Cumulative Prospect Theory (CPT), employing diverse methodologies. He and Zhou (2010) [5] explored how the value function reacts to changes in the allocation of security, revealing that the CPT utility value function defies both concavity and convexity. Bernard and Ghossoub (2010) [6] established a robust link between Omega performance metrics and ideal investor portfolios when applying CPT with a specific value function. Chao et al. (2018) [7] introduced the ARCGA method as a novel method for optimal CPT investment allocation, presenting an empirical analysis comparing choices across various criteria level.

Based on the paper by Bernard and Ghossoub (2010) [6], this study formulated a CPT portfolio choice model within a single-period economy in a market consistent of one equity and one risk-free asset, where

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three key elements of CPT are discussed: the criteria point, the asymmetric S-shaped utility function, and the distorted probability distortions. Distinguishing this study from Bernard and Ghossoub's seminal work (2010) [6], several noteworthy contributions are evident. Firstly, within the model setting, this study introduces and substantiates propositions tailored to meet the requirements for resolving the optimal portfolio problem. Notably, the inclusion of an arbitrage-free proposition is demonstrated and confirmed as applicable in the single-period economy model. Secondly, addressing the CPT optimal portfolio problem, this study provides analytic solutions, where feasible, particularly in instances where risk propensity on losses dominates risk aversion on gains (indicated by $\alpha > \beta$ in the model). Finally, the analytic optimal portfolio solutions in this study incorporate loss aversion.

The remaining parts of this study are organised as follows: Section 2 shows a behavioural portfolio choice model featuring all the three key elements of CPT; Section 3 sets up a portfolio optimisation model with the goal of maximising the expected utility and obtained the analytic solutions of optimal portfolio; Section 4 is the conclusion.

2. Model

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be the probability space, where Ω is a non-empty set representing all possible market scenarios, $\mathcal{F}$ is a σ-field on Ω, and $\mathbb{P}$ is the true probability measure. Fix an investment horizon $T \in (0, \infty)$, and let the trading dates be $\{0, T\}$. The information available to investors over time is represented as a filtration $\mathbb{F} = \{\mathcal{F}_t\}_{t \in \{0, T\}}$.

Assume a market that contains of one risk-free asset (with initial price one and the interest rate $r \geq 0$) and one equity (with return rate $R = S_T / S_0$). Based on a one period economy, the constant $S_0 > 0$ is the stock price at the current time 0 and the $\mathcal{F}$-measurable random variable S_T is the equity price at time T. Let $x_0 \in \mathbb{R}$ be the initial capital. The equity receives an investment amount $\theta \in \mathbb{R}$ while the rest, $x_0 - \theta$, is allocated to the risk-free asset. The investor's final wealth value can be expressed as:

$$W_T^{\theta} = (x_0 - \theta)(1 + r) + \theta(1 + R) = x_0(1 + r) + \theta X \tag{1}$$

where the $\mathcal{F}$-measurable random variable $X := R - r$ is the surplus yield of equity compared to risk-free rate (note that X does not depend on initial capital x_0).

Assumption 2.1. *Assumed that* $\mathbb{P}\{X \neq 0\} > 0$ *or equivalently,* $\mathbb{P}\{X = 0\} < 1$, *otherwise the risk-free asset and the equity would be indistinguishable.*

This discussion is based on no-arbitrage, In the presence of arbitrage opportunities, investors swiftly exploit them, restoring market equilibrium and eliminating arbitrage opportunities. The author gives the following assumption related to absence of arbitrage.

Assumption 2.2. *The excess return X can take both values larger than 0 and lower than 0 with positive probabilities, i.e.,* $\mathbb{P}\{X > 0\} > 0$ *and* $\mathbb{P}\{X < 0\} > 0$.

The Assumption 2.2 excludes arbitrage opportunities in the model, that is, it is not possible to make a risk-less profit out of nothing.

Proposition 2.3. *If Assumption 2.2 holds, then the model is free of arbitrage, i.e., for all* $\theta \in \mathbb{R}$, *if* $W_0^{\theta} = 0\ a.s.$ *and* $W_T^{\theta} \geq 0\ a.s.,$ *then* $W_T^{\theta} = 0\ a.s.$

In view of the limitations of EUT, Kahneman and Tversky (1992) [4] used value function and distorted weighting function to replace utility function and linear probability. Compared with EUT, CPT has three improvements: considering reference points; S-shaped utility functions; and the distorted probabilities.

