ISSN 2616-5902 Vol. 4, Issue 17: 85-93, DOI: 10.25236/AJBM.2022.041712

Research on Time Inventory Control Model of 3PL Warehousing of Furniture Products under Uncertain Lead Time

Nan Xu, Xiaogang Li

Zhejiang Wanli University, Ningbo, Zhejiang, 315100, China

Abstract: *3PL warehousing has become an important link between suppliers and retailers in the cross-border e-commerce supply chain. During the COVID-19, product shortage became one of the biggest concerns with the overseas 3PL warehouse integration using ocean shipment. Therefore it has become the focus of the cross-border e-commerce industry and research. Due to the large volume and low-value density, furniture products are generally replenished by ocean transportation. The shortage of such products can be caused by ocean transportation which brings uncertain arrival times and difficult inventory control. This paper, considering the uncertain lead time, uncertain demand, and 3PL warehouse free storage period, introduces a replenishment inventory control model. Utilizing the mathematical dynamic simulation model, it takes refrigerators as a product example into the calculations to prove the validity and accuracy. The experimental results demonstrate that the warehouse free-storage period is a vital factor affecting inventory control for cross-border e-commerce enterprises. When the lead time is uncertain, it is the most optimized solution to use the maximum lead time to determine the maximum inventory.*

Keywords: *Uncertain lead time, Random demand, 3PL warehousing, Periodic inventory control*

1. Introduction

With the continuous spread of the epidemic, the online time of consumers in various countries has increased, and the cross-border shopping boom has set off around the world. China has become the main destination country for overseas consumers' cross-border shopping, and cross-border e-commerce has achieved rapid development, making cross-border e-commerce one of the important ways to transform and upgrade China's overseas trade [1].All walks of life gradually carry out cross-border e-commerce business. According to customs survey statistics, in 2021, China's cross-border e-commerce export commodities mainly include clothing shoes and bags, home textiles and electronic products. As the largest exporter of furniture in the world, China's total annual production of furniture accounts for more than 25% of the global total. Many domestic furniture export enterprises use third-party network platforms for cross-border sales [2].Since 2017, the sales volume of furniture category has been on the rise, especially in 2020, the total sales volume nearly doubled compared with 2019, cross-border e-commerce has become one of the main ways of furniture export [3].Wayfair, an American furniture e-commerce platform, reported a nearly 84% jump in revenue to $4.3 billion in the second quarter of 2020.

With the increasing demand of overseas consumers for the timeliness of cross-border e-commerce products [4], traditional cross-border logistics is difficult to meet the needs of overseas customers due to the backward inventory mode and poor replenishment process. With the advantages of rapid logistics response, high service quality and high customer satisfaction based on localized distribution, overseas warehouses have increasingly become an important link in the supply chain of cross-border e-commerce enterprises [5-6].However, due to the large volume and heavy quality of furniture products, they cannot enter Amazon's FBA due to the limitations of specifications. Listed furniture manufacturing companies such as Lesge Stock and Henglin Stock have built overseas warehouses by themselves, but the construction of overseas warehouses is costly and complicated, which is difficult for small and medium-sized enterprises to bear [3]. Due to the cost pressure and operational risks of self-built overseas warehouses, most of the small and medium-sized cross-border e-commerce enterprises in China adopt 3PL warehousings [4].

Limited by volume and weight, furniture products are generally replenished by sea. However, due to the long transportation time and great uncertainty, the maritime replenishment mode is prone to the imbalance between supply and demand. In the environment of high volatility of cross-border e-commerce demand and uncertain replenishment lead time, inventory control has become the biggest risk of overseas warehouse mode, and also become the management difficulty and focus of cross-border furniture e-commerce enterprises.

At present, the inventory control strategy of cross-border e-commerce enterprises in the furniture industry is relatively simple. The main reason for the high inventory level, low inventory turnover rate and great financial pressure of enterprises is to use their own or industry experience to prepare goods in overseas warehouses [7-8]. However, at present, there are few studies on the storage control of cross-border e-commerce 3PL warehousings, so there are even fewer studies on furniture products.

