

Study on contagion of debt default risk of the secured party in the guarantee chain—From the perspective of small and medium-sized enterprises

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Abstract: In the process of financing, small and medium-sized enterprises obtain funds through mutual guarantee, joint guarantee and other forms to form a guarantee chain. On the guarantee chain, the debt default risk caused by the guaranteed enterprise failing to pay off the debt according to the agreed time will infect the guarantee enterprise and affect the operation of the guarantee chain capital. This paper takes the guaranteed enterprise as the research object, and based on the characteristics of the guarantee chain business process, defines the asset-liability ratio of the guaranteed enterprise, the degree of information disclosure, the profitability and the guarantee rate of the guaranteed enterprise's net assets and other influencing factors, and uses the hidden Markov with the characteristics of state transfer to construct the default risk contagion model of the guaranteed chain debt. To measure the influence degree of debt default risk contagion of the guaranteed enterprise. It is found that the asset-liability ratio and the guarantee ratio of net assets are positively correlated with the contagion intensity of the debt default risk of the guaranteed enterprise, while the degree of information disclosure and profitability are negatively correlated with the contagion intensity of the debt default risk of the guaranteed enterprise.

Keywords: Guarantee chain, Debt default risk, Small and medium-sized enterprises, Risk contagion, Hidden Markov model

1. Introduction

The Bank of China Research Institute pointed out in the 2021 Economic and Financial Outlook Report that debt default risk has gradually become the main risk that China's capital market needs to face^[1]. The risk of corporate debt default is likely to spread to the financial system, affecting the stability of the financial system and economic operation. According to Wind data, the outstanding RMB loans of financial institutions reached 213.99 trillion yuan by the end of 2022, up 11.1 percent year on year; For the whole year, yuan loans increased by 21.31 trillion yuan, an increase of 1.36 trillion yuan year-on-year, and GDP grew by only 3 percent. The possibility of corporate debt defaults has risen sharply. Moreover, among the companies that have defaulted on their debt, the proportion of defaults by small and medium-sized enterprises has increased year by year. Small and medium-sized enterprises are the main force of China's economic operation and development, but for a long time, financing difficulties and high financing have been restricting their development. Under the influence of the domestic macroeconomic downturn and the epidemic, SMEs are faced with more operational problems, financing constraints and debt repayment problems, which makes the debt default risk of SMEs higher, which may trigger a chain reaction of guarantee, thus affecting the interests of other enterprises and individuals.

Guarantee can be used not only as a means to prevent the default of the guaranteed party, but also as a way to promote the flow of funds to a large extent. As the guarantee relationship becomes more and more complex, the mutual guarantee and joint guarantee of borrowers will form corresponding guarantee chains or even guarantee chain networks. If the guaranteed party defaults in the debts of small and medium-sized enterprises, a large number of mutual guarantee and joint guarantee relationships make the default risk of the guaranteed party spread rapidly along the guarantee chain, forming a chain reaction, resulting in the accumulation of risks in the guarantee chain. Therefore, studying the contagion of default risk of the secured party in the guarantee chain of SMEs can effectively reduce the default probability of the secured party in the guarantee chain, and is conducive to preventing and resolving systemic financial risks.

Combing through the existing literature, it is found that in the research of SMEs' guarantee chain debt risk, there are few studies on default risk contagion of the secured party in the academic circle. Based on this, this paper introduces the hidden Markov process to construct the debt default risk contagion model of the secured party in the guarantee chain, and analyzes the influence of influencing factors such as the asset-liability ratio of the secured party, the degree of information disclosure, the profitability and the guarantee ratio of the secured party's net assets on the contagion intensity of the debt default risk of the secured party.

2. Literature review

(1) Research on risk contagion of guarantee chain

With the increasing influence of guarantee chain risk, many domestic scholars have studied the risk of guarantee chain from different angles. Ma Li et al. studied the causes of SME guarantee chain risk^[2]. The root causes of SME guarantee chain risk include financial institution risk and enterprise risk, and put forward corresponding prevention and control measures and suggestions; Zhang Zexu et al. analyzed the transmission process of guarantee chain crisis and found that the critical conditions for the occurrence and cessation of the transmission of guarantee chain crisis were related to the asset-liability ratio of enterprises and the amount of external guarantee of enterprises^[3]; Zhang Lecai et al. analyzed the risk mechanism of guarantee chain and proposed that a guarantee chain crisis will occur when the liquidity impact of an enterprise exceeds the buffer capacity of the enterprise itself and the guarantee enterprise^[4].

