

# Prediction of Remaining Lifetime of Hall Current Sensor Based on Weibull Distribution

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**Abstract:** The degradation mechanism of Hall current sensors is characterized by complexity, fluctuations, and nonlinearity, which poses a challenge to accurately predicting the lifetime of highly reliable and long-lived Hall current sensors. In this study, accelerated degradation tests were conducted on Hall current sensors, and performance degradation data was used to predict the sensor's reliability and remaining lifetime. Firstly, the working principle and degradation mechanism of Hall current sensors were analyzed, and the fluctuation of zero-point output voltage was identified as the performance degradation parameter. Next, the types of accelerated degradation models and test conditions were described, and a method based on the Weibull distribution was employed to estimate the pseudo-failure lifetime at various temperature stress levels under accelerated degradation testing. Finally, an accelerated model for the distribution parameters of pseudo-lifetime was derived using the Arrhenius model, thereby completing the reliability and remaining lifetime prediction of Hall current sensors under normal working conditions.

**Keywords:** Weibull distribution; Arrhenius model; Pseudo Failure Life; Remaining life

## 1. Introduction

Hall current sensors are a critical component of the new power system, with advantages such as high accuracy, good linearity, high reliability, low power consumption, and easy maintenance and replacement. They have been widely used in the field of power Internet of Things. Given their importance in ensuring the safe and stable operation of the power grid, predicting the reliability and lifetime of Hall current sensors is crucial. However, these sensors have a long lifetime and high reliability, and their performance degradation trends are difficult to observe in the short term under normal working conditions. Therefore, in reliability engineering, accelerated degradation testing is typically used to predict the reliability and lifetime of Hall current sensors by changing the environmental stress level and shortening the working cycle while keeping the failure mechanism of the product unchanged. This testing method can effectively improve the accuracy of predictions and provide a guarantee for the safe and stable operation of the power grid.

Currently, researchers from various academic fields have extensively studied accelerated degradation testing and the Weibull distribution. For example, Lv et al. modeled the performance degradation of high-power switches using the log-normal distribution and the Weibull distribution<sup>[1]</sup>. While Sun modeled the performance degradation of capacitors using the Weibull distribution and the Gaussian-Poisson joint distribution, and compared the accuracy of the two distribution types<sup>[2]</sup>. Wang analyzed the reliability of integrated circuits based on the Wiener degradation process<sup>[3]</sup>.

In this study, a certain type of Hall current sensor was selected for accelerated degradation testing under temperature stress. Based on the test data, an appropriate degradation trajectory model was chosen, and the pseudo-lifetime of the product was extrapolated according to the failure threshold. Then, the distribution parameters of the pseudo-lifetime were estimated using the Weibull distribution parameter estimation method, and the reliability and remaining lifetime of the Hall current sensor under normal working temperature conditions were predicted using the acceleration equation.

## 2. Hall current sensor failure mechanism analysis

### 2.1. Failure mechanism analysis

The Hall current sensor typically consists of a primary circuit, a magnetic concentrator, a Hall element, and a conditioning circuit. The basic working principle is based on the Hall effect principle, which is also known as the magneto-electric conversion effect<sup>[4]</sup>. When a current flows through the primary conductor, a magnetic field is generated around the conductor. The Hall element is excited by the magnetic field and generates a corresponding voltage output signal. The voltage signal is processed by an operational amplifier and output as the secondary side compensation current. When the magnetic field generated by the compensation current through the compensation winding on the secondary side balances the magnetic field generated by the primary current, the primary current can be calculated by measuring the compensation current. The most common cause of failure of the Hall current sensor is internal component failure, and thermal stress has a relatively large impact on the components. Thermal stress can cause oxidation, desoldering, and other changes to the components, resulting in measurement fluctuations of the sensor. When the accumulated fluctuation exceeds a certain threshold, the Hall current sensor will fail.