The concept of a reference point is introduced because EUT focuses on the final value of wealth, whereas CPT emphasizes changes in capital compared with a specific criteria value.

Definition 2.4. [**Criteria Level, Gains and Losses**] A *reference level* (or *benchmark*) is an $\mathcal{F}$-measurable random variable B. An investor care about the *deviation* of the final capital W_T^{θ} from a *criteria level* B. Given a market scenario $\omega \in \Omega$, when the terminal value is greater than B (i.e., $W_T^{\theta}(\omega) - B(\omega) > 0$), the investment is regarded as a *gain* and brings positive utility to the investor, whereas when it is less than B (i.e., $W_T^{\theta}(\omega) - B(\omega) < 0$), the investment is considered a *loss* and brings negative utility to the investor.

Assumption 2.5. *The criteria point B represents the sum individuals will receive at the period's end if they allocate the entire initial capital* x_0 *into the risk-free asset, i.e.:*

$$B := x_0(1 + r) \tag{2}$$

Under the Assumption 2.5, the deviation is simply a multiple of the excess return:

$$W_T^{\theta} - B = \theta X \tag{3}$$

The second is the value function. EUT believes that people are always risk averse. But in fact, Kahneman and Tversky (1992) [4] points out that people's attitudes toward losses and gains are quite different. The value function call well describes this phenomenon. We then define the value function u, also called S-shaped utility function.

Definition 2.6. [**S-shaped Utility**] The *S-shaped utility* (also known as *value function*) $u : \mathbb{R} \rightarrow \mathbb{R}$ is of the form:[1]

$$u(x) := u_+(x^+) - u_-(x^-), \text{ for all } x \in \mathbb{R}, \tag{4}$$

where the functions $u_\pm : [0, +\infty) \rightarrow [0, +\infty)$ are continuous, strictly increasing, concave, with $u_\pm(0) = 0$.

The functions u^+ and u^- represent the positive and negative utility functions defined on the wealth interval with B as the reference point, respectively.

Figure 1 reasonably describes the people's psychological characteristic. According to the valuation rule proposed by Kahneman and Tversky (1992) [4], utility function is a two-part cumulative functional, in which function u_+ represents people's satisfaction evaluation of portfolio return, and function u_- represents disappointment evaluation.

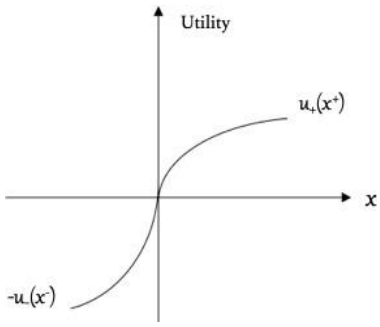

Figure 1: The value function u(x).

They also put forward the reflection effect [3], that is, the value function should be a reflection shape with the coordinate origin as the centre and deviate from the two directions of gains and losses, i.e., the S-shaped curve, as shown in Figure 1. When facing gains, the function exhibits concave shape, indicating risk aversion; while facing losses, the function demonstrates convex shape, indicating risk seeking. This is an important feature of the value function, which is also the reason why the value function is S-shaped. In addition, the slope of the value function on the losses part is greater than that on the gains part. In other words, under the corresponding gains and losses, the rate of diminishing marginal utility on losses is faster than that of diminishing marginal utility on gains. In terms of investors' behaviour, the psychological changes brought by the loss to investors are greater than the same proportion of returns, and the graph shows that the slope of the equation on the left is larger than that on the right.

CPT investors have a certain degree of distortion towards objective probabilities. Kahneman and Tversky (1992) [4] think that the weight function is a distortion of the given probability instead of a subjective probability. The weight function is non-linear in form and shape. Its primary trait is that individuals typically overrate unlikely events and underrate likely events.

Definition 2.8. [**Distortion Function**] The functions $w_\pm$: $[0,1] \rightarrow [0,1]$ are *probability distortions* on gains (+) and on loss (−) if they are continuous and strictly increasing, with $w_\pm(0) = 0$ and $w_\pm(1) = 1$.

Definition 2.10. [**CPT Functional**] For any $\mathcal{F}$-measurable random variable X, defining its *CPT functional* as:

$$V(X) := V_+(X^+) - V_-(X^-), \tag{5}$$

[1] For all $x \in \mathbb{R}$, $x^\pm := max\{\pm x, 0\}$. I.e., when $x \geq 0$, $x^+ = x$, and $x^- = 0$; when $x < 0$, $x^+ = 0$, and $x^- = -x$.

