

Rube-Goldberg Machine Report

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ABSTRACT: *This virtual Rube- Goldberg machine consists of 7 different physics principles in the duration of about 30 seconds which includes elastic and inelastic collision ,energy and momentum conservation , circular motion, torque and projectile motion in 2 dimensions. The biggest challenge to construct this Rube Goldberg Machine is to elaborately use different indexes and functions in Blender (an app used to make physics model) to support my ideal machine. During this process, I not only developed a deep understanding of many physics concepts, but also learnt IT techniques. Finally, my machine managed to complete seven consecutive steps and achieve the aim to crash the egg. (although hard to operate the last step virtually)*

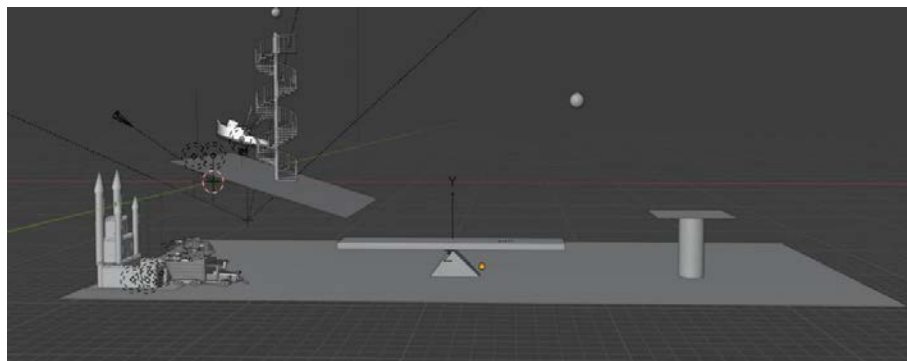
KEYWORDS : *Rube - Goldberg Machine, circular motion , energy conservation , momentum conservation , torque , moment of inertia , two - dimension projectile , angular velocity , elastic collision , inelastic collision.*

Introduction:

My task is to create an interesting machines including many physics mechanics. As all the steps are being operated virtually, it is a mission for us to keep this machine as real as possible (under conditions reality like gravity and energy loss as heat). Another task comes from the machine itself , that all external forces are forbidden, the first movement of the ball exerts knock—on effect to support the whole system. This requires continuous steps with high accuracy like domino. Finally, I keep in mind that my Rube Goldberg machine will be used for analysis and as a clear demonstration of interconnected physics principles , therefore, I need to simplify my final outcome and make it easier to trace.

Methods :

At first, I brainstormed many ideas for each step . I searched on the Internet and learn about this software Blender by many tutorials. Then, I built my initial



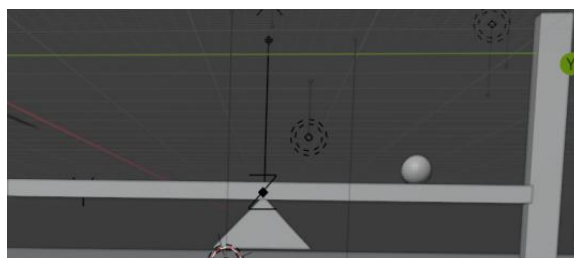
design of ball 1 falling through a spiral staircases, which then drops onto a seesaw to trigger the movement of ball 2, which originally rest on the seesaw. In this case, energy is transferred to the second ball. As it arrived on the platform, the egg would be cracked by this external force. However, as I went through the checklist and was in preparation for rendering animation, two crucial problems appear. There isn't enough steps. And the seesaw is really unstable so that routine of the second ball is unpredictable. Sometimes it's impossible for that ball landing on the platform.

So, I adjusted my plan and add more movements consisting physics principles. By putting a rectangular block beside the seesaw, stability is achieved. Under this circumstance, the second ball has to reach the same side as the first ball after this torque effect.

Then, I changed the bounciness (coefficient of restitution) of the second ball to enable more elastic collision. Following stages like the conservation of momentum was also added.

After that, considering the aim of this Rube Goldberg machine: creative, imaginative and interesting. I took advantage of this software to colour these models, using different materials and realistic climb-wall effect.

After rendering this whole animation with 1000 key frames, I put this video in Tracker to track the movement of these two balls. When analyzing each component of this machine, I need to roughly test the reliability for those balls to equip enough energy to crash the egg by an inelastic collision at last. Tracker is used to record all