### 2.2. Selection of performance degradation parameters

By analyzing the working principle and failure mechanism of the Hall current sensor, it has been found that fluctuations in the zero-point output voltage can intuitively reflect the performance degradation of the sensor. As the working time of the sensor increases, significant changes can be detected in the zero-point voltage. Therefore, the fluctuation of the zero-point output voltage of the sensor is used as a performance degradation parameter.

## 3. Related Theories

### 3.1. Accelerated degradation test and degradation trajectory model

Accelerated degradation testing refers to the process of subjecting a product to stress levels higher than those under normal working conditions to accelerate the product's failure or performance degradation without changing the failure mechanism of the product<sup>[5]</sup>. Accelerated degradation testing focuses more on the performance degradation process of the product, and evaluates the product's reliability by modeling the performance degradation trajectory or degradation amount of the product.

When predicting the lifetime and evaluating the reliability of a product based on its degradation data, the choice of different models can have a significant impact on the results. Degradation trajectory models are widely used in the field of accelerated testing due to their simplicity and intuitiveness. Common degradation trajectory models are shown in Table 1.

Table 1: Common degradation trajectory models.

Degradation model	Degenerate expressions
Linear model	$y(t) = \alpha t + \beta$
Natural logarithmic model	$y(t) = \alpha \ln(t) + \beta$
Exponential Model	$y(t) = \beta e^{\alpha t}$

In the equation,  $y(t)$  represents the degradation amount of the product,  $\alpha$  and  $\beta$  are unknown parameters, and  $t$  represents the degradation time.

### 3.2. Accelerated degradation model

Accelerated degradation models describe the relationship between the degradation amount of a product and various stress levels. By using an accelerated model, the value of the product's degradation characteristic at high stress levels can be used to infer the value at normal stress levels. The most widely used accelerated models include the Arrhenius model, the Eyring model, and the Inverse Power Law model<sup>[6-8]</sup>.

Based on the analysis of the failure mechanism of the Hall current sensor, the Arrhenius model is chosen to describe the relationship between the fluctuation of the zero-point voltage and the test temperature.

$$\varepsilon = A \exp(E_a / kT) \tag{1}$$

In the equation,  $\varepsilon$  represents the degradation amount of the product;  $A$  is a constant and is positive;  $E_a$  is the activation energy, which is an inherent property of the product material;  $k$  is the Boltzmann constant;  $T$  is the thermodynamic temperature.

For ease of calculation, the following transformation is made to equation (1):

$$\varepsilon = e^{a+b \cdot \varphi(T)} \tag{2}$$

### 3.3. Weibull distribution parameter estimation

Based on the degradation trajectory model, the time points at which each sample reaches the failure threshold under various stress levels can be calculated and defined as the pseudo-failure life. By estimating the parameters of the pseudo-failure life distribution of the samples and analyzing them, an overall reliability assessment can be obtained.

Assuming that the pseudo-life of the sample follows a Weibull distribution, the probability density function of the random variable  $t_j$  can be expressed as:

$$f(t_j | Z_j) = \frac{\theta_j}{\eta_j} \left(\frac{t_j}{\eta_j}\right)^{\theta_j-1} \exp\left[-\left(\frac{t_j}{\eta_j}\right)^{\theta_j}\right] \tag{3}$$

In the equation, the parameters  $\theta_j$  and  $\eta_j$  represent the shape and scale parameters of the corresponding Weibull distribution under the acceleration stress  $S_j$ . After calculating the pseudo-life  $t_{ij}$  of the  $i$ th sample under the  $S_j$  stress based on the failure threshold, the maximum likelihood function can be expressed as:

$$\begin{cases} \eta_i^{m_i} = \frac{1}{r} \sum_{j=1}^n t_{ij}^{m_i} \\ \frac{\sum_{j=1}^n t_{ij}^{m_i} \ln t_{ij}}{\sum_{j=1}^n t_{ij}^{m_i}} - \frac{1}{m_i} - \frac{1}{r} \sum_{j=1}^n \ln t_{ij} = 0 \end{cases} \tag{4}$$

