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Analysis of the stress state of solid propellant grains under internal pressure load

Tingjing Geng, Hongfu Qiang, Zhejun Wang*, Xueren Wang, Chunguo Yue

206 Staff Room, Xi'an High-Tech Institute, Xi'an, China
**Corresponding author: 1832994156@qq.com*

Abstract: *In order to determine the stress state of the solid rocket motor grain under the internal pressure load, based on the finite element numerical simulation calculation, the length and size requirements of the grain under the internal pressure simplified to a plane strain problem are obtained. Numerical simulation calculations are further carried out for the grains in plane strain state that meet the simplified conditions, and the radial and circumferential stress states of the grains are obtained. The numerical simulation results are compared with the viscoelastic theoretical solution of the grains under the same material parameters. The results show that when the aspect ratio exceeds 2.5, the axial strain is approximately 0, which can be equivalent to a plane strain problem. In addition, the analysis shows that under the action of internal pressure, the plane strain grains are in a bidirectional compression stress state with a radial and circumferential stress ratio of approximately 1:1. Finally, by comparing the numerical solution and the analytical solution, it is found that the relative error is within 3%, which verifies the correctness of the numerical simulation calculation results.*

Keywords: *solid propellant, finite element, plane strain, bidirectional compression*

1. Introduction

When the solid rocket motor is ignited, the high-pressure gas generated will directly act on the solid propellant grains. Once the mechanical properties of the propellant grains cannot withstand the internal pressure of the gas, cracks will occur on the inner surface of the grains, which will lead to engine channeling. The occurrence of accidents such as fire or even explosion of the whole machine[1]. Therefore, it is necessary to carry out the mechanical analysis of the solid propellant grain under the internal pressure load, combined with the mechanical performance test research under the corresponding load conditions, to accurately analyze the structural integrity of the solid rocket motor, so as to ensure the safe and reliable performance of the missile power system.

At present, scholars at home and abroad have carried out a large number of studies on the mechanical properties of solid propellant pharmaceutical columns under internal pressure. In terms of numerical calculation, due to the limitations of the finite difference method, the finite element method is generally used due to the complexity of the boundary element method[2]. Yan[3] used the finite element method to carry out the numerical simulation calculation of the engine charge with the coating under internal pressure loading. Yang[4] also used the finite element method to carry out the stress and strain analysis of the engine grain, but at the same time considered the influence of the artificial debonding layer and the change of the material's Poisson's ratio on the grain stress and strain. Wang[5] used ANSYS finite element software to analyze the structural integrity of the grain under internal pressure loading, he verified the rationality of the numerical solution under the plane strain state through the comparison of analytical solutions, but did not give a satisfactory result for plane strain. Song[6] combined the three-dimensional viscoelastic finite element method to calculate the structural integrity of the grain under low-temperature ignition conditions. Theoretical calculations, Boyd[7] ignored the deformation of the shell and obtained the analytical solution of the stress under the corresponding internal pressure load. On this basis, Wang[8] did not make such assumptions, and proposed a new viscoelastic analysis method for solving propellant grains under the action of internal pressure, including two forms of differential operator and integral operator. Yan[9] used a new viscoelastic model-fractional derivative model to conduct mechanical analysis of propellant grain expansion under uniform internal pressure, which improved the accuracy of the analytical solution, but the relevant test parameters were not easy to obtain.

In summary, current scholars mainly combine numerical calculations and theoretical solutions to qualitatively and quantitatively analyze the overall structural integrity of propellant grains under internal

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pressure. However, the study of specific stress states at characteristic locations is relatively less. Therefore, this article focuses on the finite element numerical simulation calculation of the round-hole grain under internal pressure load, gives the specific stress state of the grain, and uses the integral viscoelastic analytical solution and numerical solution that are easy to obtain with relevant parameters. Contrast, verify the correctness of the simulation conclusion.

2. Numerical simulation analysis of grain under internal pressure load

This paper makes the following assumptions on the selected solid rocket motor and round bore medicine cylinder: (1) It is assumed that the propellant material used in the grain is an isotropic linear viscoelastic material, and the Poisson's ratio is a fixed value. (2) The ablation and ablation of the surface of the inner hole are not considered. (3) Regardless of the influence of the interface such as the insulation layer on the stress state of the grain, it is assumed that the grain and the shell are directly bonded. (4) The grain and the shell are longer, and the two ends of the grain are bonded to the shell, which is in a plane strain state.