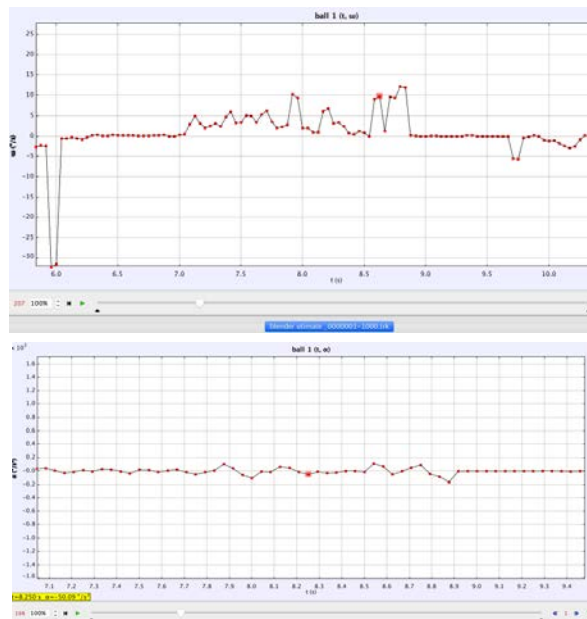
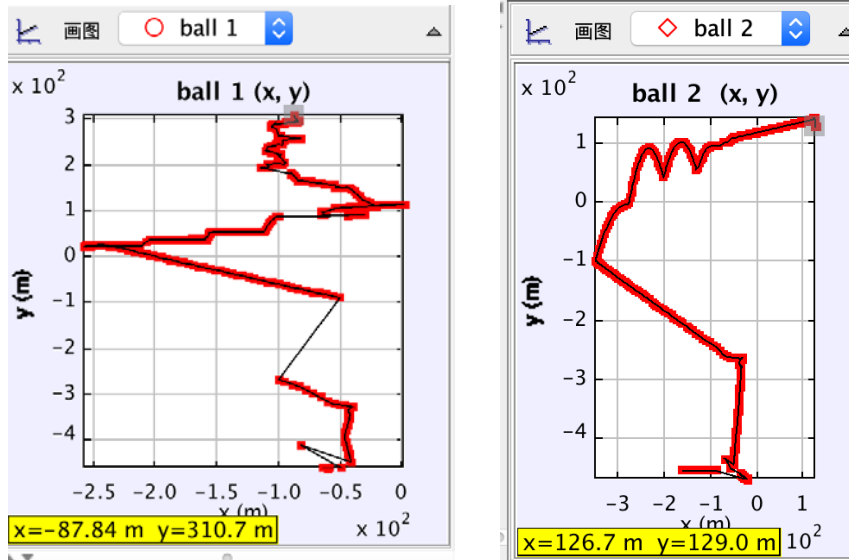


the physics indexes like position and velocity that will help our calculations when analyzing different movements.

Results

Detailed Description	Physics principle
Circular motion	<p>When the green ball is moving around a spiral staircases . Circular motion is achieved. However, in my model , the route green ball follows is not perfectly circular.</p> <p>In contrast , for every single step the ball drops, its route is limited and modified by the barrier ,which forms circular motion.</p>
Torque	<p>Torque effect happens on the seesaw, when the green ball hits one end , it exerts a force that causes the seesaw to rotate about z axis , some parts of energy is transferred to the second ball which fosters its motion</p>
Elastic collision	<p>Elastic collision is where the blue ball hits two blocks above the cylinder and bounces back. Momentum is conserved , when the blue ball changes direction (from dropping to upwards motion) ,its Kinetic Energy is nearly conserved during the collision period , which enables an elastic collision. However, this is not perfect elastic collision which means little energy is still dissipated as heat</p>

<p>Energy conservation</p>	<p>When the first green ball and the second blue ball are sliding off the pink-colored slope, it is a demonstration where gravitational potential energy is transferred to kinetic energy and thermal energy within a certain time period , total energy in the system is conserved. A part from this example, energy conservation is everywhere in the model</p>
<p>Two— dimension motion</p>	<p>Two—dimensional motion merges within the entire machine. I choose a projectile motion when the blue ball falls from the yellow block onto the slop . During this period ,its horizontal velocity is fairly constant, whilst the blue ball accelerates in its vertical direction which forms a 2-dimensional projectile motion.</p>
<p>Inelastic collision</p>	<p>For inelastic collision , one direct example is when later arrived green ball collides head on with the blue ball. In this collision , momentum is conserved , however , much energy is lost to break the egg and produce heat . As a result, total energy is not conserved.</p>
<p>Momentum conservation</p>	<p>In the same scenario as last physics quantity, but to put the focus on two balls rathe than that crashed egg . After the collision , two balls move in opposite directions ,but when analyzing their motions, I find that their horizontal velocity components canceled out, their vertical momentum components after equals that before. Thus momentum is conserved.</p>