Using the maximum likelihood estimation method based on equation (4), the parameter estimation values of the shape parameter  $\theta_j$  and the scale parameter  $\eta_j$  can be obtained. After estimating the parameters of the Weibull distribution, the reliability function of the Weibull distribution can be obtained by combining its distribution function.

$$R(t) = \exp\left[-\left(\frac{t}{\eta}\right)^\theta\right] \tag{5}$$

## 4. Instance Verification

### 4.1. Modeling the degradation process of Hall current sensors

When conducting reliability analysis and lifetime prediction of sensors based on accelerated degradation testing, the first step is to determine the degradation trajectory equation of the current sensor based on the degradation test data. Combined with the failure threshold, the pseudo-failure life of the current sensor can be calculated. Then, a hypothesis test is performed on the distribution of the pseudo-failure life, and the model parameters are estimated. By combining the accelerated degradation equation and the estimated parameters, the degradation model under normal stress is obtained. The reliability of the model is then evaluated.

### 4.2. Experimental protocol design

In this study, an accelerated degradation test was conducted on the sensors using a high and low-temperature test chamber. The appropriate acceleration stress needs to be determined first to achieve a

good acceleration effect in the test. The minimum stress level of the test should be slightly higher than the stress level under normal operating conditions to accelerate the process of product failure or performance degradation. The maximum stress of the test should be the maximum load that can be applied without changing the failure mechanism of the Hall current sensor. Based on the product's operating principles and environment, 323K, 343K, and 363K were selected as the acceleration stresses for the constant stress accelerated degradation test of the sensor. Eight samples were selected for the test under each stress level, and the test lasted for 360 hours with measurements taken every 24 hours, for a total of 15 times. The performance degradation trajectories of the product under acceleration stress are shown in Figures 1-3. The normal operating temperature of the sensor was determined to be 298K based on the product's instructions, and the product was deemed to have failed when the zero output voltage dropped by 10mV.

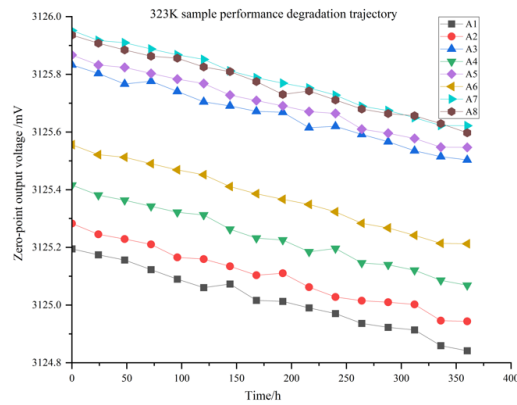


Figure 1: Sample performance degradation trajectory under 323K stress.

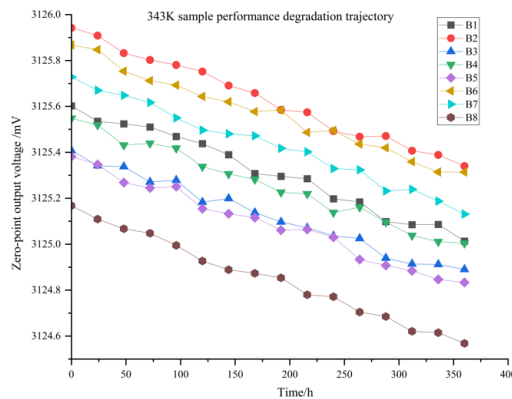


Figure 2: Sample performance degradation trajectory under 343K stress.

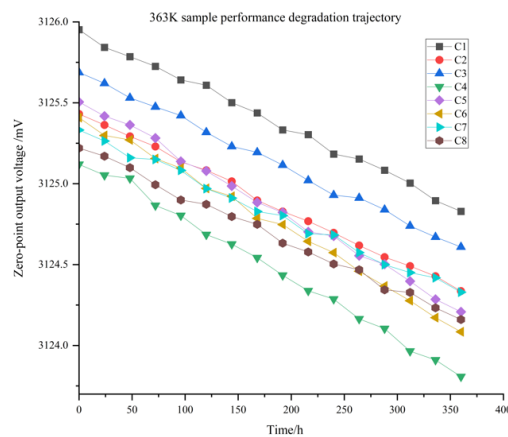


Figure 3: Sample performance degradation trajectory under 363K stress.