(2) Related research on debt default risk

The risk of debt default refers to the uncertainty that an enterprise does not pay off its debts according to the agreed time. Some scholars have analyzed the causes of debt default risk. Tan Benyan et al. found that the factors affecting the debt default risk include the financial status of enterprises, the ownership structure of enterprises and corporate governance^[5]; Wang Ning's research found that the default risk will decrease with the increase of the enterprise's solvency, profitability and operation capacity^[6]; The contagion based on related guarantee spreads the debt risk mainly through the enterprise guarantee chain. Fisman et al. elaborated the contagion path of debt default risk based on guarantee relationship^[7]. Li Mingming found that guarantee relationship and equity relationship became the transmission path of debt default risk infection^[8]. Leng et al. pointed out that the joint responsibility loan mechanism effectively increased the guarantee amount of debtors and led to risk contagion among enterprises^[9].

(3) Research methods of default risk

In recent years, the phenomenon of debt defaults has begun to appear frequently, and the government has issued several documents to remind the market to prevent debt default risks. Debt default risk has become a research focus at present. Xu Kai et al. built a mathematical model of infectious disease in a complex network to explore the internal mechanism of contagion of debt default risk caused by the response of individual members within an enterprise group^[10]. Xie Xiaofeng et al. used the double Stackelberg game to study the contagion path of debt default risk in the context of linked repo guarantee^[11]. Geng et al. use data mining technology to build a neural network model to predict corporate debt default risk^[12]. Li Zhanlei et al. build a default risk contagion model of mutual guarantee financing based on the unsteady heat conduction principle of Fourier Law, and analyze the influencing factors and degree of borrowers' associated credit risk contagion through numerical simulation^[13]. Walters N et al. studied the influencing factors of risk contagion in the banking system based on the dynamic network model and found that a lower initial ratio of net assets would lead to more serious default risk contagion^[14]. Based on KMV model and PageRank algorithm, Liu Zhiyang simulated and analyzed the inter-bank contagion risk from the perspective of inter-bank debt default^[15].

After searching the existing literature, it is found that the literature on SME debt default risk based on the guarantee chain at home and abroad mainly focuses on risk identification, risk early warning, risk assessment, investigation of influencing factors and risk sources, and most of them use KMV model to evaluate and forecast the debt default risk. The contributions of this paper are as follows: First, based on the guarantee model, the contagion of debt default risk is studied to make up for the shortcomings of existing research perspectives. Secondly, the hidden Markov process is introduced to construct the debt default risk contagion model of the secured party in the guarantee chain, and the influencing factors and degree of debt default risk contagion caused by the default of the secured party are explored.

3. Model construction

3.1 Using Hidden Markov model to study the rationality of debt default risk contagion

The Hidden Markov model (HMM) is a kind of Markov chain, including the following characteristics: (1) the state at time t_n is only related to the state at time t_{n-1} , and has nothing to do with the state before time t_{n-1} ; (2) hidden states S , which are usually not directly observable; And (3) observable states O , which can usually be directly observed.

The contagion of debt default risk of the secured party of the guarantee chain contains the following characteristics^[22]: (1) In the contagion process of debt default risk of the secured party of the guarantee chain, the state of time t_n is only related to the state of the previous time t_{n-1} ; (2) the contagion intensity of the debt default risk of the secured party of the guarantee chain cannot be directly observed; (3) The influencing factors such as asset-liability ratio, profitability, information disclosure degree and net asset guarantee ratio can be directly observed, and the risk contagion intensity of the secured party is related to these factors. Therefore, Hidden Markov applies to the study of risk contagion of debt default of the secured party in the guarantee chain.

3.2 Debt default risk contagion model

With reference to Wu Yanwen and Yan Tao et al. 's studies on the guarantee risk of SMEs and risk contagion in the guarantee circle, it is defined that the asset-liability ratio, profitability, information disclosure degree and the guarantee ratio of the guarantor's net assets of the secured party not only affect the development of SMEs, but also affect the debt default risk contagion of the secured party in the guarantee chain to a certain extent^{[16][17]}.