Ball 1 motion path

Ball 2 motion path

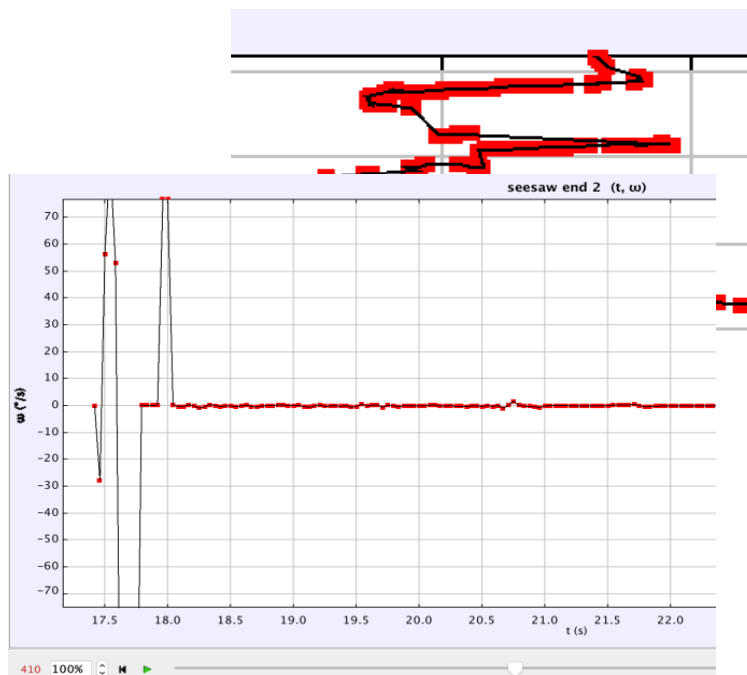
Data Analysis in Tracker

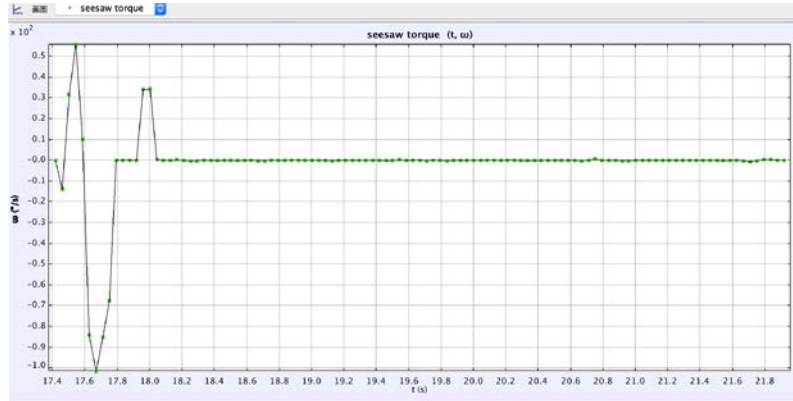
I need to declare that I didn't set axis of the system in Tracker properly so that I adjust it to a smaller magnitude in my later calculation ,so some data is adjusted .

Circular Motion (spiral staircases)

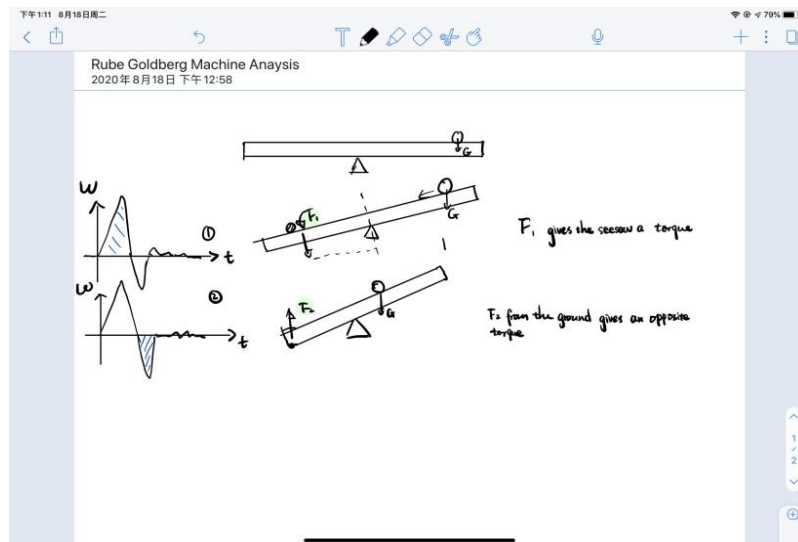


Overview motion path





During this period , the ball has experienced circular motion when falling with the path of this spiral staircases.



The ball finishes a circle orbit from 6:00 to 8:80

Angular velocity vs.time

Angular acceleration vs.time From 6:00s to 8:80s

From 6:00s to 8:80s

Analysis of Results

As we can see from the tracker graphs , the initial angular speed of green ball (on top of staircases) is 0. As the ball dropping along staircases , its angular velocity increases gradually with fluctuations. Due to many obstructions, I am not able to track every detail of this dropping ball, it reaches the maximum angular velocity, the ball completes one rotation in 2.79s (from 6.00s to 8.79s) within many frames.

There may be some inaccuracy in data because the motion is more complex than simple circular motion.

Angular velocity $\omega = \Delta\theta / \Delta t$ so $\omega = 2\pi / 2.79 = 2.25 \text{ rad/s}$ Angular acceleration $a = \Delta\omega / \Delta t$ so $a = 2.25 / 2.79 = 0.8 \text{ rad/s}^2$

Diameter of the spiral staircases is change in x indexes from two ends, which is 32.4m, thus radius is 16.2m

Centripetal acceleration $a(c) = r * \omega^2$ so that $a(c) = 16.2 * (2.25)^2 = 82.1 \text{ ms}^{-2}$

