

The solution of the volume of collaborative space of 6R two-arm robot based on the body-elemental algorithm

Haowen Gao, Yinuo Xu

Department of Automation, National University of Defense Technology, Changsha, China

Abstract: TO the problem of solving the collaborative space of a two-arm robot accurately, this paper takes the 6R two-arm robot as the research object, proposing a solution combining Monte Carlo method and body-elemental algorithm. Firstly, according to the D-H parameters of the 6R two-arm robot, we build an analytical model of the 6R two-arm robot using the Matlab toolbox. Then, we use the Monte Carlo method to quantify the respective working spaces of both arms based on this model. Finally, we calculate the volume of the cooperative space through the body-elemental algorithm, that is, finding the boundary elements and non-boundary elements of the collaboration space constantly and improving the accuracy of the volume through the continuous refinement of the boundary body elements. The results show that, when the number of sample points reaches 200,000, the solution time after refining 3 times is only about 15s, which is efficient. At the same time, the volume variance derives from 10 sets of random numbers is about $9e-10$, which is quite precise. The volume of collaborative space through this algorithm lays a theoretical foundation for the subsequent optimization of the structure of the two-arm robot.

Keywords: Two-arm robot; collaborative space; Monte Carlo method; body-elemental algorithm

1. Introduction:

In recent years, with the development of science and technology, the field of robotics has also developed rapidly, and the application scope is more and more wide. However, due to the continuous working conditions and the continuous expansion of the working range, in some working conditions, the single-arm robot has been unable to meet the needs of the use, therefore, the development of dual-arm robot is also imminent^[1]. The robot workspace refers to the set of all possible spatial positions reached by the end-effector during the robot working process, while the cooperative space refers to the set of positions reached by the end-effector at the same time. Dual-arm cooperative space is an important performance of the cooperative working ability of dual-arm robots. The calculation of the cooperative space is of great significance to the design and development of robots^[2].

At present, there are three commonly used calculation methods of robot workspace: geometric method, analytical method and numerical method^[3]. The analytical method is accurate, but it is not suitable for engineering applications because the spatial boundary conditions must be determined first and the calculation process is very complicated. However, geometric method can not accurately describe the workspace of multi-degree-of-freedom robot because it is limited by the degree of freedom of robot. The numerical method is to calculate the set of all the feature points in the robot workspace. The calculation process is simple and the calculation accuracy is relatively high. Based on these basic ideas, Wang Wei et al. used MATLAB to extract the boundary curve of cooperative space by extremum theory method and search area method, but solved it without the volume of cooperative space^[4]; Xie Shengliang et al. used Monte Carlo random number method to analyze the collaboration space of dual-arm robot, and at the same time extracted the boundary points of the collaboration space by dividing the space by column and Angle, which has high precision but large amount of calculation^[5].

For the above issues, Xu Zhenbang and his team proposed an algorithm for using body elements to find the volume of a single robot working space in 2018^[6]. In the calculation process, the boundary volume element is constantly refined, and this algorithm effectively improves the calculation accuracy of the workspace volume. However, this algorithm does not give the calculation efficiency, and it also does not use this algorithm to solve the double arm cooperative space volume. Therefore, this paper firstly uses MATLAB to accurately model two 6R robot arms, then uses Monte Carlo method to generate seed workspace and extract collaboration space based on this model, and finally obtains the volume of

collaboration space through voxel algorithm. 1 Establishment of a kinematic model of a two-arm robot

1.1. Two-arm robot model construction

The lab 6 degrees of freedom two-arm robot is shown in Figure 1, and the left arm DH parameter table is shown in Table 1.