Scarf's seminal paper considers a finite programming inventory system with fixed ordering costs, linear holding costs, and shortage costs under discrete time periods and nonstationary stochastic demand [9]. Many scholars have shown that strategy is one of the most commonly used inventory control strategies [10-12]. In the study of overseas warehouse storage model, Zidogan constructed an overseas warehouse cost control model based on EOQ model consisting of order cost, storage cost and shortage cost [13]. Wang Dan, based on Zidogan's research and combined with the actual data of enterprise products, figured out the optimal order volume and minimum total cost [6]. However, under this replenishment model, the annual turnover of overseas warehouses is only 1.2 times. Without considering the purchase cost and shortage cost, Jinhonghong constructed a random fuzzy incoming order model for periodic inventory review, and solved the optimal order quantity and order time under the minimum inventory cost [14]. In practice, overseas warehouses adopt warehouse-free period and step-based collection of storage fees to speed up inventory turnover, while cross-border e-commerce companies tend to avoid warehouse-period as replenishment cycle and carry out regular inventory control on goods in 3PL warehousings. In order to realize the scale effect and shorten the lead time, cross-border e-commerce enterprises will purchase in bulk in the domestic front warehouse, and replenish the overseas warehouse through the front warehouse. Therefore, this paper constructs a replenishment control model based on the cost structure of the resource root, but does not consider the procurement cost.

In the actual production and operation, the uncertainty of market demand and lead time is objective, and the inventory control model under random lead time and random demand is more consistent with the actual production and operation. However, many scholars have ignored the uncertainty of overseas warehouse replenishment lead time. Kaplan, R.S. proposed that the uncertain lead time has an impact on (S, s) inventory strategy under random demand [15]. Ehrhardt and R extended the research of Kaplan and R.S. and proposed efficient algorithms of dynamic inventory model [16-17], intuitively showing that random lead time has a great impact on inventory strategy. Therefore, the uncertain lead time factor is considered in this paper.

The above research shows that due to cost considerations, cross-border e-commerce enterprises will pay attention to the warehouse free period of overseas warehouses, speed up inventory turnover according to the charging characteristics of overseas warehouses, and shorten the replenishment lead time of sea transportation by using domestic front-loading warehouses. Due to the uncertainty of demand and lead time, how to scientifically and reasonably control the 3PL warehousing under the 3PL warehousing mode has become one of the focuses of cross-border e-commerce industry. Therefore, the policy based on the free warehouse period, considering the uncertainty in advance, for large volume, low density value of furniture products, building a single cycle of a single product cross-border e-commerce 3PL warehousing regular inventory control model, we examine the rationality of the model by multiple cycle, cross-border e-commerce for furniture enterprises to effectively control 3PL warehousing storage, Provide decisions and suggestions to improve the balance between supply and demand.

2. Problem description and hypothesis

To achieve the lowest total inventory cost while maintaining a certain service level, cross-border e-commerce furniture enterprises will determine the maximum inventory based on past demand, demand fluctuations and lead time. This paper assumes that both demand and lead time are uncertain and follow normal distribution, and lead time is less than the review period, namely $L < T$. In the periodic review cycle, the demand in each unit cycle is independent of each other. After the demand in the current period is realized, the remaining inventory will be sold in the next cycle.

Furniture manufacturing enterprises avoid warehouse period as the review cycle, at the beginning of each review cycle, according to the initial inventory, regular replenishment from the domestic front warehouse to the 3PL warehousing inventory level to the highest inventory level.This paper does not consider the risk of stock shortage in front warehouse. Capital holding costs are incurred after goods are shipped from the front warehouse; Replenishment will incur replenishment costs; The operation of goods in the 3PL warehousing, such as warehousing, warehousing, labeling, packaging and other operational matters, produces operational costs; The net stock of the 3PL warehousing after the free period will incur storage costs; During the cycle demand is greater than the common overseas warehouse inventory will incur Shortage costs, as shown in Table 1.