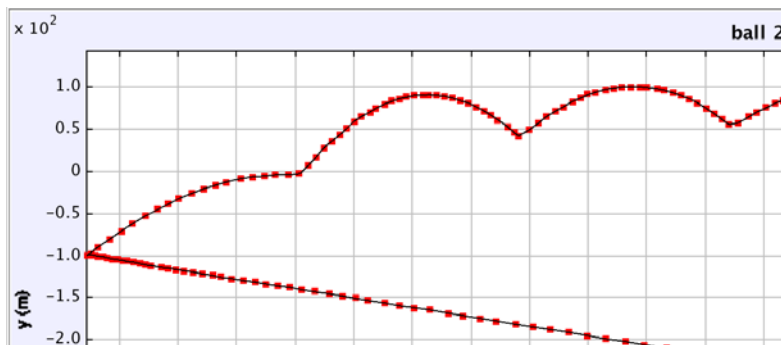
Torque (Seesaw)

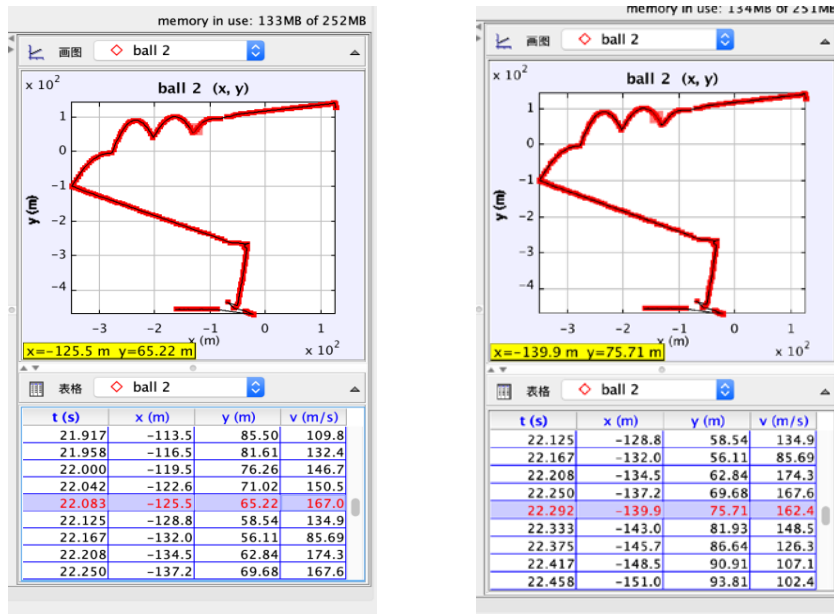
Right end of the seesaw left end of the seesaw

Total angular velocity of the seesaw (impact around 17.4s-17.8s)

Analysis of Results :

The movement of this seesaw in my machine is an anti clockwise rotation around its z axis until it touches the floor. As soon as it touches the floor , the ground provides it with an opposite force (clockwise movement) to make it stop at the new equilibrium point. This





why the seesaw experiences a positive angular velocity first and then a negative one.

Mass of the seesaw board is 10 kg . Length of the seesaw is 1.619m (length in trader is too big : 161m so I adjust them to make it realistic)

Moment of Inertia (Seesaw) , which is the amount of torque needed for a certain angular acceleration .

$$\text{Seesaw Center : } I_1 = 1/12 M * L^2 = 1/12 * 10 * (1.619)^2 = 2.1843 \text{ kgm}^2$$

$$\text{Left End of Seesaw (Treated as a pointed mass) } = I_2 = M * (0.5L)^2 = 1 * (0.81)^2 = 0.6553 \text{ kg m}^2$$

$$\text{Right. End of seesaw } = I_3 = M * (0.5L)^2 = 0.01 * (0.81)^2 = 0.00656 \text{ kg m}^2$$

$$\text{Total moment of inertia is } I = I_1 + I_2 + I_3 = 2.1843 + 0.6553 + 0.00656 = 2.8461 \text{ kg m}^2$$