Hidden Markov model is usually expressed as $\lambda = (S, O, \pi, A, B)$: hidden state S , observable state O , initial state probability matrix π , hidden state transition probability matrix A , observation state transition probability matrix B .

The HMM model is formed by the transfer process between the hidden state (risk contagion intensity state) and the observable state (risk contagion influencing factors) in the research on the debt default risk contagion of the secured party of the SME guarantee chain, as shown in Figure 1.

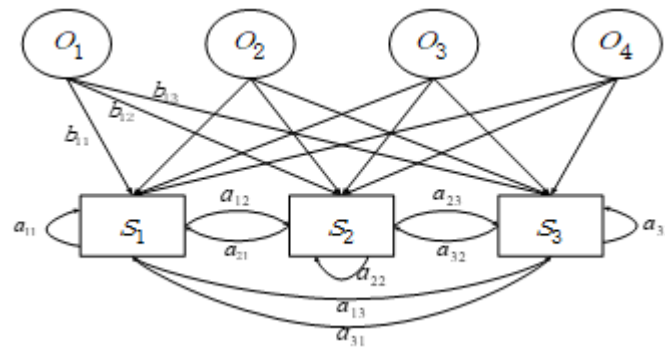


Figure 1: HMM model of debt default risk contagion problem

In the HMM model,

1) $S = \{s_1, s_2, s_3\}$ is the set of risk contagion intensity states. Because of the dynamic nature of risk infection, it is used to represent the state of risk infection intensity of $\{X_t | X_t \in S\}$ SMEs at time t . Among them, s_1 represents small and medium-sized enterprises in a low risk state, s_2 represents small and medium-sized enterprises in a medium risk state, and s_3 represents small and medium-sized enterprises in a high risk state.

2) $O = \{o_1, o_2, o_3, o_4\}$ is a collection of single risk contagion affecting factor states. Where, with the increase of the subscript, the status value of the risk infection influencing factor increases.

3) $\pi = [\pi_1, \pi_2, \pi_3]$ is the initial state matrix of small and medium-sized enterprises. Where, $\pi_1 = P(X_0 = s_i)$, $1 \leq i \leq 3$ represents the probability that SMEs are in the state s_i of risk contagion intensity at time 0.

4) $A = [a_{ij}]$ is the risk state transition matrix. Where, $a_{ij} = P(X_{t+1} = s_j | X_t = s_i)$, $1 \leq i, j \leq$

3 indicates that the state of risk infection intensity of SMEs at time t is s_i , and the probability of transformation into s_j at time t+1.

5) $B = [b_j(k)]$ is the observed state probability matrix. Where, represents the probability that the state of risk infection influencing factors is o_k when $b_j(k) = P(o_k|s_j)$, $1 \leq k \leq 4, 1 \leq j \leq 3$ SMEs are in the state s_j .

6) $Q = \{q_1, q_2, q_3, q_4\}$ is the set of risk contagion influencing factors, respectively representing asset-liability ratio, profitability, information disclosure degree and net asset guarantee ratio.

In the model $\lambda = (S, O, \pi, A, B)$, according to the forward algorithm, the forward probability is defined as the observation sequence up to time t is $Q = \{q_1, q_2, \dots, q_t\}$, and the hidden state at time t is the i among all the hidden states (denoted as), then the forward probability can be denoted as s_i .

$$\alpha_t(i) = P(q_1, q_2, \dots, q_t, x_t = s_i | \lambda) \tag{1}$$

Where, the observation sequence $Q = \{q_1, q_2, \dots, q_t\}$ represents the observation probability when the state of risk infection intensity at time t is s_i , and the state of risk infection influencing factor is o_k .

The initial state is:

$$\alpha_1(i) = \pi_i b_i(q_1) \quad i = 1, \dots, m \tag{2}$$

Where, m represents the number of hidden states, π_i represents the initial probability of each hidden state, and $b_i(q_1)$ represents the probability of the i-th hidden state generating the observed state o_1 . Then the formula for recursion is:

$$\alpha_{t+1}(j) = (\sum_{i=1}^m \alpha_t(i) a_{ij}) b_j(q_{t+1}) \quad i = 1, \dots, m \tag{3}$$

Where, a_{ij} represents the state transition probability from implied state i to implied state j.