Figure 1: 6R two-arm robot

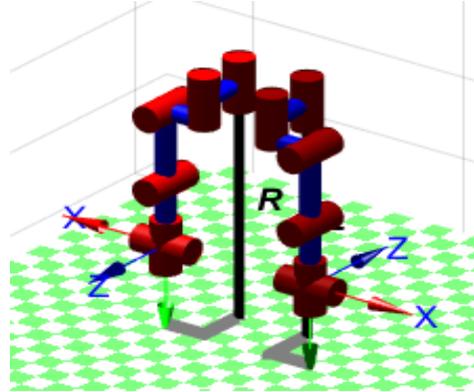


Figure 2: A model of a two-arm robot based on MATLAB

Table 1: DH parameter table for the left arm of a two-arm robot

Joint	α_{i-1}/rad	a_{i-1}/m	θ_i/rad	d_i/m
1	0	a_0	θ_0	0
2	$\text{Pi}/2$	a_1	θ_1	d_2
3	0	a_2	θ_2	0
4	$\text{Pi}/2$	0	θ_3	d_4
5	$-\text{pi}/2$	0	θ_4	0
6	$\text{Pi}/2$	0	θ_5	0

Take the lab-armed robot for example, each joint uses the Japanese HARMONIC reducer FHA series motor, then we calculate and find that the range of connecting rod lengths described above, as shown in Table 2:

Table 2: Maximun and minimun values for each connecting rod length

Rod parameters	a_0	a_1	d_2	a_2	d_4
Maximun/m	0.16	0.17	0.082	0.307	0.256
Minimun/m	0.08	0.09	0.002	0.227	0.186

1.2. The analysis and solving of forward kinematic

The robot forward kinematics refers to obtaining the position of the end of the actuator relative to the base when given all of the joint positions as well as connecting rod parameters. Establish a coordinate system using the modified D-H method^[7-8], and derive the forward kinematic equations, then we get the matrix of posture transformations between two adjacent connecting rods:

$${}^{i-1}T_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & a_{i-1} \\ \sin\theta_i \cos\alpha_{i-1} & \cos\theta_i \cos\alpha_{i-1} & -\sin\alpha_{i-1} & -\sin\alpha_{i-1}d_i \\ \sin\theta_i \sin\alpha_{i-1} & \cos\theta_i \sin\alpha_{i-1} & \cos\alpha_{i-1} & \cos\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

Multiplying this matrix sequentially to solve the positive kinematic equation for the end actuator of the robotic arm:

$${}^0T = {}^0T(\theta_0) \times {}^1T(\theta_1) \times {}^2T(\theta_2) \times {}^3T(\theta_3) \times {}^4T(\theta_4) \times {}^5T(\theta_5) = \begin{bmatrix} R & P \\ 0 & 1 \end{bmatrix}, P = [p_x \ p_y \ p_z] \quad (2)$$

Where p_x, p_y, p_z denote the positions of the end actuator of the robotic arm and wo get:

$$p_x = \cos\theta_1 a_1 + \sin\theta_1 d_2 + \cos\theta_1 a_2 \cos\theta_2 + d_4 \cos\theta_1 \sin(\theta_2 + \theta_3) \quad (3)$$

$$p_y = \sin\theta_1 a_1 - \cos\theta_1 d_2 + \sin\theta_1 a_2 \cos\theta_2 + d_4 \sin\theta_1 \sin(\theta_2 + \theta_3) \quad (4)$$

$$p_z = a_2 \sin \theta_2 - d_4 \cos(\theta_2 + \theta_3) \quad (5)$$

2. The algorithm for collaborative space volume

Numerical methods can effectively play the computational advantages of computers, generating a collection of feature points for the workspace. Based on this, we generate spatial point cloud maps of the left and right arms of the robot through the Monte Carlo method, then we divide the collaborative space through the body-elemental algorithm and get the volume finally.