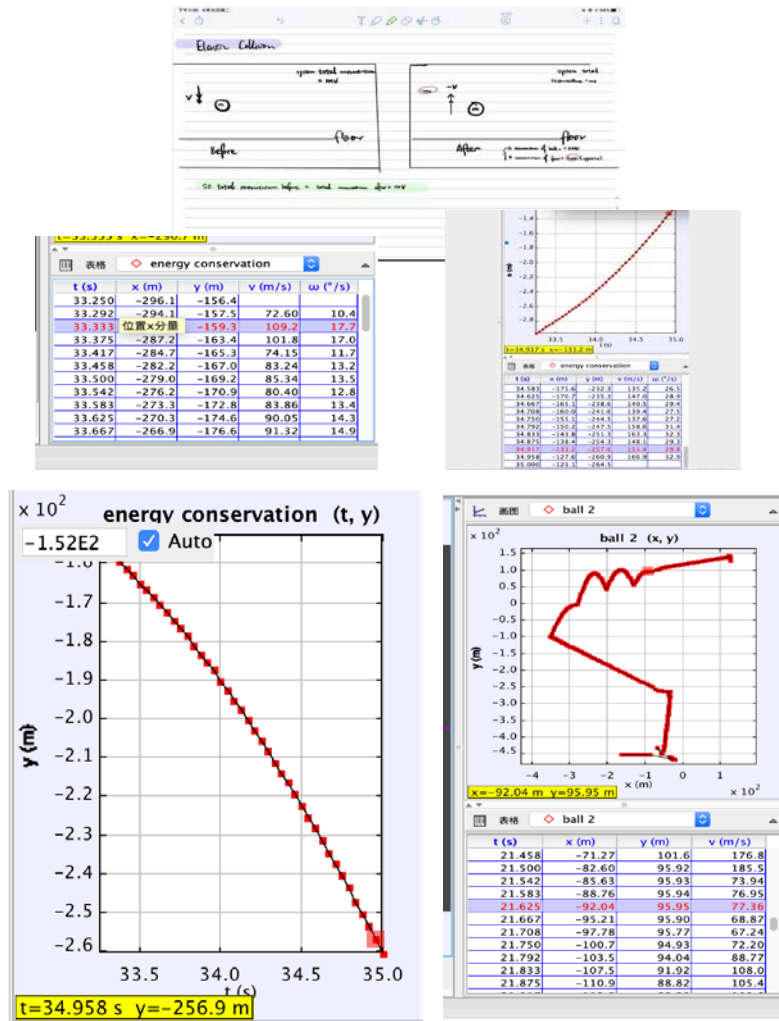
Angular velocity = 60 degree per second , which is 1.04 rad / s (shown on the tracker graph) for anti clockwise motion

Angular velocity = -100 degree per second , which is - 1.7 rad /s for clockwise motion Time for seesaw to rotate = 0.23s

$$\text{Angular acceleration} = 4.5 \text{ rad/s}^2 \text{ (for anti clockwise) } = 7.39 \text{ rad / s}$$

$$\text{Torque} = F * r = I * a \text{ (which is moment of inertia * angular acceleration) } = 12.8 \text{ Nm (kg m}^3 \text{ /s}^2 \text{)}$$

Elastic collision



This is elastic collision in my model , when ball 2 hits those yellow blocks and bounces back

These two sets of values indicates velocities of this ball before and after its collision with the ground.

In an elastic collision , energy is conserved so that $\frac{1}{2} mv^2$ (before) = $\frac{1}{2} mv^2$ (after) Momentum is conserved so $m*v$ (before) = $m*v$ (after)

Mass before = Mass After (0.01kg)

Coefficient of restitution = (relative velocity after collision) / (relative velocity before collision)
) = 162.4/ 167 = 0.972

Analysis of results

Velocity of the ball before(167m/s) nearly = —velocity of the ball after (162.4m/s) with little heat energy loss

For Momentum Conservation : If we take the ground and the ball 2 as a whole system, total momentum before is the mass times velocity of the ball , which is $0.01 * 167 = 1.67$ kg m/s Total momentum after = Total momentum bef Total momentum after in this model is the momentum of the ball and the momentum of the floor.

From the diagram above , we can see an equilibrium in momentum within the system during the collision. Similarly, even without calculations , we can see energy is conserved as mass and velocity are exactly the same.

Energy Conservation

This is when the green ball falls on the pinky slope , some parts of its gravitational potential energy is transferred to kinetic energy.

This is its variation in height with t

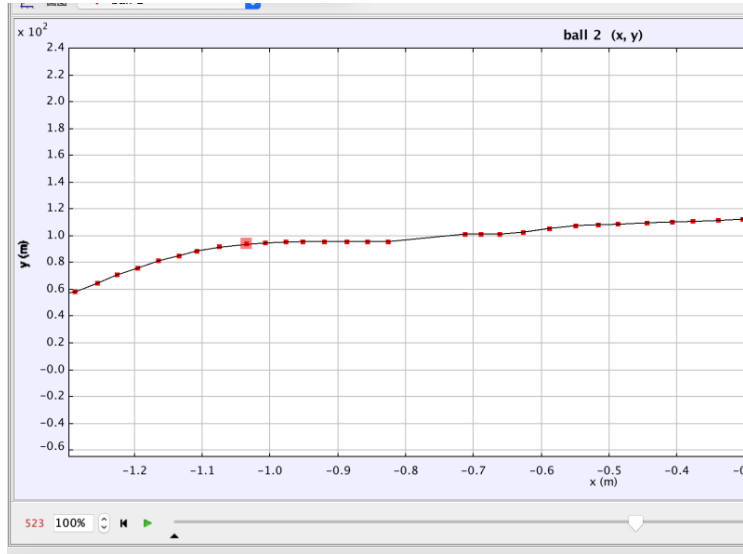
Ball index at Position 1 and 2

Analysis of Results

From the data, we can see a change in height from -152.4 m to -256.9 m , so change is 103.4 m in this process.