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With

$$V_{\pm}(X^{\pm}) := \int_0^{+\infty} w_{\pm}(\mathbb{P}\{u_{\pm}(X^{\pm}) > y\})dy, \tag{6}$$

and the convention $+\infty - \infty := -\infty$.

***Remark* 2.11.**

(1) Note that $V_{\pm}(X^{\pm})$ is the Choquet integral of $u_{\pm}(X^{\pm})$ with respect to the capacity $w_{\pm} \circ \mathbb{P}$:

$$V_{\pm}(X^{\pm}) = \int_0^{+\infty} \omega_{\pm}(\mathbb{P}\{u_{\pm}(X^{\pm}) > y\})dy = \int u_{\pm}(X^{\pm})d(w_{\pm} \circ \mathbb{P}) \tag{7}$$

(2) Note that $\mathbb{E}_{\mathbb{P}}$ denotes the expectations under the physical probability measure. The CPT functional $V(X)$ is the generalisation of the expected utility $\mathbb{E}_{\mathbb{P}}[u(X)]$ of EUT, because $V(X) = \mathbb{E}_{\mathbb{P}}[u(X)]$ if the probability measure is not distorted (i.e., $w_{\pm}(p) = p$ for all $p \in [0, 1]$, whence $(w_{\pm} \circ \mathbb{P})(A) = w_{\pm}(\mathbb{P}(A)) = \mathbb{P}(A)$ for all $A \in \mathcal{F}$).

CPT optimal portfolio mainly discusses whether there is an investment strategy to maximise investors' expected utility, as given by the CPT functional.

Definition 2.12. [**CPT Optimal Portfolio Problem**] The *optimal portfolio selection for a CPT investors* with initial capital $x_0 \in \mathbb{R}$ is:

$$v(x_0) := \sup_{\theta \in \mathcal{A}(x_0)} V\left(W_T^{\theta} - B\right) \tag{8}$$

where $\mathcal{A}(x_0)$ is a set of *admissible portfolios*. A portfolio $\hat{\theta} \in \mathcal{A}(x_0)$ is *optimal* if:

$$v(x_0) = V\left(W_T^{\hat{\theta}} - B\right). \tag{9}$$

Remark 2.13.

(1) If there are no investment constraints, then $\mathcal{A}(x_0) = \mathbb{R}$.

(2) If short-selling is forbidden, but borrowing is still allowed, then $\mathcal{A}(x_0) = [0, +\infty)$.

(3) If short-selling and borrowing are both forbidden, then $\mathcal{A}(x_0) = [0, x_0]$.

Having established the general Cumulative Prospect Theory (CPT) model, our next steps involve solving the CPT optimal portfolio problem and examining the properties of the CPT optimal portfolio, given the specific selection of the piecewise power utility as the value function.

3. CPT Optimal Portfolio

In section 3, this study employs the piecewise power utility function introduced by Kahneman and Tversky (1992) [4] as the value function of portfolio optimisation problem of CPT investors, and try to find the optimal allocation θ in risk assets in three cases : $\boldsymbol{\alpha = \beta}$, $\boldsymbol{\alpha < \beta}$ and $\boldsymbol{\alpha > \beta}$, respectively.

Assumption 3.1. *The value function is defined as the piecewise power utility:*

$$u_+(x) := x^{\alpha}; \;\; u_-(x) := \lambda x^{\beta} \tag{10}$$

where $0 < \alpha \leq 1$, $0 < \beta \leq 1$ *and* $\lambda \geq 1$.

Note that α, $\beta > 0$ ensures that $u_{\pm}(\cdot)$ are strictly increasing, while $\alpha, \beta \leq 1$ ensures that $u_{\pm}(\cdot)$ are concave. The parameter λ represents the level of loss aversions. Considering the Arrow-Pratt's measures of RRA, the piecewise power utility functions $u_{\pm}(\cdot)$ are Constant Relative Risk Aversion (CRRA) utilities because

$$RRA_{u_+(x)} = -\frac{xu_+''(x)}{u_+'(x)} = 1 - \alpha,$$

$$RRA_{u_-(x)} = -\frac{xu_-''(x)}{u_-'(x)} = 1 - \beta,$$

and is unbounded above (i.e., $\lim_{x \to +\infty} u_{\pm}(x) = +\infty$).