2.1. Analysis of the collaborative space

The Monte Carlo method through random sampling numerical method to solve mathematical problems, this paper, by using the algorithm to solve the collaboration space and its solving process is: first, to produce a large number of monte carlo random Numbers represent different joint variable combination, and then will produce a combination of variable substitution to the mechanical arm forward motion equation (2) the collection of work space, a series of points. The specific steps for generating the point set of the left arm workspace are as follows:

Step 1: Using the Matlab random function rand() to generate N groups of random numbers representing individual joint variables:

$$\theta_i = \theta_{imin} + rand(N, 1) * (\theta_{imax} - \theta_{imin}) \quad (6)$$

Where θ_{imin} and θ_{imax} denote the minimum and maximum values of the ith joint corner respectively, and the joint angle range of a specific robotic arm is listed below:

Table 3: Joint corner range

Joints	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
Upper limits $\theta_{max}/(^\circ)$	90	90	180	180	90	180
Lower limits $\theta_{min}/(^\circ)$	-90	-120	0	-180	-90	-180

Step 2: Determining a set of joint lengths and substituting the random values of joint angles produced by step1 into the equation of forward motion, then getting the position coordinates of the end actuator of the left arm.

Step 3: Displaying all location coordinate points in a 3D Cartesian coordinate system via Matlab, then getting a workspace for the left arm.

The right arm workspace can be obtained using the same method. As the number of random points increases, the result of volume of the collaboration space is more accurate. When the number of random points in both arms is N= 10000, we plot its point cloud shown in Figure 3:

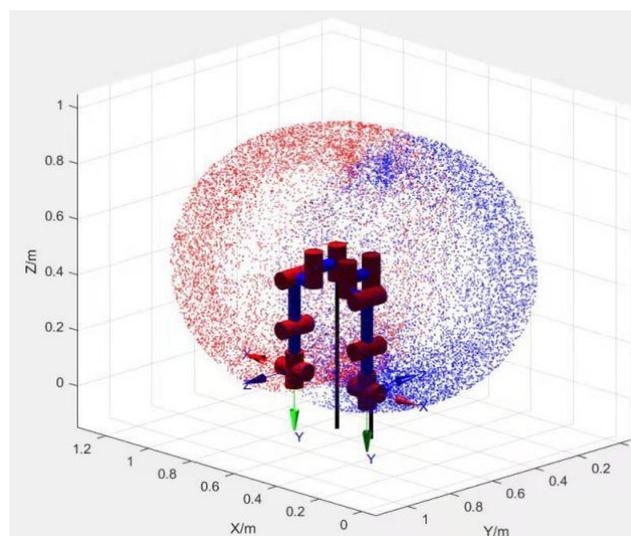


Figure 3: Two-arm robot collaborative space point cloud map

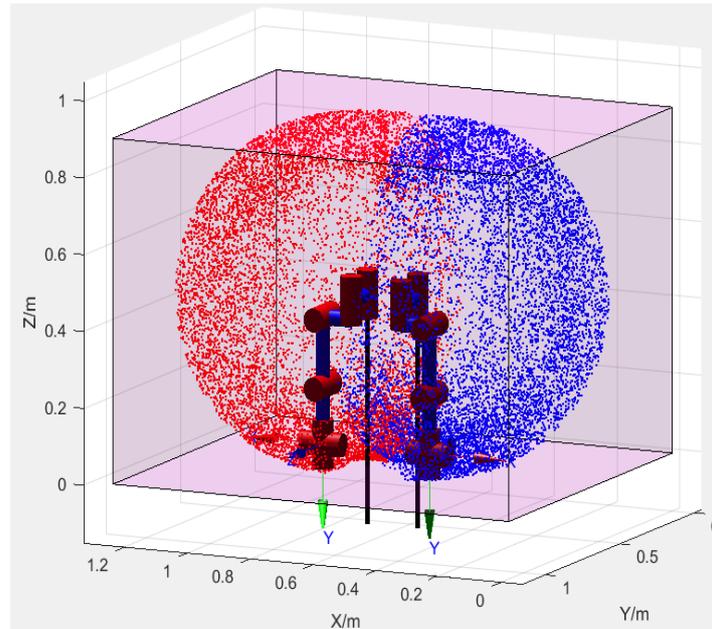


Figure 4: Cuboid envelope

2.2. The body-elemental algorithm

Visual voxel is the smallest unit in three-dimensional space. In calculation, a certain number of voxel is generally used to simulate the shape of the actual object. In the process of solving, because the volume of a single voxel is known, the volume of the actual object can be approximately equivalent to finding the sum of all the volume of the voxel. But because the shape of actual object is very complicated, there must be some error in using body element simulation to approximate its volume. As the error occurs in the boundary part of the actual object, the degree of the fit of the boundary body element greatly determines the accuracy of the volume solution. Obviously, the smaller the volume of the defined body element is, the better the simulation of the real objects is. But at this point efficiency is inversely proportional to precision, so refining the body element to improve the accuracy will inevitably lead to a decrease in computational efficiency. Therefore, in order to effectively improve the calculation accuracy and ensure high calculation efficiency, we divide the body element into boundary element and non-boundary element. When simulating a real object using body elements, we firstly use larger volumes to approximate the shape of the body, then we find the body element located in the boundary part of the object and refine it to reduce the boundary error. The specific steps to solve the volume of the collaborative space of a two-arm robot using the body-elemental algorithm are as follows:

Step 1: Wrapping the entire workspace of the left and right arm point clouds in a single cuboid envelope, as shown in Figure 4. Since the coordinates of all point clouds are known, the maximum value of the cuboid envelope in the direction of the three axes $x_{max}, y_{max}, z_{max}$ and the minimum $x_{min}, y_{min}, z_{min}$ can be found, and the side length of the box is:

$$l_x = |x_{max} - x_{min}| \quad (7)$$

$$l_y = |y_{max} - y_{min}| \quad (8)$$

$$l_z = |z_{max} - z_{min}| \quad (9)$$

Step 2: Dividing the cuboid envelope in step1 into m, n, p isometric parts along the three coordinate axes. The cuboid envelope is divided into $m \times n \times p$ individual elements and the volume of each element is:

$$V_0 = \frac{l_x}{m} \times \frac{l_y}{n} \times \frac{l_z}{p} \quad (10)$$

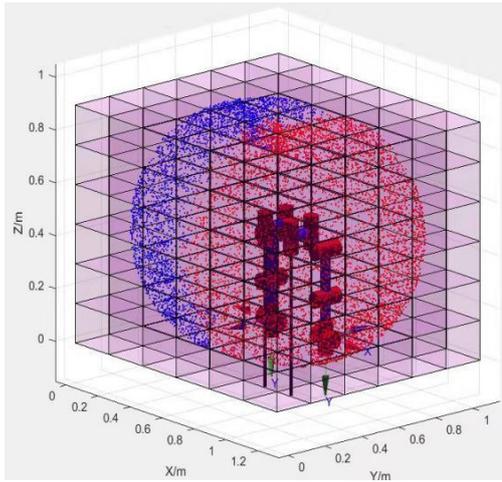


Figure 5: The entire envelope is divided collaboration into small cube

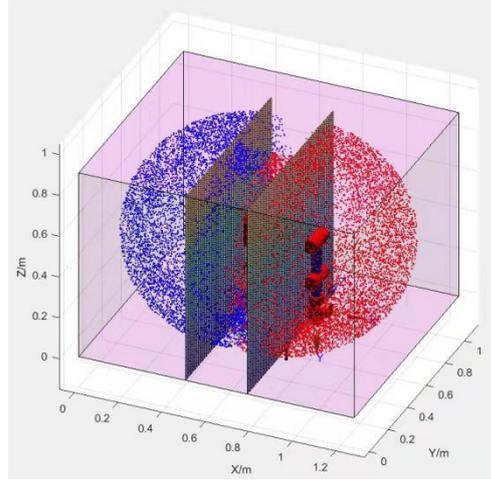


Figure 6: Possible areas of the elements space after cropping

Step 3: Using algorithm 1 to traverse the body elements in step 2, finding the element that contains both of the left and the right arm point cloud. The remaining body elements don't belong to the collaboration space, which can be deleted directly.