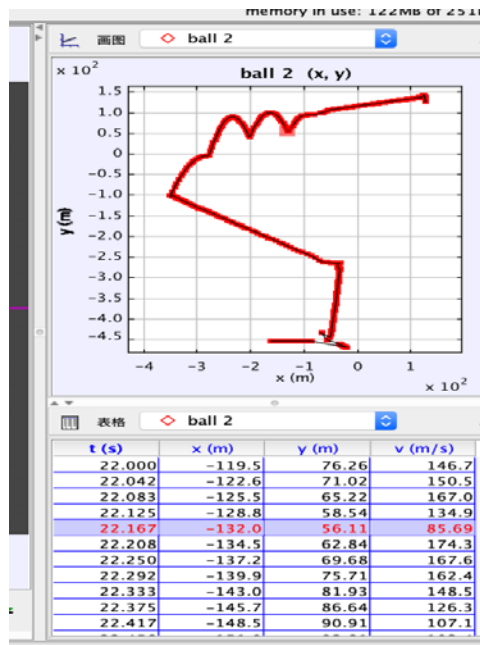
The y axis in this diagram has -584 m as a minimum, if we consider this as a floor so that it has 0m.

The height in position 1 is 431



Position 1

Position 2



Total energy in position 1 is that $GE + KE + \text{Rotational KE} = 0.5 * 109.2^2 + 11.025 (\text{ see blow }) * (0.31 \text{ rad/s})^2 + 431 * 10 = 11726 \text{ J}$

Total energy in position 2 is that $KE + \text{Rotational KE}$

Rotational KE is the kinetic energy caused by rotation of the ball. $= I \omega^2$
 Diameter of the ball is 10.5 m, so the radius is 5.25m.

Moment of inertia of the blue ball (sphere) $= \frac{2}{5} MR^2 = \frac{2}{5} * 1 * (5.25)^2 = 11.025 \text{ kg m}^2$

ω (angular velocity = 29.8 degree per second , which is 0.52 rad per second)
 Rotational Kinetic energy therefore $= 11.025 * (0.524)^2 = 3.02 \text{ J}$

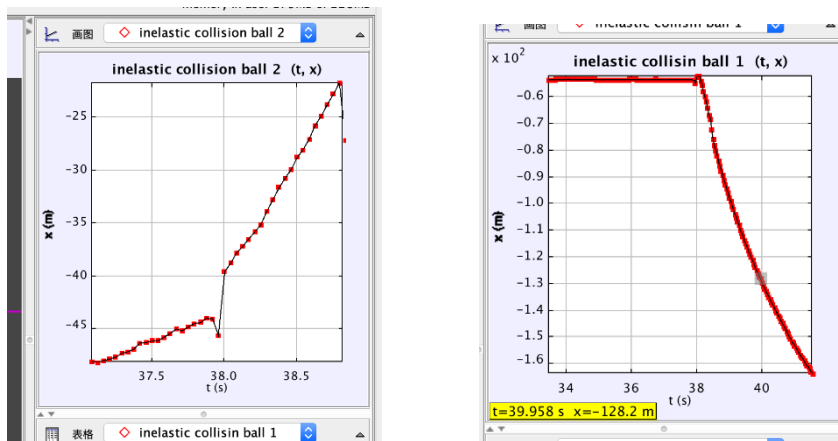
KE in position 2 is $0.5 * 151.4^2 = 11460 \text{ J}$

Total energy in position 1 nearly = total energy in position 2 (11726 J vs 11460 J)

Two— dimension projectile

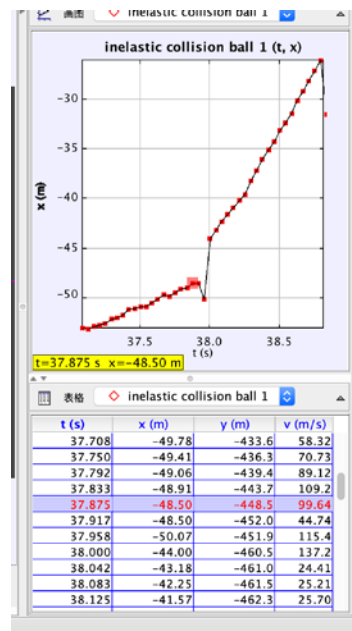
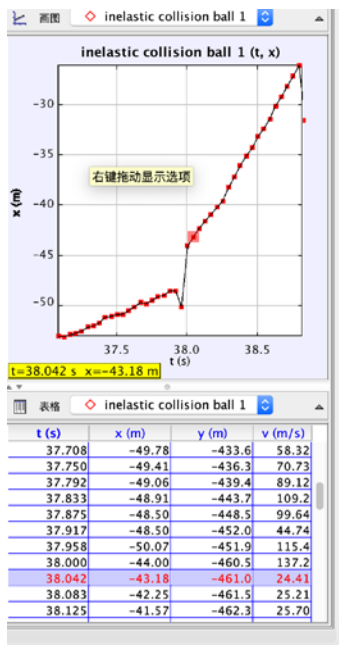
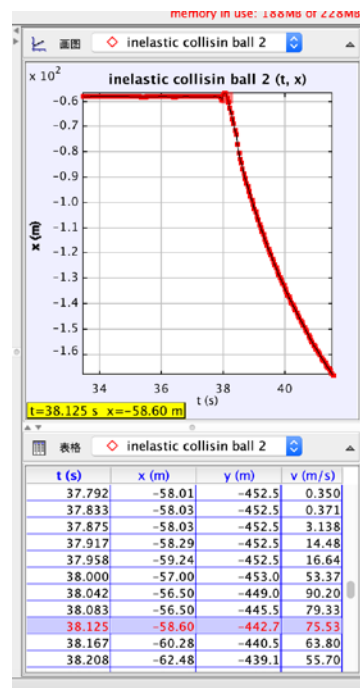
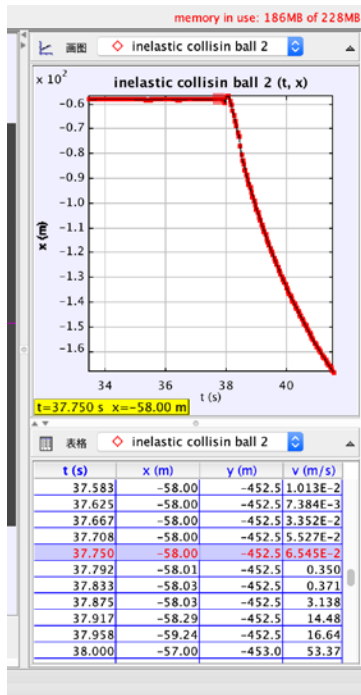
This is two— dimension projectile motion of the blue ball when it moves from the stage of seesaw into a downward sloping curve.