Table 1: Model parameter table

parameter	Parameter Description
S	Maximum inventory, decision variable
T	Review period, this paper to avoid the warehouse period as the review period
L	Replenishment lead time,The lead time follows a normal distribution $N(\overline{L},\lambda_L^2)$, $\overline{L}$ is the mean of the uncertain lead time, λ_L is the standard deviation of the uncertain lead time
R	Review the quantity demanded in the cycle interval,Demand is normally distributed $N(\overline{R},\sigma_R^2)$, $\overline{R}$ is the average demand during the review period, σ_R is the standard deviation of requirements for the review period,The demand density function is $\varphi(R,T)$,The cumulative distribution function is $F(R,T)$
R_{T+L}	The total quantity demanded during the review period and the lead period is the total quantity demanded during the decision period
R_L	Review the quantity demanded in the cycle interval
I	Opening inventory
Q	Periodic replenishment quantity
Q_{NS}	Out of stock quantity
TC	Total inventory cost
C_1	Inventory holding cost in the decision cycle, including storage cost and inventory holding cost.Among them, c_{11} is the storage cost per unit commodity, c_{12} is the capital occupancy cost per unit commodity inventory
C_2	Operational costs in the decision cycle, c_2 is the operating cost per unit commodity
C_3	Replenishment cost in the decision cycle, c_3 is the replenishment cost per unit commodity
C_4	The shortage cost in the decision cycle, c_4 is the shortage cost per unit item
α	Service level coefficient, can be written $F^{-1}(,\ T)$
$\varphi(,T)$	Review the demand density function over the period
$F(,T)$	The cumulative distribution function of demand over the review period

3. Model building and solving

3.1. Construction of replenishment model

Under the condition of ensuring customer service level, cross-border e-commerce furniture enterprises, aiming at minimizing the total inventory cost in the decision-making cycle $(T+L)$, decide the optimal solution of the maximum inventory level of the 3PL warehousing.This paper considers that the total inventory cost in the decision cycle consists of storage cost C_{11}, capital occupancy cost C_{12}, operation cost C_2, replenishment cost C_3 and shortage cost C_4.

$$TC = C_{11} + C_{12} + C_2 + C_3 + C_4$$

3.1.1. Storage cost C_{11}

The storage cost is related to the storage capacity and storage time after the free storage period. For the convenience of processing, this paper assumes that the period from the current arrival point to the next arrival point is the warehouse exemption period, namely a review period T. The storage cost is related to the storage capacity and storage time after the free storage period. For the convenience of processing, this paper assumes that the period from the current arrival point to the next arrival point is the warehouse exemption period, namely a review period. The time between the next arrival point and the sale of the goods will be the billing time for storage costs.

$$C_{11} = \frac{1}{2} c_{11} \int_0^S (S - R_{T+L}) \varphi(R_{T+L}, T+L) d_{R_{T+L}} \quad (1)$$

ISSN 2616-5902 Vol. 4, Issue 17: 85-93, DOI: 10.25236/AJBM.2022.041712

3.1.2. Capital occupancy cost C_{12}

Capital occupancy cost is the interest rate cost taken from the issuing of the front warehouse to the selling out, which is related to the initial and final storage reserves and in-transit inventory in the decision-making cycle. Inventory holding cost is calculated by the average storage capacity.

Therefore, the inventory holding cost is:

$$C_{12} = \frac{1}{2} c_{12} \int_0^S (2S - R_{T+L}) \varphi(R_{T+L}, T+L) d_{R_{T+L}} \tag{2}$$

3.1.3. Operating cost C_2

3PL warehousings are charged according to pieces and items, and the operation costs mainly include the cost of goods in and out of storage, on and off shelves, and entrusted delivery, as well as replacement packaging and labeling. This part of the cost is related to the sales volume during the decision period.

When $R_{T+L} \leq S$,the sales volume is equal to the demand, then the operation cost is: $c_2 \int_0^S R_{T+L} \varphi(R_{T+L}, T+L) \mathrm{d}_{R_{T+L}}$,

When $R_{T+L} > S$,the sales volume is the maximum inventory, so the operation cost is: $c_2 \int_{S+1}^{\infty} S \varphi(R_{T+L}, T+L) \mathrm{d}_{R_{T+L}}$,

Therefore, the total operation cost is:

$$C_2 = c_2 \int_0^S R_{T+L} \varphi(R_{T+L}, T+L) \mathrm{d}_{R_{T+L}} + c_2 \int_{S+1}^{\infty} S \varphi(R_{T+L}, T+L) \mathrm{d}_{R_{T+L}} \tag{3}$$

3.1.4. Replenishment cost C_3

Replenishment cost is composed of order cost and transportation cost. Since this paper considers that the furniture manufacturer replenishes goods from the front warehouse to the 3PL warehousing, the order cost does not include the various costs incurred by the purchase of goods. Since furniture manufacturers adopt multi-product LCL transportation and this paper only studies the replenishment model of a single product, the single order cost shared by each product is low, so the order cost is not considered in this paper. The transportation cost is the Marine freight incurred in the replenishment process from the front warehouse to the 3PL warehousing, which is related to the current replenishment volume.