Algorithm 1: Finding the inner body element of the collaboration space

```

Data_l : Points cloud data of the left arm
Data_r : Points cloud data of the right arm
Data : An all-zero matrix used to store the location of the collaboration space
V : The possible collaboration space
1  for i = 1,...,N do
2      if Data_r[i] ∈ V
3          Data[i] ← Data[i] + 10000
4      end if
5      if Data_l[i] ∈ V
6          Data[i] ← Data[i] + 1
7      end if
8  end for
9  Data ← mod(Data,10000) & (Data>10000)
10 return Data
    
```

A body element in the collaboration space means that it contains both of the left and right arm point cloud. We create a binary database ,Data, to store elements in the collaboration space. Because the collaboration space is relatively small compared to the entire cuboid envelope, directly dividing the entire cuboid envelope will inevitably lead to most of the elements being non-collaborative space elements, wasting computing resources and computational precision. Define x_{lmin} as the minimum x-value in the left arm point cloud data, x_{rmax} as the maximum x-value in the right arm point cloud data, and an area V that could become a collaboration space. As shown in Figure 6, its scope is:

$$x_{lmin} \leq x < x_{rmax} \quad (11)$$

$$y_{min} \leq y < y_{max} \quad (12)$$

$$z_{min} \leq z < z_{max} \quad (13)$$

Traversing the data of point cloud located in area V, if a point is from the left arm workspace and is inside a body element, the database, Data, adds 10000 to the value at the position of the body element; if

it is from the right arm workspace and is inside a body element, then the database, Data, adds 1 to the value at the position of the body element. In the end, the body element whose Data value is greater than 10,000 and cannot be divisible by 10,000 can be seen as the collaborative body. This algorithm utilizes the techniques of matrix manipulation, greatly reducing the number of loops traversed in the algorithm, thus improves the computational efficiency.

Step 4: Using algorithm 2 to find the boundary elements for the remaining elements in step 3, using n_1 to represent the number of elements in the boundary portion of the collaboration space, using n_2 to represent the number of elements in the non-boundary portion of the collaboration space, then the volume of boundary parts and non-boundary parts can be expressed as:

$$V_1 = V_0 \times n_1 \quad (14)$$

$$V_2 = V_0 \times n_2 \quad (15)$$

Algorithm 2: Finding boundary and non-boundary elements

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Data_b: The array that stores the ordinal of the bounding body element
Data_n: The array that stores the ordinal of the non-bounding body element
Data: The binary matrix that stores the location of the collaboration space
1  for i = 1,...,n do
2      num ← the ordinal of the body elemen in the collaboration space
3      Condition 1 ← num is the boundary ordinal of the layer
4      Condition 2 ← Data(num+1)=0
5      Condition 3 ← Data(num-1)=0
6      Condition 4 ← Data(num+n)=0
7      Condition 5 ← Data(num-a)=0
8      if Condition 1 to Condition 5 at least one is true ,we have
9          Data_b ← num
10     else
11         Data_n ← num
12     end if
13 end for
14 return Data_b,Data_n
    