Expression $1 - \alpha$ describes the degree of risk aversion on gains of CPT investors while expression $1 - \beta$ describes the degree of risk propensity on losses of CPT investors. The smaller the α ($1 - \alpha$ becomes

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larger), the more the investor becomes risk-averse on gains; the smaller the β ($1-\beta$ becomes larger), the more the investor becomes risk seeking on losses. Moreover, the scenario $1-\alpha > 1-\beta$, or equivalent, $\alpha < \beta$, indicates that the risk aversion when the investor is gaining is dominating the risk propensity when the investor is losing. On the contrary, the scenario $1-\alpha < 1-\beta$, or equivalent, $\alpha > \beta$, represents that the risk seeking on losses is much stronger than the risk aversion on gains, which is not in accordance with experimental conclusion in Tversky and Kahneman (1992) [4]. The coefficient λ represents the degree of loss-averse, with higher λ value indicating greater aversion to losses among CPT investors. Tversky and Kahneman (1992) [4] estimated the value of λ=2.25. Giorgi and Hens (2006) [8] explains λ as, "Investors dislike losses by a factor of 2.25 as compared to their liking of gains". It can be also understood as when investors suffer the same amount of losses and gains, the degree of suffering of losses is 2.25 times as much as the pleasure brought by gains. This is consistent with the fact shown in Figure 1.

Afterwards, a crucial quantity is introduced to promote the resolution of the CPT optimal portfolio selection. Building upon the insights of Bernard and Ghossoub (2010) [6], the definition of the CPT-ratio is provided.

Definition 3.2. [**CPT-ratio**] The *CPT-ratio* of X is defined as:

$$\Omega(X) \coloneqq \frac{\mathbb{G}(X)}{\mathbb{L}(X)} \tag{11}$$

where

$$\mathbb{G}(X) \coloneqq (\mathcal{C}) \int (X^+)^\alpha \, d(w_+ \circ \mathbb{P}) = \int_0^{+\infty} w_+(\mathbb{P}\{(X^+)^\alpha > y\})dy \tag{12}$$

$$\mathbb{L}(X) \coloneqq (\mathcal{C}) \int (X^-)^\beta \, d(w_- \circ \mathbb{P}) = \int_0^{+\infty} w_-\big(\mathbb{P}\{(X^-)^\beta > y\}\big)dy \tag{13}$$

Remark 3.3.

(1) Imposed $\mathbb{G}(X) \geq 0$ and $\mathbb{L}(X) > 0$ so that the CPT-ratio is well defined and belongs to $[0, +\infty)$.

(2) Neither function $\mathbb{G}(X)$ nor $\mathbb{L}(X)$ depends on optimal allocation θ.

The CPT-ratio quantifies the ability of risk assets to bring gains to CPT investors. The higher the CPT-ratio, the stronger the ability of risk assets to bring gains than to losses, and the more willing investors are to invest in risk assets. On the contrary, the lower the CPT-ratio, the weaker the ability of risk assets to bring gains than to lose, the less invest in risk assets. Thus, the optimal allocation in risk assets depends on the CPT-ratio.

In the remainder of this chapter, the analytical expression of optimal allocation is provided. Adhering to the approach outlined by Carole Bernard and Mario Ghossoub (2010) [6], the analytical results will be explored in two distinct scenarios: one where short-selling is prohibited but borrowing is allowed, and the other where both short-selling and borrowing are forbidden.

3.1. Constraints on Short-selling and Permissible Borrowing

Assumption 3.4. *Constraints on Short-selling and Permissible Borrowing, i.e., $\mathcal{A}(x_0) \in [0, +\infty)$.*

Theorem 3.5. *Let $x_0 \in \mathbb{R}$, and let Assumptions2.2, 2.5, 3.1, 3.4 hold.*

(1) If $\alpha < \beta$, then the unique optimal portfolio is:

$$\hat{\theta} := \left(\frac{\alpha}{\lambda\beta}\right)^{\frac{1}{\beta-\alpha}} \Omega(X)^{\frac{1}{\beta-\alpha}} \tag{14}$$

(2) If $\alpha = \beta$, then:

(a) there is no optimal portfolio when $\Omega(X) > \lambda$;

(b) any portfolio $\theta \in [0, +\infty)$ is optimal when $\Omega(X) = \lambda$;

(c) the unique optimal portfolio is $\hat{\theta} = 0$ when $\Omega(X) < \lambda$.