his graph shows the motion path of the ball



Position 1 (start of motion)

Position 2 (end of motion) Mass of the projectile ball = 0.01kg



KE of projectile ball 2 = KE initial - GPE = $\frac{1}{2}mv^2 - mgh = 0.5 * 77.36^2 * 0.01 - 0.01 * 10 * 24 = 27.5 \text{ J}$

This is a virtual machine, thus air resistance can be ignored, so the horizontal velocity V_x of the ball wouldn't change. $0.5 * m * V(x)^2 = 27.5 \text{ J}$ $V_x = 74.1 \text{ m/s}$

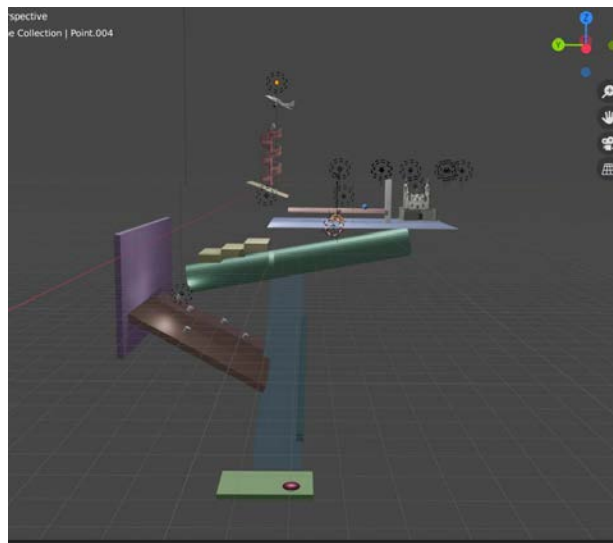
The blue projectile ball falls a distance of 3.9 m, in order to calculate the time, $+ 0.5gt^2 = 3.9$. $T = 0.34 \text{ s}$

Horizontal displacement = $V_x * T = 74.1 * 0.37 = 27.4 \text{ m}$ (not exactly fits in the data : -132-

(-92)=40m in x direction because I didn't take consideration of rotational kinetic energy so results may have some inaccuracy)

Inelastic Collision & Momentum Conservation

Inelastic collision happens where the later arrived green ball hits long- waiting blue ball and they move in opposite direction. Some of its energy is used to crack eggs, so energy is not conserved in two- ball system, however momentum is conserved.



Ball 1 motion path during inelastic collision x vs time

Ball 2 motion path x vs time

Weirdly, after my predicted calculations for many times , The system treat two ball as the same mass 1 kg. change in velocity for the moving ball 1, from 99.6m.s before the collision to 24.41 m/s after the collision.

This is indexes for stationary ball 2 to gain velocity and momentum during he collision from nearly 0 to 75.5m/s

Total momentum before = mass(ball 1) * velocity (ball 1) = 1* 99.64 = 99.64 kg m/s
Total momentum after = new momentum of ball 1 + new momentum of ball 2

= 1 * 24.41 (m* v) + 1* 75.5 = 99.9 kg m/s (some new rotational kinetic energy increase its speed)

So momentum is conserved in this collision process.

V (1) = 99.64m/s V (2) = 24.4m/s V(3) = 75.5m/s

Total energy in the system before collision = ball 1 energy = mostly KE+ some GE= 1/2 m(1)*v(1

)² + m1* g *h = 0.5 *1* 99.64 ² + 27 (height) *1* 10 = 5234 J

Total energy in the system after collision = ball 1 energy + ball 2 energy = 1/2 m(1) v(2)² + 1/2 m (2) v(3)² = 0.5 * 1* 24.4² + 0.5 * 1* 75.5² = 3147 J

Change in energy in this process is 5234- 3147 = 2086 J ,which is used to crack the egg

Conclusion Steps layout

Circular Motion :

From circular motion , we calculate many important indexes like angular velocity , angular acceleration and centripetal acceleration. Its angular velocity and angular acceleration are not too large as 2.25 rad /s and 0.8 rad /s² as it also has another downward motion . However , its centripetal acceleration is quite big , 82.1 m/s², nearly 9 times the gravity , thus the ball will have an obvious visible horizontal circular motion .