When $I > 0$,namely $R_{T+L} \leq S$, the replenishment quantity is $S - I$,and the replenishment cost is: $c_3 \int_0^S (S - I) \varphi(R_{T+L}, T+L) d_{R_{T+L}}$

When $I < 0$,namely $R_{T+L} > S$,the replenishment quantity is S ,and the replenishment cost is: $c_3 \int_{S+1}^{\infty} S \varphi(R_{T+L}, T+L) d_{R_{T+L}}$

Therefore, the total cost of replenishment is:

$$\begin{aligned} C_3 &= c_3 \int_0^S (S - I) \varphi(R_{T+L}, T+L) d_{R_{T+L}} + c_3 \int_{S+1}^{\infty} S \varphi(R_{T+L}, T+L) d_{R_{T+L}} \\ &= c_3 S - c_3 \int_0^S I \varphi(R_{T+L}, T+L) d_{R_{T+L}} \end{aligned} \tag{4}$$

3.1.5. Shortage cost C_4

Shortage cost refers to the loss caused by cross-border e-commerce when the goods are in short supply during the peak sales season, including the direct profit lost due to the shortage, and the potential loss caused by customers' trust in the brand (the decline of store score and ranking). The shortage cost is related to the shortage quantity $R_{T+L} - S$.

Therefore, the shortage cost is:

$$C_4 = c_4 \int_{S+1}^{\infty} (R_{T+L} - S) \varphi(R_{T+L}, T+L) d_{R_{T+L}} \tag{5}$$

The total inventory cost of the 3PL warehousing time inventory control model can be obtained by

ISSN 2616-5902 Vol. 4, Issue 17: 85-93, DOI: 10.25236/AJBM.2022.041712

summation of equations (1), (2), (3), (4) and (5):

$$\begin{aligned} TC = & \frac{1}{2}c_{11}\int_0^S (S-R_{T+L})\varphi(R_{T+L},T+L)d_{R_{T+L}} + \frac{1}{2}c_{12}\int_0^S (2S-R_{T+L})\varphi(R_{T+L},T+L)d_{R_{T+L}} + \\ & c_2\int_0^S R_{T+L}\varphi(R_{T+L},T+L)d_{R_{T+L}} + c_2\int_{S+1}^{\infty} S\varphi(R_{T+L},T+L)\mathrm{d}_{R_{T+L}} + c_3 S - \\ & c_3\int_0^S I\varphi(R_{T+L},T+L)d_{R_{T+L}} + c_4\int_{S+1}^{\infty}(R_{T+L}-S)\varphi(R_{T+L},T+L)d_{R_{T+L}} \end{aligned} \tag{6}$$

3.2. *Inventory control model solving*

Assumption: There exists a unique optimal solution S^* with the highest inventory, which minimizes the total inventory cost in the decision cycle.

Proof: Calculate the first and second derivatives of S with respect to TC in formula (6):

$$\begin{aligned} TC' = & \frac{1}{2}c_{11}\int_0^S \varphi(R_{T+L},T+L)d_{R_{T+L}} + c_{12}\int_0^S \varphi(R_{T+L},T+L)d_{R_{T+L}} \\ & c_2\int_{S+1}^{\infty}\varphi(R_{T+L},T+L)d_{R_{T+L}} + c_3 - c_4\int_{S+1}^{\infty}\varphi(R_{T+L},T+L)d_{R_{T+L}} \end{aligned} \tag{7}$$

For simple operation, let $F(S)=\int_0^S \varphi(R_{T+L},T+L)d_{R_{T+L}}$, formula (7) be obtained by simplification

$$\begin{aligned} TC' &= \frac{1}{2}c_{11}F(S) + c_{12}F(S) + c_2[1-F(S)] + c_3 - c_4[1-F(S)] \\ &= (\frac{1}{2}c_{11} + c_{12} - c_2 + c_4)F(S) + c_2 + c_3 - c_4 \end{aligned} \tag{8}$$