$P_t(i)$ is the probability that the debt default risk intensity of the secured party is in the risk state s_i at time t:

$$P_t(i) = \frac{\alpha_t(i)}{\sum_{i=1}^m \alpha_t(i)} \tag{4}$$

R is the contagion intensity value of the debt default risk of the secured party at time t:

$$R = \sum_{i=1}^m s_i P_t(i) \tag{5}$$

According to the above formula, the risk contagion intensity at time t is positively correlated with the probability of being in a state s_3 at time t. Therefore, the influence of influencing factors q_i at time t on the probability of being in state s_3 at time t is the influence of influencing factors on the intensity of risk infection.

4. Numerical simulation

This paper uses Python for numerical simulation analysis. The HMM model parameters are randomly assigned. The specific parameters are as follows:

$$\pi = [0.5247 \quad 0.3139 \quad 0.1614]$$

$$A = \begin{bmatrix} 0.5338 & 0.3175 & 0.1487 \\ 0.1789 & 0.4632 & 0.3579 \\ 0.0937 & 0.2721 & 0.6342 \end{bmatrix}$$

When the influencing factor is the net asset guarantee ratio:

$$B = \begin{bmatrix} 0.6531 & 0.2137 & 0.0843 & 0.0489 \\ 0.1387 & 0.3468 & 0.3462 & 0.1683 \\ 0.0279 & 0.1986 & 0.2053 & 0.5682 \end{bmatrix}$$

When the influencing factor is asset-liability ratio:

$$B = \begin{bmatrix} 0.7647 & 0.1347 & 0.0794 & 0.0212 \\ 0.1457 & 0.4672 & 0.3127 & 0.0744 \\ 0.0209 & 0.1185 & 0.3019 & 0.5587 \end{bmatrix}$$

When the influencing factor is profitability:

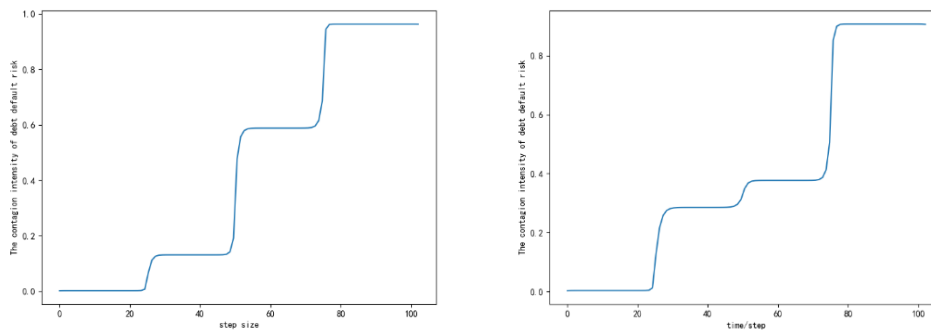
$$B = \begin{bmatrix} 0.0209 & 0.0785 & 0.2419 & 0.6587 \\ 0.1457 & 0.3127 & 0.4672 & 0.0744 \\ 0.7647 & 0.1347 & 0.0794 & 0.0212 \end{bmatrix}$$

When the influencing factor is the degree of information disclosure:

$$B = \begin{bmatrix} 0.1719 & 0.1567 & 0.1387 & 0.5327 \\ 0.2163 & 0.2351 & 0.2871 & 0.2615 \\ 0.7535 & 0.1453 & 0.0803 & 0.0209 \end{bmatrix}$$

If the step size is 100, the occurrence frequency of each risk contagion influencing factor increases with the step size, and the change trend of the probability in the state s_3 is obtained, that is, the change trend of the contagion intensity of the debt default risk of the secured party^[18]. As shown in Figure 2 and Figure 3:

As can be seen from Figures 2 and 3, net asset guarantee ratio and asset-liability ratio are positively correlated with the contagion intensity of debt default risk of chain of guarantee, while profitability and information disclosure degree are negatively correlated with the contagion intensity of debt default risk of chain of guarantee.