```

Layer elements within the collaboration space along the z-axis. The boundary body elements are defined as: At least one of the elements in the same layer adjacent to the element is a non-collaborative space element; The adjacency her is only considering the four positions of the body element before and after the body element. Considering the array of 5×8 elements of size as shown in Figure 7, the ordinal numbers of each body element are shown in the figure. When using algorithm 2 to filter boundary body elements, to each collaboration space body element in turn, that is, the non-white position in the figure we judge 5 conditions respectively (lines 3, 4, 5, 6, and 7 in algorithm 2). The first condition determines whether the element is at the edge of the array, if so, it must be a bounding element, otherwise, the judgment of the latter four conditions is required, that is, whether there are non-collaborative space entities in the adjacent 4 entities of the body element. Only one of the above 5 conditions is met, the body element can be considered as a boundary body element (as shown in the orange part of Figure 7), then we write its ordinal number into the array Data_b. Otherwise, it is considered to be a non-boundary body element, and we write its ordinal number into the array Data_n.

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40

Figure 7: Schematic diagrams of searching for boundary and non-boundary elements

Step 5: Refine the boundary part of the body element several times. The specific method is shown in Figure 8, and we use $i(1, \dots, N)$ to represent the number of times that the boundary body element is refined. After the i th refinement, body elements are divided into $(i + 1)^3$ identical small body elements. After each refinement, a traversal search of the point cloud in the collaboration space is performed, then we locate the element that still contains the point cloud. This process is cycled until the precision requirements of the setting are met.

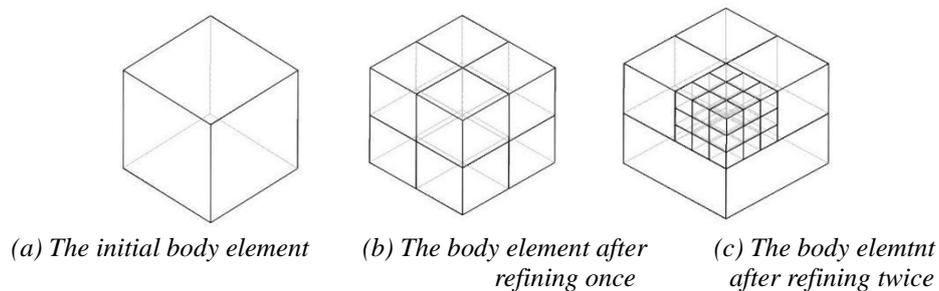


Figure 8: The schematic diagram of boundary body elements refinement

Step 6 Find the volume of the collaboration space. Using V_1^i to represent the sum of the boundary volume after i th refinement, using V_2^i to represent the sum of the non-boundary volume after i th refinement, then the total volume of the collaboration space after i th refinement V^i is:

$$V^i = V_1^i + V_2^i \tag{16}$$

3. Results and analysis

Take the double 6R robotic arm selected in this experiment as an example, we use the algorithm of the part 2 to solve the volume of its collaboration space. We firstly use the Monte Carlo method to generate $N=200,000$ random points for the two-arm model and obtain a cuboid enveloper, then we follow the Algorithm 1 to crop the whole space and find the possible collaboration space and then filter out the collaboration space elements. After that, we use Algorithm 2 to find the boundary elements and non-boundary elements of the collaboration space and follow the subdivision method of the boundary elements in step 6 to obtain volume of the collaboration space V^3 by recursively refining it 3 times. The final result for 10 sets of random numbers is: $3.3577e-3m^3$ for average and $9.2569e-10$ for the variance.

4. Conclusion

This article refers to the body-elemental algorithm proposed by the predecessors, then we improve and apply it to solving the collaboration space. In order to improve the computational efficiency of the solution process, this paper finds a possible cooperative space region in the spatial region (that is, there must be no cooperative space point cloud in other locations), and uses matrix calculation method instead of cyclic traversal, which greatly improves the computational efficiency. In order to improve the solving accuracy, this paper adopts an algorithm to distinguish boundary elements from non-boundary elements, and then reduces the errors caused by the complex shape of the collaborative space by continuously refining the elements of the boundary part. Take the 6R two-arm robot for example, this

paper shows the feasibility and effectiveness of the body-elemental algorithm in solving the collaborative space. The final results show: when the number of sample points is 200,000, the algorithm show high efficient, with 15s for tree refinements. And the variance of the calculated volume is only $9.26e-10$ corresponding to 10 sets of random numbers, which shows high precision. The collaborative space volume obtained by this method can provide theoretical support for the structural optimization of subsequent two-arm robots.

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