$$TC'' = \frac{1}{2}c_{11} + c_{12} - c_2 + c_4$$

The stock shortage affects the continuous operation of the store, so the unit cost of the stock shortage is greater than the product price, and the price set by the enterprise includes the cost of commodity procurement, logistics and so on, so the price is greater than the costs, namely $c_4 > p > c_2$. Therefore, it can be seen that the second derivative $TC''>0$ of the total replenishment cost, that is, the total inventory cost, is a concave function, and when the first derivative is 0, the total cost is minimized and the optimal solution with the highest inventory exists. According to equation (8), the service level coefficient can be derived as

$$F(S) = \frac{c_4 - c_2 - c_3}{\frac{1}{2}c_{11} + c_{12} - c_2 + c_4} = \frac{2(c_4 - c_2 - c_3)}{c_{11} + 2c_{12} - 2c_2 + 2c_4} \tag{9}$$

When the service level is $F(S^*)$, the total inventory cost in the decision cycle is the lowest, that is, when the service satisfaction rate is $F^{-1}[\frac{2(c_4 - c_2 - c_3)}{c_{11} + 2c_{12} - 2c_2 + 2c_4}]$, the optimal solution is the highest inventory:

$$S^* = \mu_{T+L} + F^{-1}[\frac{2(c_4 - c_2 - c_3)}{c_{11} + 2c_{12} - 2c_2 + 2c_4}]\sigma_{T+L} \tag{10}$$

4. Example analysis and management enlightenment

Model checking is an indispensable step in simulation experiments, and the validity of the model is proved by this, so as to draw conclusions and scientific management strategies. The example analysis in this paper is based on the real operation data, and the constants are assigned according to the parameters of 3PL warehousing and Amazon products. Considering that the United States has always been the largest export market of China's cross-border e-commerce overseas, this paper takes the American market of Ningbo cross-border e-commerce as an example and selects the 3PL warehousing company A located in the Port of Newark. According to the company's warehouse free period, that is, the replenishing cycle T is 30 days. Since the sea transportation time from Ningbo to Newark port is 22~25 days, and the customs

ISSN 2616-5902 Vol. 4, Issue 17: 85-93, DOI: 10.25236/AJBM.2022.041712

clearance and shipping schedule fluctuation time are 3~5 days, it is assumed that the replenishment lead time meets the normal distribution of 25~30 days.

Affected by the COVID-19, working from home has led to a significant increase in overseas furniture and electrical appliances, which has driven the sales growth of refrigerators, air fryers, electric lifting tables and other products. According to field research, cross-border e-commerce enterprises have inventory turnover once every three months on average, in which the goods in transit time is one month and the storage time is two months on average. Considering the one-month free storage period, the unit storage cost is assigned with the storage period as one month. Considering the direct and indirect losses caused by the backorder, the unit backorder cost is assigned at 1.2 times the selling price. In this paper, refrigerator is taken as an example product, and the operation cost and replenishment cost are converted and valued according to the operation data and product parameters of the 3PL warehousing. The specific parameter assignment is shown in Table 2:

Table 2: Each parameter is assigned

parameter	The refrigerator
Review cycle T (day)	30
Product specification (m3)	0.75*0.45*0.5
Weight (kg)	8
Bank lending rate	5.41%
Unit purchase cost p (yuan)	380
Consider unit storage costs for the free period c_{11} (yuan per piece)	26.5
Unit storage costs are not considered for the free period c_{11}' (yuan per piece)	53
Unit inventory capital occupancy cost c_{12} (yuan per piece)	20.558
Unit operating cost c_2 (yuan)	77
Unit replenishment cost c_3 (yuan)	90
unit shortage cost c_4 (yuan)	3192

4.1. Influence of lead time value on optimal solution of maximum inventory

When both the lead time and the demand are uncertain, the selection of the lead time value will affect the decision of the maximum inventory in the inventory control strategy, and then affect the total inventory cost. The maximum inventory obtained by the inventory control model is the optimal solution under single period decision. In order to verify the model results, this paper will conduct a comparative verification analysis of the total inventory cost under different lead time values from the perspectives of monthly average demand and demand fluctuation through multi-period dynamic simulation.