#### 4.3. Degradation model selection

From the degradation trajectory plots of the zero output voltage of the Hall current sensor under different temperature stresses, it can be observed that the actual degradation trajectory of the sensor's performance parameter is approximately a straight line. Therefore, a linear model was selected as the degradation model for the sensor's performance. In this study, the least squares method was used to estimate the unknown parameters in the linear degradation model. As shown in Table 1, the linear degradation trajectory of the sensor can be expressed as:

$$y_{ij} = \alpha_i t + \beta_i \quad (6)$$

In the equation,  $y(t_{ij})$  represents the degradation data of the  $i$ th sample at time  $t_{ij}$ .  $\alpha_i$  and  $\beta_i$  are the degradation model parameters for the  $i$ th sample, which can be estimated using the least squares method:

$$\begin{cases} \hat{\alpha}_i = \frac{\sum_{j=1}^{15} (t_j - \bar{t})(y_{ij} - \bar{y}_{ij})}{\sum_{j=1}^{15} (t_j - \bar{t})^2} \\ \hat{\beta}_i = \bar{y}_{ij} - \hat{\alpha}_i \bar{t} \end{cases} \quad (7)$$

In the equation,  $\hat{\alpha}_i$  and  $\hat{\beta}_i$  are the least squares estimates of the parameters  $\alpha_i$  and  $\beta_i$ , respectively.  $\bar{t}$  and  $\bar{y}_{ij}$  are the mean total degradation time and mean degradation amount, respectively. The degradation trajectory model for each sample can be obtained as:

$$y_{ij} = \hat{\alpha}_i t + \hat{\beta}_i \quad (8)$$

According to the definition of degradation failure, when the amount of product performance degradation over time reaches the failure threshold  $D$ , the product fails, and the corresponding time is the pseudo-failure life of the product. The pseudo-failure life of each product can be obtained:

$$T_i = \frac{D - \hat{\beta}_i}{\hat{\alpha}_i} \quad (9)$$

Taking eight samples of A1-A8 under 323K stress for accelerated testing as an example, the pseudo-failure life of the samples can be obtained from the above equation as shown in Table 2.

*Table 2: Sample pseudo-failure life at 323K (50°C).*

Sample	Pseudo Failure Life $T_i(h)$
Sample A1	10399.83
Sample A2	11009.8
Sample A3	11006.91
Sample A4	10494.86
Sample A5	10851.57
Sample A6	10040.43
Sample A7	10476.1
Sample A8	10845.36

#### 4.4. Pseudo-failure life hypothesis testing and parameter estimation

The pseudo-failure life of degraded products generally follows a normal or Weibull distribution. Suppose the pseudo-failure life distribution of components is Weibull distribution. The Weibull distribution was examined by Quantile-Quantile Plot (Q-Q Plot) on the data of 8 samples at 323K (50°C), and the results are shown in Figure 4.

In the Q-Q Plot, it is reasonable to assume that the samples obey the Weibull distribution if all the sample points are near a straight line. As can be seen from Figure 4, all the sample points are near a straight line, so it is reasonable to assume that the sensor pseudo-failure life obeys the Weibull distribution.

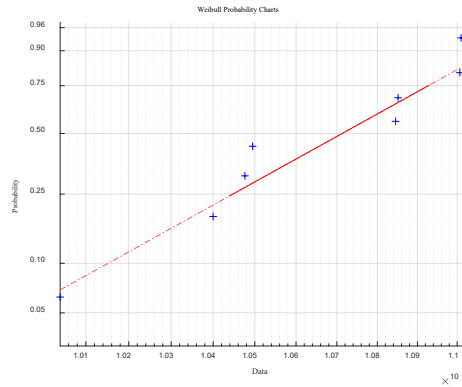


Figure 4: Sample pseudo-failure life Weibull distribution probability diagram.