(3) If $\alpha > \beta$, then there is no optimal portfolio.

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3.2. Constraints on both Short-selling and Borrowing

Assumption 3.6. *Constraints on both Short-selling and Borrowing, i.e.* $\mathcal{A}(x_0) = [0, x_0]$.

The "forbidden borrowing" here means that investors can only buy, and can only use their own funds (the initial funds x_0), not from other parties to borrow.

Theorem 3.7. *Let Assumptions2.2, 2.5, 3.1, 3.6 hold.*

(1) If $\alpha < \beta$*, then the optimal portfolio is*[2]

$$\bar{\theta} := min(\left(\frac{\alpha}{\lambda\beta}\right)^{\frac{1}{\beta-\alpha}} \Omega(X)^{\frac{1}{\beta-\alpha}}, x_0) \quad (15)$$

(2) If $\alpha = \beta$*, then:*

(d) the unique optimal portfolio is $\bar{\theta} = x_0$ *when* $\Omega(X) > \lambda$*;*

(e) any portfolio $\theta \in [0, x_0]$ *is optimal when* $\Omega(X) = \lambda$*;*

(f) the unique optimal portfolio is $\bar{\theta} = 0$ *when* $\Omega(X) < \lambda$.

(3) If $\alpha > \beta$*, then:*

(g) the unique optimal portfolio is $\tilde{\theta} = x_0$ *when* $x_0 > \theta^*$*;*

(h) the optimal portfolios are $\tilde{\theta} \in \{0, x_0\}$ *when* $x_0 = \theta^*$ *;*

(i) the unique optimal portfolio is $\tilde{\theta} = 0$ *when*$x_0 < \theta^*$.

Here, $\theta^* > 0$ *is the unique portfolio (possibly not admissible) such that* $V(\theta * X) = V(0)$.

Based on the experimental results of American experimenters, Kahneman and Tversky (1992) [4] points out that CPT investors are more risk-aversing facing gains than risk seeking facing losses. That is, for a piecewise power utility, case $\alpha < \beta$ is more consistent with their experimental results. However, an interesting report is, the experimental results in ZENG (2007) [9] shows that case $\alpha > \beta$ can applied in Chinese subjects. ZENG follows Tversky and Kahneman's research paradigm to experiment 38 graduate students in Sun Yat-sen University of China and draws the opposite conclusion: with a medium to high probability, the risk seeking on losses is always greater than the risk aversion of gains, as their estimates show α=1.21, β=1.02. The probability distortion ZENG applied shows γ=0.55 and δ=0.49. Moreover, the α and β values estimated by Kahneman and Tversky(1992) [4] are less than 1, which makes the value function S-shaped and has a weakening sensitivity, while the experimental results of ZENG (2007) [9] show that α and β is greater than 1 and the value function is inverted S-shaped, which shows that it has a stronger sensitivity, violating the claim of the prospect theory that the value function has a weakening sensitivity. Apparently, ZENG (2007) [9] did not impose the hypothesis of $\alpha, \beta \leq 1$ in his report, and he also mentioned that ones may make appropriate adjustments to the cumulative prospect theory when applying it to Chinese subjects.

4. Conclusions

This study presents a Cumulative Prospect Theory (CPT) portfolio selection model within a single-period economy. CPT posits that individuals assess assets relative to benchmarks, prioritize risk aversion during gains and risk propensity during losses, and exhibit loss aversion, reflected in steeper value functions for losses than gains. Additionally, individuals are likely to overrate the impact of unlikely events and underrate likely events. The portfolio optimization model aims to maximize distorted expected utility, providing analytic solutions under two assumptions: short-selling is forbidden and borrowing is allowed, and both short-selling and borrowing are forbidden.

However, it is noted that the report has certain limitations. Specifically, the sensitivity analysis for the optimal allocation concerning the α and β in the power utility function is not determined analytically. Future research aims to extend the single period economy model to multi-periods model and try to solve

[2] To distinguish, in this case of $\alpha < \beta$ and $\alpha = \beta$, the optimal portfolio is represented by $\hat{\theta}$ in Theorem 3.5, while it is represented by $\bar{\theta} := (\hat{\theta}, x_0)$ in Theorem 3.7. But in the case of $\alpha > \beta$, the optimal portfolio is represented by $\tilde{\theta}$ both in Theorem 3.5 and 3.7.

the problem of S-shaped utility maximisation.

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