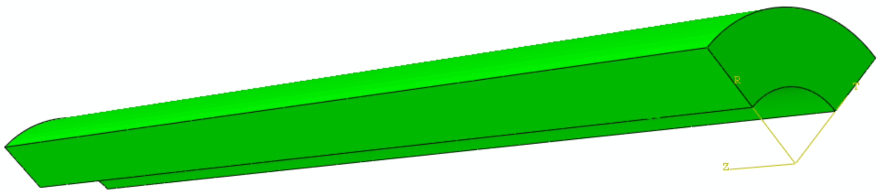
2.1. Model establishment

In studying stress-related computation under internal pressure load, many scholars generally reduce the internal pressure problem to a planar strain problem, but the plane strain problem is characterized by infinitely long or limited long but axial displacement, namely $\varepsilon_z = 0$ [10]. Therefore, it is necessary to perform simulation calculations for three-dimensional engine models at different lengths to determine the engine length dimensions that meet the requirements of hypothetical (4). In this paper, five finite element models with different length whose specific dimensions are shown in Table 1.

Table 1: Simulation model length and dimensions

L/D	0.5	1	1.5	2	2.5
L / mm	219.5	439	658.5	878	1097.5

The size of the grain model part refers to a certain solid engine, the outer diameter is *R*=439 mm, and the inner diameter is *r*=219.5 mm. The thickness of the shell is 3 mm, and the length is the same as the length of the grain. Due to the symmetry of the engine, in order to reduce the amount of calculation, this article only takes a 1/4 model for simulation calculation. The schematic diagram of the model is shown in Figure 1.



1Figure 1: Schematic diagram of the1 / 4 model

2.2. Material parameters and boundary conditions

In the numerical simulation calculation, the propellant used in the grain adopts the line viscoelastic constitutive based on the Prony series, with the series n taking 6, and the parameters of each order are shown in Table 2. Other parameters for the material properties: Poisson ratio $v_p = 0.495$.

The material parameters of the shell are: the elastic modulus $E_c = 2\times10^5$ MPa and the Poisson ratio $v_c = 0.3$.

The grid is divided into C3D8RH units, a total of 35574 units. The analysis step took two steps, Step1 is the static general analysis step and Step2 is the viscosity analysis step. The boundary condition is to impose symmetrical restraint on the symmetry plane of the model, restrain the axial displacement at both ends of the grain and the shell, and apply uniform pressure load on the surface of the inner hole of the grain. The amplitude expression is $P(t) = P(0)(1-e^{-60t}) = 7(1-e^{-60t})$, and the loading time lasts for 0.1s.

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Table 2: Parameters of Prony series of the constitutive relationship of HTPB composite solid propellant

i	∞	1	2	3	4	5	6
E_i/ MPa	2.561	0.3171	0.9502	3.255	1.576	1.335	0.3928
τ_i / s	—	0.001	0.1	1	10	100	1000

2.3. Influence of aspect ratio on the axial strain of grain

Select the typical characteristic location, that is, the inner hole element in the middle plane, for strain analysis. The radial strain ε_r, circumferential strain ε_θ and axial strain ε_z obtained by numerical calculation of five different aspect ratios of the engine model are shown in Table 3. Among them, Q is the proportionality coefficient, which is used to describe the relative magnitude of the axial strain ε_z and the maximum absolute value of the strain $|\varepsilon|_{\max}$. Since $|\varepsilon_r|$ is always greater than $|\varepsilon_\theta|$, so $Q=|\varepsilon_r/\varepsilon_z|$.

Table 3: Typical characteristic position strain values of the grain under different length ratios

L/D	ε_r	ε_θ	ε_z	Q
0.5	-0.0526	0.0357	0.0032	17.52
1	-0.0543	0.0305	0.0021	27.19
1.5	-0.0516	0.0290	0.0014	36.85
2	-0.0503	0.0287	0.0005	85.83
2.5	-0.0497	0.0285	0.00015	312.9

It can be seen from Table 3 that with the continuous increase of the grain length-to-diameter ratio, the proportional coefficient Q value generally shows a gradual increase trend. When L/D is less than 2.5, the magnitude of Q is still below 10^2, but when it exceeds 2.5, the magnitude of Q reaches 10^2, indicating that the axial strain ε_z is very different from the strain in the other two directions, and it can be approximately 0 at this time, which can be simplified as a plane strain problem. Since the aspect ratio of the tactical missile engine is generally greater than 2.5, the internal pressure problem of the tactical missile rocket engine ignition can be simplified into the internal pressure problem under the two-dimensional plane strain state in engineering.