In order to explore the influence of lead time value on total inventory cost, the optimal solution of maximum inventory and total inventory cost under lead time are calculated as average lead time $\bar{L}$, the sum of average lead time and single standard deviation $\bar{L}+\sigma_L$, and the maximum lead time $L_{\max}$, respectively. The optimal solution of lead time value is verified by numerical simulation. From the perspective of monthly average demand, keep the other parameters of the refrigerator unchanged, so that the demand fluctuation is 0.2, so that the monthly average demand is 75, 100, 200, 300, 400, 500. From the perspective of demand fluctuation, let the monthly average demand be 1000 and the demand fluctuation ratio be 0.1, 0.2, 0.3, 0.4 and 0.5 respectively. Considering the "dimensional disaster" problem in simulation decision-making, this paper sets the maximum inventory step 25 according to $S_{\bar{L}}$, $S_{\bar{L}+\sigma_L}$ and $S_{L_{\max}}$. Specific simulation comparison results are shown in Table 3 and Table 4:

Table 3: Comparison of maximum inventory decisions under different monthly average demands

Monthly demand	75	100	200	300	400	500
$S_{\bar{L}}$	180	240	480	720	960	1200
$S_{\bar{L}+\sigma_L}$	186	247	495	743	990	1237
$S_{L_{\max}}$	189	251	503	754	1005	1256
S^*	190	250	505	755	1010	1270

It can be seen from Table 3 that under different monthly average demands, the optimal solution of the maximum inventory of refrigerator is closest to the maximum inventory determined by the value of lead

ISSN 2616-5902 Vol. 4, Issue 17: 85-93, DOI: 10.25236/AJBM.2022.041712

time. Except when the monthly average demand is 100, S^* is less than $S_{L_{\max}}$, the other four groups of data show that S^* is slightly greater than $S_{L_{\max}}$, and the relative deviations between S^* and $S_{L_{\max}}$ of the six groups of data are 0.26%, 0.20%, 0.20%, 0.07%, 0.25% and 0.55%, respectively.

Table 4: Comparison of maximum inventory under different demand fluctuations

Demand fluctuation ratio	0.1	0.2	0.3	0.4	0.5
$S_{\bar{L}}$	2170	2399	2637	2877	3118
$S_{\bar{L}+\sigma_L}$	2240	2474	2716	1961	3206
$S_{\bar{L}+\sigma_L}$	2275	2512	2756	3002	3250
S^*	2250	2525	2775	3025	3275

As can be seen from Table 4, when the demand fluctuation of refrigerator is 0.1, the gap between $S_{\bar{L}+\sigma_L}$ decided by lead time $\bar{L}+\sigma_L$ and the optimal solution obtained by simulation is the smallest, with a deviation of 0.22%. However, when the demand fluctuation is 0.2 to 0.5, the optimal solution determined by the maximum lead time is close to the maximum inventory, and the deviation between them is 0.26%, 0.34%, 0.38%, 0.38%, respectively. When the demand fluctuation is 0.1, the deviation between $S_{L_{\max}}$ and S^* is 0.55%.

According to the above data analysis, under different monthly demand and demand fluctuations, the maximum inventory of the inventory control model constructed in this paper in the maximum lead time decision is the most close to the simulation results, and the relative deviation is small, indicating that cross-border e-commerce enterprises use the maximum lead time to make the maximum inventory decision is the most reasonable.

4.2. Influence of lead time value on minimum total inventory cost

The decision of the optimal solution of the highest inventory ultimately affects the total inventory cost. This paper makes a comparative analysis of the total inventory cost under different lead time decisions to test the rationality of the inventory control model decision. Specific simulation comparison results are shown in Table 5 and Table 6:

Table 5: Comparison of total inventory cost under the perspective of monthly average demand and different lead time value decisions

Monthly demand	75	100	200	300	400	500
$TC_{\bar{L}}$	16638	22140	43722	65842	91618	111671
$TC_{\bar{L}+\sigma_L}$	16050	21190	42253	63197	87362	108047
$TC_{L_{\max}}$	15757	20799	41601	62383	85893	106186
TC^*	15659	20897	41668	62416	85799	105342

Table 6: Total inventory cost pairs under different lead time value decisions from the perspective of demand fluctuation

Demand fluctuations	0.1	0.2	0.3	0.4	0.5
$TC_{\bar{L}}$	205660	218964	237130	252902	268671
$TC_{\bar{L}+\sigma_L}$	198804	211618	220108	233608	248458
$TC_{L_{\max}}$	199582	207896	216190	225639	238351
TC^*	198746	208204	216569	224704	234453