After conducting a hypothesis test on the pseudo-failure life, it is necessary to estimate the parameters of the Weibull distribution for the pseudo-failure life of each sample under different stress levels. According to the equation, the shape and scale parameter estimates for the Weibull distribution for the pseudo-failure life of each sample under different stress levels are shown in Table 3.

Table 3: Estimates of pseudo-failure life distribution parameters.

Temperature stress	Shape parameters $\theta$	Scale Parameters $\eta$
323K	41.6168	10789.4331
343K	24.6618	6466.2575
363K	14.2725	3323.0211

#### 4.5. Reliability assessment and remaining life prediction

Based on the analysis of the failure mechanism of the Hall current sensor under temperature stress, it can be concluded that the Arrhenius model is the most suitable accelerated degradation model for the sensor. By substituting the shape and scale parameter estimates for the Weibull distribution of the pseudo-failure life of each sample under different stress levels into equation (2) and using nonlinear fitting, the relationship between the model parameters and temperature stress can be obtained. The expression for this relationship is shown in Table 4.

Table 4: Weibull distribution and temperature stress expressions.

Weibull distribution parameters	Relationship between parameters and temperature
Shape parameters $\theta$	$\theta = e^{55.73-9.074 \times \ln T}$
Scale Parameters $\eta$	$\eta = e^{64.04-9.4754 \times \ln T}$

By substituting the normal operating temperature of the current sensor (298K) into the relationship between the overall parameters and stress levels in Table 4, the degradation data of the sensor under normal operating temperature follows a Weibull distribution with a shape parameter of  $\theta=56.519$  and a scale parameter of  $\eta=23336.183$ . By substituting these parameters into equation (5), the reliability function of the current sensor under normal temperature stress can be obtained:

$$R(t) = \exp \left[ - \left( \frac{t}{23336.183} \right)^{56.519} \right] \quad (10)$$

Based on the parameter estimation results of the pseudo-failure life of the samples under different stress levels, and with the help of the reliability function, the reliability curves of the Hall current sensor under different temperature levels can be obtained, as shown in Figure 5.

From Figure 5, it can be seen that the reliability of the sensor product under 298K stress significantly decreases when the test time reaches 20,000 hours. According to the usage specifications of the Hall current sensor, when the reliability decreases to 0.5, the sensor cannot perform current detection work normally, and the probability of sensitivity reduction greatly increases. Combining the reliability curve in Figure 5, the time corresponding to a reliability of 0.5 can be taken as the remaining life of the sensor sample at the initial time. Thus, the test time when the reliability of the Hall current sensor under 298K stress decreases to 0.5 can be obtained as 23,064 hours.

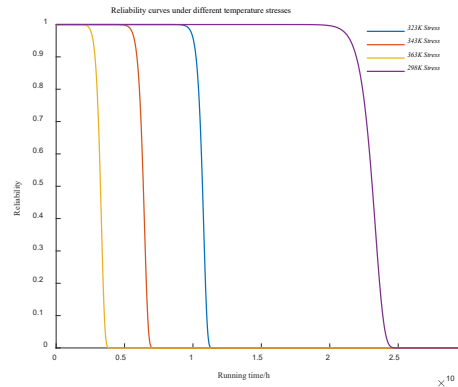


Figure 5: Reliability curves under different temperature stresses.

## 5. Conclusions

This article analyzes the working principle and failure mechanism of the Hall current sensor, considering the characteristics of sensor performance degradation. A linear model is used to establish a degradation trajectory model for the sensor zero output voltage. Based on the Weibull distribution, the parameters of the pseudo-failure life under different temperature levels are estimated, and the reliability function and reliability curve under normal stress are obtained by the accelerated degradation equation. The article predicts the product's lifespan. By fitting the failure data using the established model, the results are consistent with the failure statistics provided by the company, which further proves the correctness of the proposed method for predicting the lifespan of the Hall current sensor using the Weibull distribution.

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