2.4. Numerical simulation analysis of grains under plane strain

According to engineering practice, the stress state of grain under internal pressure load under plane strain.Using symmetry, only 1/4 model was considered for numerical simulation calculation, the material properties remained consistent, and the grid divided the CPE4RH (four-node plane strain quadrangle) units, with a total of 21,564 units. The analysis step took two steps, Step1 is the static general analysis step and Step2 is the viscosity analysis step. The boundary condition is to impose symmetrical constraints on the symmetry plane of the model, add a completely fixed constraint on the outside of the shell, and apply a uniform pressure load on the surface of the inner hole of the grain. The amplitude expression is $P(t)=P(0)(1-e^{-60t})=7(1-e^{-60t})$, and the loading time lasts 0.1 s.

The distribution cloud diagram of radial stress and circumferential stress obtained under the cylindrical coordinate system is shown in Figure 2.

As can be found from Figure 2, both the radial and peripheral stress distributions are distributed according to the ring, mainly due to the uniform internal pressure load and the symmetric boundary setting.

Select the inner diameter $r1$=219.5 mm, intermediate radius $r2$=329.25 mm, outer diameter $r3$=439 mm takes three typical positions to extract the change of radial and peripheral stress over time during the pressure loading period within 0.1 s, as shown in Figure 3.

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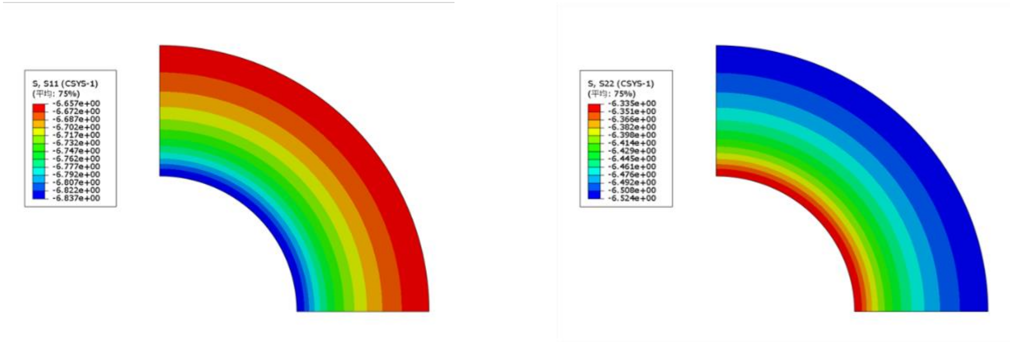
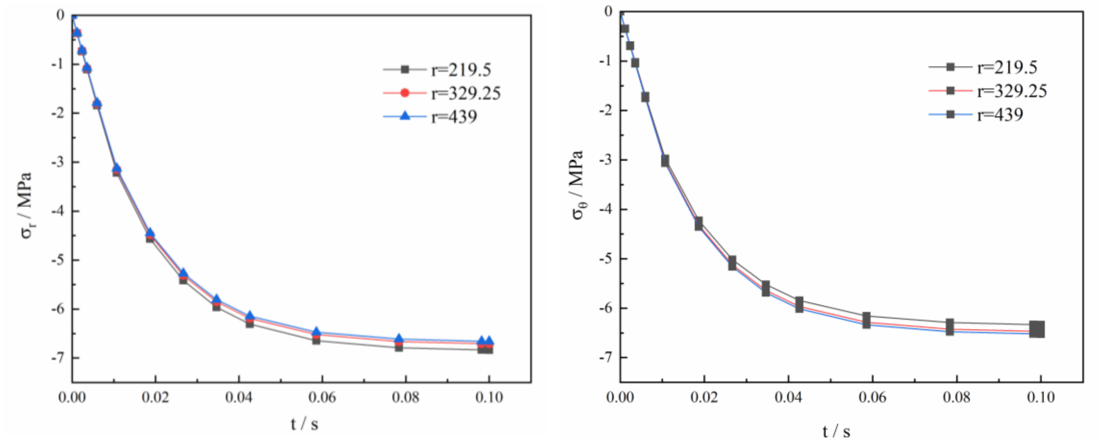