According to Table 5, when the average monthly demand of refrigerators is different, compared with the total inventory cost decided by $S_{\bar{L}}$ and $S_{\bar{L}+\sigma_L}$, the total inventory cost $TC_{L_{\max}}$ under decision $S_{L_{\max}}$ is the smallest, and it is the closest to the minimum total inventory cost TC^* obtained by numerical simulation, with the relative deviation below 0.1%. As can be seen from Table 6, when the demand fluctuation of refrigerator is 0.1, the total cost gap between $TC_{\bar{L}+\sigma_L}$ and TC^* is the smallest, and the difference between them is only 0.03%. When the demand fluctuation is between 0.2 and 0.5, the total inventory cost is the smallest when the lead time reaches the maximum value, and the relative

ISSN 2616-5902 Vol. 4, Issue 17: 85-93, DOI: 10.25236/AJBM.2022.041712

deviation between $TC_{L_{\max}}$ and TC^* is small. Except for the fluctuation of 0.5, it reaches 0.21%, and all the others are below 0.1%. The above data show that the deviation between $TC_{L_{\max}}$ generated by cross-border e-commerce enterprises' decision $S_{L_{\max}}$ with the maximum lead time $L_{\max}$ and TC^* corresponding to the simulation result S^* is small. Therefore, it is feasible and reasonable for the inventory control model constructed in this paper to decide the maximum inventory amount according to $L_{\max}$.

4.3. Consider the effect of warehouse free period on total inventory cost

The warehouse free period can reduce the total inventory cost. Cross-border e-commerce enterprises also develop corresponding replenishment mechanisms for the warehouse free period, but there is a lack of research on the cost impact of the warehouse free period. In order to explore the impact of warehouse free period on total inventory cost, this paper simulated and compared the changes of total inventory cost of refrigerators under the sales environment with average monthly demand of 500 and demand fluctuation of 0.2. The following will be analyzed from two perspectives. One is the comparative analysis of the total inventory cost of 3PL warehousings with and without warehouse free period policy. Second, under the policy of warehouse free period, cross-border e-commerce enterprises consider the impact of warehouse free period on the maximum inventory and total inventory cost in the decision-making process. TC_1 is the total inventory cost considering the decision of warehouse free period, TC_2 is the total inventory cost without considering the decision of warehouse free period, TC_3 is the total inventory cost of overseas warehouse without warehouse free policy. The optimal solution of the maximum inventory under the three conditions and the corresponding total inventory cost simulation results are shown in Tables 7 and 8:

Table 7: Comparison of the effects of considering the free storage period on the maximum inventory decision

	Consider the free storage period	The free period is not considered
$S_{\bar{L}}$	1200	1193
$S_{\bar{L}+\sigma_L}$	1237	1231
$S_{\bar{L}+\sigma_L}$	1256	1249
S^*	1270	1270

It can be seen from Table 7 that without considering the free storage period, the maximum inventory decision of refrigerator at different values of lead time decreases, because without considering the free storage period, the increase of unit storage cost reduces the safety inventory coefficient, which leads to the relative decrease of the maximum inventory decision value of refrigerator.

Table 8: Considers the impact of warehouse free period on total inventory cost

S	1210	1220	1230	1240	1250	1260	1270	1280	1290
TC_1	110692	109712	108733	107754	106774	105795	105342	105680	106018
TC_2	114353	113495	112637	111779	110921	110063	109736	110207	110677
TC_3	114353	113495	112637	111779	110921	110063	109736	110207	110677

As can be seen from Table 8, when cross-border e-commerce enterprises choose 3PL warehousings with warehouse free period policy, TC_1 is reduced by 38049 yuan compared with TC_3, saving 36.12%. This shows that cross-border e-commerce enterprises that do not consider the warehouse free period will lead to an increase in unit storage cost and storage volume, which will lead to a significant increase in inventory storage cost and a decrease in the maximum inventory of decisions, which will lead to a significant increase in backorder cost. For the case that there is a warehouse exemption policy for 3PL warehousings, but cross-border e-commerce enterprises do not consider when making decisions, the cost of TC_1 is 4394 yuan lower than that of TC_2, saving 4.17%. This shows that, under the policy of free warehouse period for 3PL warehousings, the impact of backorder cost on total inventory cost is great, and the maximum inventory quantity decided by considering the free warehouse period is closer to the optimal solution of simulation, which reduces the total inventory cost and results in a better decision.