Formula :

1. angular velocity = change in angle / time
2. Angular acceleration = change in angular velocity / time
3. Centripetal acceleration = radius * angular velocity²

Torque :

Seesaw in my machine carries the job as a torque, the ball 1 (green) falls on it and exerts a force on it that makes it rotate. It firstly experiences an anti-clockwise acceleration and then the floor stops it for counter motion , which means it has a positive angular velocity followed by a negative one.

I calculated the torque effect on it

Formula :

Torque = force * perpendicular distance on them = moment of inertia * angular acceleration

Moment of inertia for a seesaw (can be seen as a thin rod) is $1/12 MR^2$, but to find total torque on the seesaw , we also need pointed mass torques of two balls which can be calculated using $1/2 MR^2$, , than angular acceleration can be derived from the graph straightly . Finally, torque on the seesaw was 12.8 Nm at last , which should be enough to support the process going.

Elastic Collision :

It happens when the blue ball (ball 2) bounces on yellow blocks. It has not perfect elastic collision , because the coefficient of restitution = 0.972 , however ,both its energy and momentum are conserved.

Formula:

Momentum Conservation : total momentum before = total momentum after

$$M_i V_i = M_f V_f$$

In this model , its initial momentum is mv (of the blue ball) , then as it hits the floor , it has a change of momentum of $-2mv$ (it bounces back with same magnitude but different direction velocity).

Simultaneously, the floor has a change in momentum of $2mv$. The total momentum in the system is therefore $2mv - mv = mv$, which means momentum is conserved

Energy Conservation : In this situation, it refers to kinetic energy merely , which is $1/2 M V_i^2 = 1/2 M V_f^2$

2 , before collision , the velocity of ball is 167m/s , its velocity after collision is 162.4 m/s , supposing their mass is constant , the energy is roughly conserved.

Energy Conservation :

In my machine, energy conservation is everywhere. The most representative example is when the green ball falls on dark pinky slope. As it falls , its gravitational potential energy is transferred to Kinetic energy but also rotational kinetic energy .

Formula :

GE= KE+ rotational KE $m g h = 1/2 m v^2 + I \omega^2$ Gravitational
Potential Energy = $m g h$

Rotational Kinetic Energy = moment of inertia * (angular velocity)²

In this example , the ball has initial energy of 11726 J and its final energy was 11640 J , which is not far part ,and tiny bit is lost due to heat.

Momentum Conservation .

This concept is the fundamental rule for two collisions in my Robe Goldberg Machine , both the elastic and inelastic ones ,it exists as long as there is no external force.

Formula :

$M_i v_i = M_f v_f$

In my machine , the first collision where the blue ball bounces back, the whole system has conserved momentum of 167 kg m/s.

In the second collision , its total momentum is still the same as their horizontal components cancel out and their vertical component is exactly the momentum before the collision. = 99 kg m/s

Two — dimension projectile

I choose time when the blue ball falls from first platform onto the block above the cylinder. Its horizontal velocity is constant as no resultant force acting on it. , Due to gravity ,the ball accelerates in the vertical direction , which forms a projectile motion.

Formula: $s = ut + 1/2 g t^2$ (suvat equation)

$KE = 1/2 m v^2$.

KE of the ball = KE initial - GPE

Firstly , we can calculate KE of the ball and get the ball's horizontal velocity, which is 74.1m/s velocity as it hasn't drop yet . Then time(0.37 s) of this dropping process is determined by using $s = u + v + t$ equations and known height it falls. Finally , $V_{\text{horizontal}} * \text{Time}$ is horizontal displacement(27.4 m) of this projectile motion.

Inelastic Collision :

It happens when the later arrived green ball hits the stationary blue ball, which spares some energy to crash the egg, thus energy is not conserved.

Its momentum is conserved ,of 99kg m/s in this collision. (although some weird reasons make all ball have masses of 1 kg , but one supposed to be 0.1 kg)

$$M_i V_i = 1 * 99.64 = 99.64 \text{kgm/s}$$

$$M_f V_f = 1 * 24 + 1 * 75 = 99.9 \text{kgm/s}$$

However , energy has some loss from 5234J to 3147 J (by equation $\frac{1}{2} m v^2$) , which indicates this is an inelastic collision. Overall , my Rube - Goldberg Machine contains seven main Physics Mechanics concepts and from energy and momentum conservation , two dimension projectile , torque , circular motion to elastic and inelastic collision. During the process of building the model , I notice the importance of IT and details in Physics , by analyzing it , my models backed up by deeper physics ideas and formulas like centripetal acceleration , coefficient of restitution. All of these contributes to a successful virtual Rube- Goldberg Machine and encourage me to apply more beautiful physics equations to daily life machine in the future.

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