(a) Influence of net asset guarantee ratio on contagion intensity of debt default risk
 (b) Influence of asset-liability ratio on contagion intensity of debt default risk

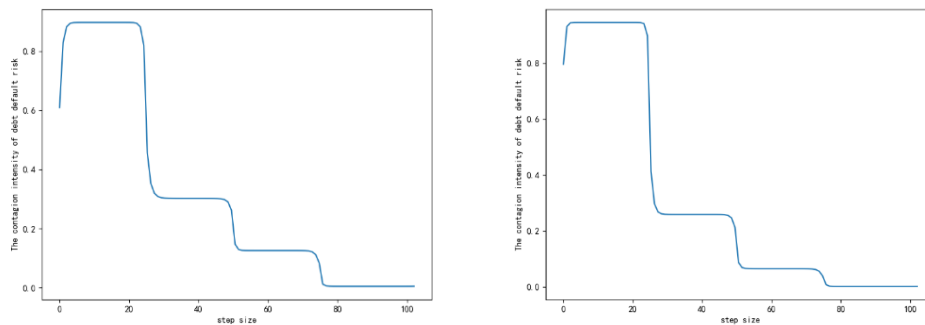
Figure 2: Influence of contagion intensity of debt default risk of the secured party

(1) Guarantee rate of net assets

If the secured party fails to repay the debt on time, as the guarantor, the higher the proportion of the guarantee amount to the net assets of the SMEs, the smaller the ability to repay the debt, and the greater the default risk. Although some small and medium-sized enterprises provide a small amount of guarantee, but their own strength is weak or insufficient funds, will affect the normal operation of the enterprise. When the amount of guarantee exceeds the net assets, if the guaranteed party defaults, the guaranteed party may not be able to undertake the joint and several guarantee liability, thus increasing the contagion intensity of debt default risk. Therefore, in order to reduce the contagion intensity of debt default risk, the guarantor should guarantee within the range of its net assets.

(2) Asset-liability ratio

Risk contagion occurs when the asset-liability ratio is greater than 1- the preferred payment amount of the estate/the market value of the total assets. The higher the asset-liability ratio, the greater the risk contagion intensity of chain of guarantee debt default. As the secured party is the main body of the default, after the secured party in the guarantee chain of SMEs goes bankrupt, the risk will be transmitted to the enterprise guaranteed by it along the guarantee chain. If the enterprise guaranteed by it cannot resist the risk, the risk will continue to be transmitted according to the previous step.



(c) Influence of profitability on contagion intensity of debt default risk
 (d) Influence of information disclosure degree on contagion intensity of debt default risk

Figure 3: Influence of contagion intensity of debt default risk of the secured party

(3) Profitability

In guarantee business, the profitability of the secured party is one of the main factors affecting the contagion intensity of default risk. The stronger the profitability of the secured party, the higher the operation level of the corresponding enterprise, the larger the income and expenditure surplus, the stronger its solvency, and the smaller the contagion intensity of the debt default risk of the secured party. Improving the profitability of enterprises can restrain the increase of default risk intensity, which is mainly reflected in the improvement of internal financing ability and default risk prevention ability in the guarantee mode.

(4) Information disclosure degree

The higher the degree of information disclosure of the insured party, that is, the higher the degree of information disclosure of the insured party's financial status, related party matters, major investment and major debt, the lower the intensity of the risk of default of the insured party's infected debt. When making a guarantee, the guarantor usually needs to fully understand the information of the secured party. If the secured party discloses its own information to the guarantor to a greater extent, it is subject to stricter constraints in information disclosure, which is conducive to improving the risk judgment ability of the guarantor and reducing the contagion intensity of debt default risk.

5. Conclusions and Suggestions

The results of this paper show that profitability and the degree of information disclosure are negatively correlated with the debt risk contagion of SMEs by the guarantor, that is, the stronger the profitability of SMEs, the higher the degree of information disclosure, the lower the debt default risk contagion; The guarantee ratio of net assets and the asset-liability ratio are positively correlated with the contagion intensity of the debt default risk of the secured party. The risk contagion can be reduced from cooperation and information sharing, guarantee management mechanism and risk contagion blocking mechanism.

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