(a) Cloud map of the radial stress distribution (b) Cloud map of the circumferential stress distribution

2Figure 2: Stress Distribution Cloud Map (MPa)

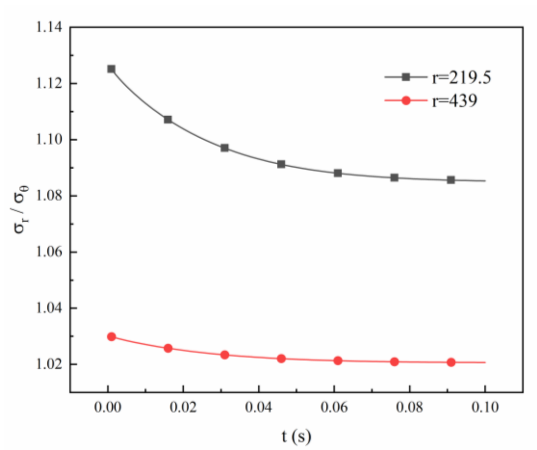


(a) Radial stress change curve over time (b) Curve of peripheral stress change over time

3Figure 3: Numerical stress change curves over time

It can be seen from Figure 3 that the radial stress and circumferential stress of the grains are always compressive stresses, and their absolute values gradually increase and become stable over time. However, the magnitude of the radial stable pressure decreases with the increase of the radius, that is, the closer the position to the internal pressure, the greater the stress value. In contrast, the axially stable stress value decreases as the radius increases, and the stress value near the thin shell becomes larger.

Calculate the stress ratio σ_r / σ_θ between the inner hole and the thin shell in the two areas where the stress maximum points appear, and the result is shown in Figure 4. The stress ratio of the stress maxima, the inner hole and the thin shell, are shown in Figure 4.



4Figure 4: Time curve of stress ratio between inner hole and outer diameter

It can be seen from Figure. 4 that as the pressure gradually stabilizes, the radial and circumferential radial stress ratio of the grain at the inner hole stabilizes at 1:1.08, while the stress ratio at the outer diameter point tends to be 1:1.02.

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In summary, it can be found that the round-hole grains are approximately in a bidirectional compressive stress state with a radial and circumferential stress ratio of 1:1 under the state of gradual pressurization.

3. Theoretical validation

For the radial and peripheral stress caused by the gradual pressurization of the circular pore cylinder at two ends, Wang[11] derives the corresponding analytical expression of stress based on the elastic-viscoelastic correspondence principle:

$$\sigma_r(r,t) = P(0)\left\{\frac{-\lambda^2}{\lambda^2-1}[(1-\frac{a^2}{r^2})\alpha + \frac{1}{\lambda^2}(\frac{b^2}{r^2}-1)](1-e^{-nt})\right\} + \frac{\lambda^2}{\lambda^2-1}(1-\frac{a^2}{r^2})\beta\frac{b}{h}\frac{E_\infty}{E_c}f_E(t) \tag{1}$$

$$\sigma_\theta(r,t) = P(0)\left\{\frac{-\lambda^2}{\lambda^2-1}[(1+\frac{a^2}{r^2})\alpha - \frac{1}{\lambda^2}(\frac{b^2}{r^2}+1)](1-e^{-nt})\right\} + \frac{\lambda^2}{\lambda^2-1}(1+\frac{a^2}{r^2})\beta\frac{b}{h}\frac{E_\infty}{E_c}f_E(t) \tag{2}$$

Where:

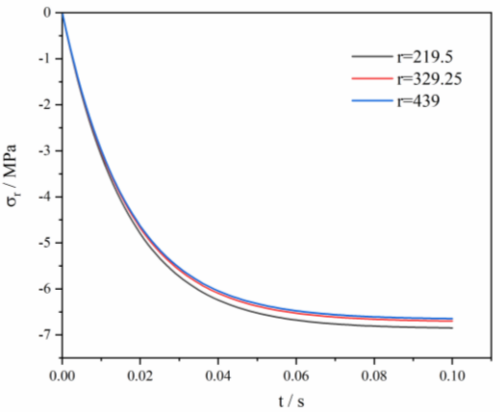
$$f_E(t) = \frac{n}{E_e}e^{-nt}\int_0^t E(\tau)e^{n\tau}d\tau$$

$$\alpha = \frac{2(1-\nu_p)}{1+(1-2\nu_p)\lambda^2}$$

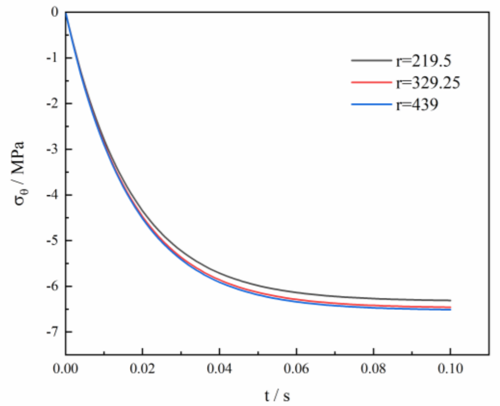
$$\beta = \frac{2(\lambda^2-1)}{[1+(1-2\nu)\lambda^2]^2}\frac{1-\nu}{1+\nu}(1-\nu_c^2)$$

a, b are the inner and outer diameters of the grain, h is the thickness of the shell, $\lambda = b/a$, E_∞ is the permanent modulus of the propellant, and E_c is the elastic modulus of the shell.

Bringing the radius value of the typical position and the material parameters in Section 2.2 into equations (1) and (2), the stress analytical solutions of three characteristic locations can be obtained, and the curve of its variation with time is shown in Figure 5.



(a) Radial stress change curve over time



(b) Curve of peripheral stress change over time

5Figure 5: Analytical stress vs. time curve

It can be seen from Figure 5 that the radial and circumferential stresses obtained by theoretical calculations are still less than 0, showing a stress state of radial and circumferential bidirectional compression. The theoretical analytical solution changes with time are also consistent with the numerical simulation results. In order to further verify the correctness of the simulation results, a quantitative comparison between the numerical solution and the analytical solution of the maximum stress point, that is, the radial and circumferential stress at a point at the inner hole, is shown in Table 4.

Table 4: Compared results between numerical and analytical solutions

	Radial stress σ_r			Peripheral stress σ_θ		
t /s	Numerical solution	Parse solution	δ / %	Numerical solution	Parse solution	δ / %
0.02	-4.8015	-4.7970	0.09	-4.4047	-4.3456	1.34
0.04	-6.3081	-6.2418	1.05	-5.8435	-5.7100	2.28
0.06	-6.6441	-6.6770	0.49	-6.1556	-6.1357	0.32
0.08	-6.7896	-6.8081	0.27	-6.2910	-6.2681	0.36
0.1	-6.8366	-6.8476	0.16	-6.3348	-6.3092	0.41

It can be seen from Table 4 that the relative error between the numerical solution and the analytical solution is very small, and the maximum relative error is only 2.28%, indicating that the simulation calculation results are consistent with the theoretical calculation results, and verify the correctness of the numerical simulation results.

4. Conclusion

(1) Based on the finite element numerical simulation software ABAQUS, the numerical simulation calculation of five three-dimensional grain models with different lengths under internal pressure loads was carried out. The relative size determines whether the three-dimensional problem can be reduced to a two-dimensional plane problem. It was found that when the aspect ratio *L*/*D* exceeds 2.5, the axial strain is approximately 0, which can be equivalent to a plane strain problem.

(2) On the basis of reasonable simplification into a plane strain problem, the stress distribution of grains under internal pressure load is simulated and calculated. The results show that the radial stress and the circumferential stress are always compressive stresses, and the stress ratio is approximately 1:1.

(3) Comparing the numerical simulation calculation results with the viscoelastic theoretical solution of grains under the action of internal pressure, it is found that the calculation results of the numerical solution and the analytical solution are relatively close, and the relative error is within 3%, which verifies the numerical simulation calculation results in this paper. The correctness can provide a reference for the subsequent experimental research on the mechanical properties of solid propellants under 1:1 bidirectional compression.

Acknowledgements

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