The above data show that cross-border e-commerce enterprises need to take the warehouse exemption policy as an important consideration when they open overseas warehouses. When choosing the 3PL warehousing with the policy of free warehouse period, considering the free warehouse period to make inventory decision is more conducive to maintaining the balance between supply and demand of 3PL warehousing and reducing the total inventory cost.

5. Conclusion

Aiming at furniture products with large volume and low value density, this paper studies the regular replenishment problem of 3PL warehousings of cross-border e-commerce enterprises under the uncertain demandMeanwhile, the warehouse free period of overseas warehouses is added to the replenishment decision model, and the inventory control model with lead time less than the review period is adopted. Cross-border e-commerce enterprises dynamically adjust the inventory and replenishment quantity of goods according to the inventory level, market demand and replenishment lead time during the review period, so as to achieve the minimum total inventory cost under a certain customer service level. The research shows that cross-border e-commerce enterprises need to consider the the warehouse free period 3PL warehousings as an important factor; In the uncertain lead time environment, cross-border e-commerce enterprises adopt the maximum lead time to determine the maximum inventory, and the periodic inventory decision result of 3PL warehousings is more reasonable.

References

[1] Shen Tong. Research on logistics model of cross-border retail e-commerce in China from the perspective of import [J]. Journal of Commercial Economics, 2020, (18): 109-112.

[2] Ren Chong, Zhang Jitao. Research on the Development of B2C Cross-border E-commerce in Furniture Industry [J]. China Forest Products Industry, 2020, 57(09): 99-101.

[3] Hu Jianan, Xu Xiangyang. Current Situation Analysis and Prospect Forecast of Cross Border E-commerce in Furniture Industry [J]. China Forestry Economics, 2021(05): 66-69.

[4] Quan Yin. Necessity and practical approach of overseas shared warehouse application of cross-border e-commerce [J]. Practice in Foreign Economic Relations and Trade, 2020 (05): 32-35.

[5] Meng Liang, Meng Jing. Analysis on the Mode Choice of "Oversea Location" of Chinese Cross border E-commerce Enterprises--based on Consumer Goods Exporting Trade [J]. China Business and Market, 2017, 31(06): 37-44.

[6] Wang Dan. Research on overseas warehouse storage optimization of a cross-border e-commerce enterprise [D]. Guangzhou: South China University of Technology, 2018.

[7] Li Xiaogang, Wang Qifeng. Research on Symbiotic Coupling Mode and Mechanism of Cross-border E- commerce Logistics Industry Chain based on 3PL warehousing [J]. China Business and Market, 2018, 32(09): 41-48.

[8] Maximiliano Udenio and Kai Hoberg and Jan C. Fransoo. Inventory agility upon demand shocks: Empirical evidence from the financial crisis [J]. Journal of Operations Management, 2018, 62(1): 16-43.

[9] Scarf H. The Optimality of (S, s) Policies in the Dynamic Inventory Problem [J]. Mathematical Methods in the Social Sciences,1960.

[10] Silver, E.A. Operations Research in Inventory Management: A Review and Critique. Oper. Res., 1981, 29, 628-645.

[11] Babai M Z, Syntetos A A, Teunter R. On the empirical performance of (T, s, S) heuristics [J]. European Journal of Operational Research, 2010, 202(2): 466-472.

[12] Lagodimos A G, Christou I T, Skouri K. Computing globally optimal (s, S, T) inventory policies [J]. Omega, 2012, 40(5): 660-671.

[13] Zi Daogen. Logistics Cost Control of Cross-border E-commerce Based on Overseas Warehouse Mode [J]. Logistics Technology, 2015, 34(16): 175-177+180.

[14] Jin Honghong. Study on Ordering Optimization of XCross-Border E-Commerce Company Under Overseas Warehouse Mode [D]. Xiamen: Xiamen University, 2019.

[15] Robert S. Kaplan. A Dynamic Inventory Model with Stochastic Lead Times [J]. Management Science, 1970, 16(7).

[16] Richard Ehrhardt. (s, S) Policies for a Dynamic Inventory Model with Stochastic Lead Times [J]. Operations Research, 1984, 32(1).

[17] Zhao Baojing. The 1.5 times principle of inventory management [J]. Market Weekly (New Logistics), 2009 (05): 60.